SERGO TOPURIA

(To the 80th Birthday Anniversary)

This year Professor Sergo Topuria, Honored Scientist, Doctor of Physical and Mathematical Sciences, would have been 80 years old and marked 55 years of his scientific and pedagogical activities. He was a prominent Georgian mathematician, a remarkable representative of the Georgian mathematical school, an excellent teacher and educator. He was one of those Georgian mathematicians who in the 60s of the past century made the first bold steps in mathematical research and thereby won general recognition and respect.

Sergo Topuria was born on December 27, 1931 and died this year, on March 15th. He was a person with lofty ideals and adhered to high moral and civic principles. His path in life, high professional competence and general public recognition are an evidence of his faithful service to the country and people.

The results obtained by S. Topuria reflect the onward development of the studies carried out by such famous mathematicians as B. Luzin, I. Privalov, A. Zygmund, G. Hardy, I. Marcinkiewisz, I. Stein, V. Shapiro, E. Gobson and others. He was deeply respected and his scientific works were highly appreciated by modern well-known mathematicians S. Nikolski, P. Ulyanov, S. Stechkin, N. Muskhelishvili, I. Vekua and others.

In 1953, Sergo Topuria graduated with honors the physical and mathematical faculty of Sukhumi Pedagogical Institute and continued his education as a post-graduate student under the supervision of well-known Georgian mathematician, Corresponding Member of the Georgian Academy of Sciences, Professor Vladimer Chelidze. In 1959, he defended his Master's thesis "On Some Tauber Type Theorems for Multiple Series and Multiple Integrals"at A. Razmadze Mathematical Institute of the Georgian Academy of Sciences.

In 1960, Sergo Topuria was elected head of the higher mathematics and theoretical mechanics chair of the Georgian Subtropical Agriculture Institute in the city of Sukhumi. In 1966, he moved to Tbilisi and took up work as docent at higher mathematics chair no. 3 at Georgian Polytechnical Institute. From 1967 to the last day of his life he headed higher mathematics chair no. 63 at the above-mentioned institute, which later was reorganized into Georgian Technical University. Due to his outstanding organizational capacity and strenuous efforts, in the course of many years this chair had been one of the leading chairs of Georgian Technical University. Concurrently, for many years he was delivering a special course of lectures for students of the mechanical-and-mathematical faculty of I. Javakhishvili Tbilisi State University. Along with teaching, organizational and social activities, he carried out scientific research with enthusiasm typical of him and, in 1973, he defended his Ph.D. thesis on the topic "Some Problems of the Boundary Properties of Harmonic Functions, the Theory of Fourier-Laplace and Fourier multiple trigonometric series".

In 1975, the title of professor was conferred on Sergo Topuria and, in 1978, the title of an Honored Scientist.

Sergo Topuria was known as a highly skilled specialist in the function theory. Comprehensive studies were carried out by him in multidimensional harmonic analysis. His scientific results are related to the following main directions: summability of multiple trigonometric series for various types of convergence; Tauber type theorems for multiple series and integrals; summation of Fourier-Laplace and differentiated Fourier-Laplace series by the linear method; representation of various measurable and almost everywhere finite functions of many variables by multiple trigonometric series and Laplace series; the boundary properties of harmonic functions in multidimensional domains.

Sergo Topuria established an analogue of S. Bernstein's inequality for a spherical polynomial in the space $L_p(S^3)$, $1 < p < \infty$.

Sergo Topuria carried out a detailed study of the question of summability of Fourier-Laplace series and their differentiated series (in terms of various types of convergence). In particular, he proved analogues of the theorems of P. Siolini, I. Stein and G. Sunuochi on almost everywhere convergence of the Cesaro means $(C, \propto)$ of Fourier_laplace series for a critical exponent.

Furthermore, he proved the theorems on the summability in the sense of the Abel and $(C, \propto)$ methods of Fourier-Laplace series and their differentiated series in the case where the angular part of the Laplace operator written in polar coordinates is used as a differentiation operator. He also obtained the analogues of the theorems of G. Riesz and I. Stein on convergence the metric of a space of Cesaro means for a critical exponent when $1 < p < \infty$. He also studied the question of strong summability of Fourier-Laplace series. Here he obtained the theorems which are specific analogues of the Hardy-Littlewood , Marcinkiewicz and Stein theorems.

S. Topuria established that if f(x) is a measurable and a.e.finite function defined on the spherical surface, then there exists a Laplace series which is summable almost everywhere to a function $f(x)$ by the A^* method as well as by the Rudin-Riemann method (this is an analogue of Luzin's theorem).

S. Topuria obtained quite a number of results related to the boundary properties of the differentiated Poisson integral for various domains (circle, ball, half-plane, half-space, bicylinder) and its application. He solved the Dirichlet problem for the ball and the half-space in the case where a boundary function is measurable and finite a.e., i.e. in a completely general case, and also he proved the existence of an angular boundary value of a harmonic function with the so-called B property in the ball.

S. Topuria was the author of over 100 scientific works, including 3 monographs. Over 30 manuals and hand-books were written by him and published under his supervision, of which the higher mathematics manual in 5 volumes is especially noteworthy. These volumes make up a complete course on higher mathematics (the theory and a collection of problems). Mention should also be made of the manual in 2 volumes for university entrants. This two-volume manual has already run through 5

editions, has remained very popular for nearly 30 years and is successfully used in senior classes in secondary schools.

S. Topuria's scientific papers and manuals are distinguished by a simple and clear presentation of facts and ideas, refined argumentation, a multitude of original examples and counter-examples – these qualities produce a favorable impression on readers.

Vladimer Khocholava

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