Abstracts

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&

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Georgian National Academy of Sciences
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საქართველოს მეცნიერებათა აკადემია
შოთა რუსთაველის ბათუმის სახელმწიფო უნივერსიტეტი
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ნიკოლოზ მუსხელიშვილი
მეცნიერი და საზოგადო მოღვაწე

“თბილისის უნივერსიტეტი? ეს ხომ მირაჟია! შეუძლებელია ქართულ ენაზე, რომელსაც სათანადო ტრადიცია არა აქვს, ამეტყველება და გადმოცემა ისეთი მეცნიერებების უმარტივესი ტერმინებისაც კი, როგორიცაა მათემატიკა, ქიმია, ბიოლოგია!” – გაიძახოდნენ თბილისში უნივერსიტეტის დაარსების მოწინააღმდეგენი. ასეთები კი მრავლად იყვნენ. დიდმა ივანე ჯავახიშვილმა და მისმა თანამოაზრეებმა გააქარწყლეს სკეპტიკოსთა ეს მოსაზრება და უკვე 1918 წლის ნოემბერში პროფესორმა ანდრია რაზმაძემ დახვეწილი ქართულით წაიკითხა პირველი ლექცია მათემატიკურ ანალიზში. ასევე მომხიბლავად აამეტყველა მათემატიკა მოსკოვის უნივერსიტეტის კურსდამთავრებულმა, ჯერ სულ ახალგაზრდა არჩილ ხარაძემ, ხოლო ასტრონომია – გენერლის ფორმაში გამოწყობილმა ანდრია ბენაშვილმა. მალე მათ მხარში გიორგი ნიკოლაძე და ნიკოლოზ მუსხელიშვილი ამოუდგნენ. შალვა ნუცუბიძე იგონებს: “ანდრია რაზმაძე, იმ დროს ფიზიკა-მათემატიკის ფაკულტეტის დეკანი, შემოვიდა ჩემთან. მას თან ვიღაც გამოკვეთილი, ენერგიული სახის ახალგაზრდა შემოჰყვა. – ეს ახალახან ჩამოვიდა პეტროგრადიდან, სადაც მეცნიერულ მუშაობას ეწეოდა. მინდა თანხმობა, რომ მოვიწვიოთ თანამშრომლად. ნიჭიერი და ენერგიული ახალგაზრდა ჩანს. ანდრია რაზმაძის თხოვნა, ცხადია, დაკმაყოფილებული იქნა. იმ ახალგაზრდა მეცნიერმა სავსებით გაამართლა ანდრია რაზმაძის ყოველივე მოლოდინი. ეს იყო დღეს ცნობილი მეცნიერი ნიკოლოზ მუსხელიშვილი”.

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ანდრია რაზმაძე, გიორგი ნიკოლაძე, ნიკოლოზ მუსხელიშვილი, არჩილ ხარაძე – ეს ის “დიდი ოთხეულია”, რომელიც ქართული მათემატიკური სკოლის სათავეებთან დგას. ძნელი წარმოსადგენია იმ სამუშაოს მოცულობა, რაც ამ ოთხეულმა ათიოდე წლის მანძილზე შეასრულა – ინტენსიურ პედაგოგიურ მოღვაწეობასთან ერთად მათ მოუხდათ ქართული სამეცნიერო მათემატიკური ტერმინოლოგიის დადგენა და დაზუსტება, მშობლიურ ენაზე პირველი ორიგინალური სახელმძღვანელოების დაწერა და გამოცემა, მათემატიკის სხვადასხვა დარგში მეცნიერული კვლევების საფუძ-ვლების ჩაყრა ...

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1929 წელს მოულოდნელად გარდაიცვალა ანდრია რაზმაძე, ხოლო 1931 წელს – გიორგი ნიკოლაძე. მთელი ტვირთი ნიკოლოზ მუსხელიშვილს, არჩილ ხარაძეს და თბილისის უნივერსიტეტის კურსდამთავრებულ რამდენიმე ახალგაზრდა მათემატიკოსს დააწვა. ამ უკანასკნელთ თავდადებისა და ერის სამსახურის საუკეთესო მაგალითს აძლევდნენ უფროსი კოლეგები, რომლებსაც ერთი წუთითაც არ შეუწყვეტიათ სამეცნიერო კვლევაძიება და პედაგოგიური საქმიანობა თბილისის სახელმწიფო უნივერსიტეტსა და საქართველოს პოლიტექნიკურ ინსტიტუტში.

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სხვადასხვა დროს ნიკო მუსხელიშვილი იყო უნივერსიტეტის პოლიტექნიკური ფაკულტეტის დეკანი, ფიზიკა-მათემატიკის ფაკულტეტის დეკანი, საქართველოს პოლიტექნიკური ინსტიტუტის პრორექტორი, თეორიული მექანიკის კათედრის გამგე, თბილისის უნივერსიტეტთან მისი თაოსნობით დაარსებული ფიზიკის, მათემატიკისა და მექანიკის ინსტიტუტის დირექტორი.

პარალელურად მისთვის ჩვეული ენერგიითა და ენთუზიაზმით განაგრძობდა პედაგოგიურ მოღვაწეობას. უფროსი თაობის მათემატიკოსებს ახლაც ახსოვთ მისი ლექციები ანალიზურ გეომეტრიაში, თეორიულ მექანიკაში, დიფერენციალურ განტოლებათა თეორიაში ...

ნიკო მუსხელიშვილის კალამს ეკუთვნის ანალიზური გეომეტრიის ორიგინალური სახელმძღვანელო, რომელიც რამდენჯერმე გამოიცა და თავის დროზე ერთ-ერთ ძირითად საუნივერსიტეტო სახელმძღვანელოდ ითვლებოდა. ასევე ორიგინალობით გამოირჩევა მისი “თეორიული მექანიკის კურსი”, რომლის ორი ნაწილი – “სტატიკა” და “კინემატიკა”, შესაბამისად, 1926 და 1928 წელს გამოვიდა. შემდგომში ეს წიგნები ხელმეორედაც გამოსცეს.

მათემატიკურ ტერმინოლოგიაზე მუშაობა ბატონმა ნიკომ სამშობლოში ჩამოსვლისთანავე დაიწყო. “მათემატიკური ტერმინოლოგიის” (1944, რუსულ-ქართული ნაწილი) წინასიტყვაობაში მისი რედაქტორი, პროფესორი ვუკოლ ბერიძე წერს: “განსაკუთრებით დიდი ამაგი

დასდო ტერმინოლოგიას აკადემიკოსმა ნ. მუსხელიშვილმა, რომელმაც თითოეული სიტყვა შეამოწმა და შეეცადა, რაც შეიძლება სრული შესატყვისობა ყოფილიყო მათემატიკურ ცნებასა და მის მიერ გამოხატულ ტერმინს შორის''. აქ უნდა აღინიშნოს, რომ ახლა საყოველთაოდ გავრცელებული ტერმინი ''ტოლობა'' და მისგან ბუნებრივად ნაწარმოები ტერმინები ''უტოლობა'' და ''განტოლება'', სწორედ ბატონმა ნიკომ შემოიღო ჯერ კიდევ 20-იან წლებში.

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1922 წელს თბილისში ფრანგულ ენაზე გამოვიდა ნიკო მუსხელიშვილის წიგნი ''კოშის ინტეგრალების გამოყენება მათემატიკური ფიზიკის ზოგიერთი ამოცანისთვის''. შეიძლება ითქვას, რომ ეს იყო თავისებური წინამორბედი მისივე ფუნდამენტური მონოგრაფიისა ''დრეკადობის მათემატიკური თეორიის ზოგიერთი ძირითადი ამოცანა'' (1933), რომელსაც ავტორის მიერ 1931-32 წლებში ლენინგრადში სეისმოლოგიური ინსტიტუტის თანამშრომლებისთვის და, აგრეთვე, ფიზიკა-მათემატიკის ინსტიტუტისა და ლენინგრადის უნივერსიტეტთან არსებული მექანიკისა და მათემატიკის ინსტიტუტის ასპირანტებისთვის წაკითხული ლექციები დაედო საფუძვლად. მონოგრაფიამ სულ მალე მოიპოვა პოპულარობა და მისი ავტორი დრეკადობის თეორიის გამოჩენილ სპეციალისტად იქნა აღიარებული. ნ. მუსხელიშვილი იმავე, 1933 წელს, სსრკ მეცნიერებათა აკადემიის წევრ-კორესპონდენტად, ხოლო 1939 წელს – ნამდვილ წევრად აირჩიეს. ამავე დროს ის სსრკ მეცნიერებათა აკადემიის საქარ-თველოს ფილიალის თავმჯდომარეა.

1941 წელს, როცა საქართველოს მეცნიერებათა აკადემია შეიქმნა, მის პრეზიდენტად ერთხმად აირჩიეს ნიკოლოზ მუსხელიშვილი. 1941 წლის 27 თებერვალს აკადემიის პირველსავე სხდომაზე ნ. მუსხელიშვილმა თავისი გამოსვლა ასეთი სიტყვებით დაასრულა: ''ჩვენთვის სამწუხაროა, რომ დღევანდელი საზეიმო განწყობილება იწამლება იმის შეგრძნებით, რომ ჩვენს შორის არ იმყოფება მეცნიერი, რომელიც ყველაზე აღფრთოვანებული იყო ამ დღესასწაულის მოლოდინში; ჩვენთვის ეჭვს გარეშეა, ივანე ჯავახიშვილი ცოცხალი რომ ყოფილიყო, ამ მაღალ თანამდებობას, რომლის დაკავების პატივი მე მაქვს, სწორედ ის დაიკავებდა''.

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''დრეკადობის მათემატიკური თეორიის ზოგიერთი ძირითადი ამოცანა'' 1935 წელს მეორე, შევსებული გამოცემით გამოვიდა და მის ავტორს 1941 წელს სტალინური პრემია მიენიჭა. იმავე პრემიით აღინიშნა ბატონი ნიკოს მეორე ცნობილი მონოგრაფიაც ''სინგულარული ინტეგრალური განტოლებანი'' (1946). მანამდე კი, 1945 წლის ივნისში, აკადემიკოს ნიკოლოზ მუსხელიშვილს სოციალისტური შრომის გმირის წოდება მიანიჭეს.

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ნ. მუსხელიშვილის ორივე მონოგრაფია საზღვარგარეთ მრავალ ენაზეა თარგმნილი და გამოცემული. ბევრი საქები რეცენზია დაიწერა მათზე ... აი, რას წერდა ინჟინერ-მექანიკოსთა ამერიკის საზოგადოების ყოველთვიური ორგანო ''გამოყენებითი მექანიკის მიმოხილვა'', რომელიც ისტონში გამოდის: ''დღეს თამამად შეიძლება ითქვას, რომ დრეკადობის თეორიაში დაიწერა ახალი ტრაქტატი. მუსხელიშვილის ნაშრომი მკითხველს აცნობს ამ დარგის უახლეს გამოკვლევებს 1915 წლიდან მოყოლებული (როცა მისი ორიგინალური გამოკვლევა გამოვიდა) 1945 წლამდე. ამ ხნის განმავლობაში რუსეთის ჟურნალებში გამოქვეყნდა ასობით სტატია მუსხელიშვილის მიღწევების გამოყენებით ... ამჟამად მუსხელიშვილის ნაშრომის გავლენა აშკარად შეიმჩნევა არარუსულ ლიტერატურაშიც, სადაც მრავალმა ავტორმა პირველად მხოლოდ ახლა აღმოაჩინა ეს ნაშრომი ... თქმა არ უნდა, რომ ნაშრომის შინაარსი ბრწყინვალეა... მეორე ესოდენ დიდი მნიშვნელობისა და ფართო მასშტაბის წიგნის შექმნას ერთი თაობა მაინც დასჭირდება''.

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მრავალი პრემიითა და ჯილდოთი იქნა აღნიშნული ბატონი ნიკოს მეცნიერული ღვაწლი. კერძოდ, 1969 წელს ტურინის აკადემიამ მას მიანიჭა საერთაშორისო პრემია ''მოდესტო პანეტი'' - ოქროს მედალი და ფულადი ჯილდო. ჩვენი თანამემამულე პირველი საბჭოთა და მსოფლიოს მეექვსე მეცნიერი იყო, რომლის მეცნიერული მიღწევები ამ მაღალი ჯილდოთი აღინიშნა. ''ეს ჯილდო სრულიად მოულოდნელი იყო ჩემთვის, - უთხრა გაზეთ ''კომუნისტის'' კორესპონდენტს მეცნიერმა, - დიდად აღფრთოვანებული ვარ ამ მაღალი აღიარებით, ტურინის აკადემია ერთ-ერთი ძველი აკადემიაა იტალიაში, მრავალი ჩემი შრომა დაკავშირებულია იტალიელი მათემატიკოსების შრომებთან. იტალია იყო პირველი

უცხოეთის ქვეყანა, სადაც ჩემი ნაშრომი დაიბეჭდა დიდი იტალიელი მათემატიკოსის ვიტო ვოლტერას წარდგინებით''.

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ქართველმა ხალხმა ნ. მუსხელიშვილს სამუდამო განსასვენებელი მთაწმინდაზე დაუმკვიდრა. მეცნიერის ხსოვნის უკვდავსაყოფად დაწესდა მისი სახელობის პრემია, მისი სახელი ეწოდა გამოთვლითი მათემატიკის ინსტიტუტს, აიგო ძეგლი ჭავჭავაძის პროსპექტზე. ძეგლი დგას იმ სახლის მახლობლად, რომელშიც 1941 წლიდან 1976 წლამდე, გარდაცვალებამდე ცხოვრობდა ბატონი ნიკო. სახლს მემორიალური დაფა აქვს და ჩვენი აკადემიის პირველი პრეზიდენტის ჰორელიეფი ამშვენებს.

ნ. მუსხელიშვილის ვაჟს, ტექნიკურ მეცნიერებათა დოქტორს, პროფესორ გურამ მუსხელიშვილს ყურადღება არ მოუკლია მამის სამუშაო ოთახისთვის. ესეც თავისებური მემორიალია, კედელზე ფოტოსურათებია გამოფენილი, სპეციალურ სტენდებზე ჯილდოებია, თაროებზე - წიგნები ... ბატონი გურამი და მისი შვილები - ოლღა და მარინა სათუთად უვლიან სახელოვანი მამისა და პაპის ნივთებსა და მის ხელშენახებ წიგნებს ...

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აი როგორ იგონებს თავის სახელოვან მამას ბატონი გურამი: ''დავიწყებ შორიდან. პეტერბურგში ცხოვრებისას მამა მუსხელოვად იწერებოდა. იქ შეძენილ წიგნებს და ადრე გამოქვეყნებულ სტატიებს ასეც აწერია: Мусхеловъ. იმ დროს ბევრი ქართველი მეცნიერი და მოღვაწე თავიანთ გვარს რუსულ ყაიდაზე წერდა. საქართველოში ჩამოსვლისთანავე მამამ გვარის გადმოქართულების თაობაზე ივანე ჯავახიშვილს მიმართა. ბატონ ივანეს მისთვის ჩვეული თავაზიანობით უთქვამს: ''გვარის არჩევა თქვენი ნებაა, მაგრამ უნდა მოგახსენოთ, რომ მუსხელს მუსხელიშვილი ჯობია, ქართული გვარია და არაფერი დაეწუნება''. სხვათაშორის, ჩვენი წინაპრები სწორედ მუსხელებად იწერებოდნენ. მაგრამ ბატონი ივანეს ავტორიტეტის გავლენით მამამ გვარად მუსხელიშვილი აირჩია...

მამას ღამით უყვარდა მუშაობა. მახსოვს, ერთხელ ის საღამო ხანს მიუჯდა საწერ მაგიდას, დილით სკოლაში წასვლისას ვნახე, რომ იგი კვლავ მაგიდასთან იჯდა და მუშაობდა, სკოლიდან დავბრუნდი და იმავე სურათს შევესწარი.

ჯერ კიდევ სტუდენტობისას მამას ჩვევად ჰქონია მისთვის საინტერესო სამეცნიერო შრომების არა მარტო აღნუსხვა, არამედ სპეციალურად განკუთვნილ რვეულში მათი მოკლე შინაარსის ჩაწერაც. ამ ჩვევისთვის მას არასოდეს უღალატნია. მისი პირველი უცხოური მივლინების მიზანი ლიტერატურის გაცნობა და შეძენა იყო. მან ბევრი წიგნი ჩამოიტანა გერმანიიდან და საფრანგეთიდან. იმხანად, ოციან წლებს ვგულისხმობ, ჩვენ ძალიან ღარიბნი ვიყავით სპეციალური ლიტერატურით.

ხშირად მეკითხებიან, რატომ არ წავედი მამის კვალზე, რატომ ვამჯობინე, ფიზიკოსი გავმხდარიყავი. მე ძალიან მინდოდა, რადიოინჟინერი გამოვსულიყავი, მაგრამ ომის წლები იყო, ლენინგრადში წასვლა ვერ შევძელი და უნივერსიტეტში შევედი ფიზიკამათემატიკის ფაკულტეტზე. პირველ კურსზე მომეწონა მათემატიკა და ამ განხრით ვაპირებდი სწავლის გაგრძელებას, მაგრამ მამამ მითხრა, ''კარგი ხელი გაქვს, უმჯობესია, ფიზიკოსი გამოხვიდე''-ო. მეც დავუჯერე. სხვათა შორის, მას თვითონ უყვარდა ფიზიკა, კლასიკოსების შრომებსაც სიამოვნებით ეცნობოდა. აი, აინშტაინის ''ფარდობითობის თეორია'' გერმანულად, ეს კი - ნიუტონის ''ნატურფილოსოფიის მათემატიკური საფუძვლები'', აგრეთვე გერმანულად. იგი ნიკო კეცხოველს შეუძენია 1921 წელს და შემდეგ მამასთვის უჩუქებია. შეხედეთ, რა ლაკონიური წარწერა აქვს: ''ნიკოს ნიკოსგან. 1946''.

თბილისის უნივერსიტეტი დავამთავრე ფიზიკის სპეციალობით. არ იფიქროთ, ნიკო მუსხელიშვილის შვილობა რაიმე შეღავათს მაძლევდა ... გამოგიტყდებით, ჩემი მასწავლებლები არ ''მზოგავდნენ'', დიდ მოთხოვნებს მიყენებდნენ. მახსოვს, რა მკაცრი იყო გამოცდაზე მამას მოწაფე და უახლოესი მეგობარი ილია ვეკუა, რომელიც ხშირად დადიოდა ჩვენთან და, ბუნებრივია, კარგადაც მიცნობდა. არც სხვები ჩამორჩებოდნენ, ზოგჯერ საათობით მცდიდნენ. შემდეგ გავიგე, მამა ეუბნებოდა თურმე, "რაც შეიძლება მკაცრად გამოსცადეთ''-ო ... თვითონ მამამ ორი საგანი წამიკითხა, ანალიზური გეომეტრია და დიფერენციალური განტოლებანი. პირველში ''ფრიადი'' მივიღე, ერთი ფორმულა ჩემებურად გამოვიყვანე და ეს მოეწონა. მეორეში კი ... მეორეს მე და ჩემი მეგობარი ვაბარებდით ერთად აქ, ამ ოთახში. ჩემმა მეგობარმა ხუთ წუთში ''ხუთიანი'' მიიღო, მე კი ხუთ წუთშივე ... ''ორიანი'', თანაც ძალიან ორიგინალურად: ჩემი პასუხი რომ მოისმინა, მამა ადგა და გავიდა ოთახიდან, რითაც მაგრძნობინა, რომ გამოცდა დამთავრებული იყო.

მამა წიგნებს ძალიან ფაქიზად ეპყრობოდა, მე და ჩემი შვილებიც მიგვაჩვია ამას. ალბათ ნახეთ, რომ ბევრი წიგნი მაგარ ყდაში აქვს ჩასმული. აქაც არის არაერთი ასეთი

წიგნი. მამას ძალიან უყვარდა პოეზია და ამიტომ, ბუნებრივია, ხშირად კითხულობდა ლექსებს, პოემებს. უყვარდა რუსთაველი, ბარათაშვილი, პუშკინი. მაინც ვფიქრობ, რომ მისი საყვარელი პოეტი ბარათაშვილი იყო.

აღტაცებით კითხულობდა დოსტოევსკის, ლესკოვს. შვილიშვილებს სიამოვნებით უკითხავდა პუშკინსა და გოგოლს.

ისიც უნდა გითხრათ, რომ ბევრს კითხულობდა ფრანგულად, განსაკუთრებით ანატოლ ფრანსს, ალფონს დოდეს და აგრეთვე კონან დოილის ფრანგულ თარგმანებს. ბევრი სხვა მწერალიც უყვარდა, ძნელია რომელიმეს გამოყოფა, მაგრამ გეტყვით, რომ სიამოვნებას გვრიდა დიკენსის რომანების კითხვა. ძალიან მოსწონდა ტარლეს "ნაპოლეონი".

მამა გულგრილი არ იყო ქართული ფოლკლორისადმი, განსაკუთრებით ანდაზებისა და შაირებისადმი. ხშირად შაირობდა კიდეც თავის მეგობრებთან ნიკო კეცხოველთან და მიხეილ ჭიაურელთან. ცხადია, ამ მახვილსიტყვაობისას მე ოთახიდან გამომიშვებდნენ ... რაც შეეხება ანდაზებს, ბევრი იცოდა, მაგრამ გამორჩეულად ერთი მოსწონდა: "ბადრიჯანს რომ ფრთები ჰქონდეს, მერცხალი იქნებოდაო".

იგი ნადირობის ტრფიალი იყო. განსაკუთრებით მწყერზე უყვარდა ნადირობა მანგლისისა და დედულეთის, მაწევანის მიდამოებში. მისი გატაცება ანუ, როგორც ახლა ამბობენ, ჰობი დურგლობაც იყო. მისი ბევრი ნახელავი გვქონდა სახლში, ზოგიერთი ახლაც შემოგვრჩა..."

...

აი, რა უწინასწარმეტყველა ქართულ მათემატიკურ სკოლას 1939 წელს დიდმა რუსმა მექანიკოსმა და მათემატიკოსმა, აკადემიკოსმა ალექსეი კრილოვმა:

"Дорогие друзья мои! Купрадзе, Векуа, Микеладзе, Рухадзе, Горгидзе, Нодия! В Москве я виделся с Н.И. Мусхелишвили, избранным единогласно в действительные члены Академии наук СССР. Николай Иванович является основоположником блестящей грузинской школы математиков, а вы – его ближайшими и первыми сотрудниками – пионерами этого дела.

Я имел удавольствие выпить с Николай Ивановичем за ваше здоровье и за процветание грузинского математического общества и выразить уверенность, что подобно тому, как живительная влага лоз цинандальских и мукузанских виноградников по своим натуральным достоинствам превосходит продукцию лоз Бордо и Шатерна, так и продукция грузинской школы математики, созданная гением Н. И. Мусхелишвили и его видными соратниками – Купрадзе, Векуа, Микеладзе, Рухадзе, Горгидзе, Нодия будет быстро развиваться и сравниваться по своим научным достойнствам с продукцией школ Лагранжа и Коши".

ხოლო აკადემიკოსი ა. იშლინსკი 1997 წელს წერდა:

"Грузинский народ вправе гордиться всемирным признанием заслуг грузинской школы математиков и механиков, сложившейся в этом столетии, основоположником которой является Н.И. Мусхелишвили. Среди представителей этой школы много прославленных имен: И.Н. Векуа, В.Д. Купрадзе, К.К. Марджанишвили, А.Я. Горгидзе и другие".

ნიკოლოზ მუსხელიშვილის ცხოვრებისა და მოღვაწეობის მოკლე ქრონოლოგია

ნიკოლოზ (ნიკო) მუსხელიშვილი დაიბადა 1891 წლის 16 თებერვალს ქ. თბილისში სამხედრო ინჟინრის გენერალ ივანე მუსხელიშვილისა და დარია საგინაშვილის ოჯახში. ბავშვობის დიდი ნაწილი მან თეთრი წყაროს რაიონის სოფელ მაწევანში გაატარა, სადაც მისი ბაბუა (დედის მამა) ალექსანდრე საგინაშვილი ცხოვრობდა.

1909 წელს ნ. მუსხელიშვილმა დაამთავრა თბილისის მეორე კლასიკური გიმნაზია და იმავე წელს შევიდა სანკტ-პეტერბურგის უნივერსიტეტის ფიზიკა-მათემატიკის ფაკულტეტზე.

1914 წელს მან წარჩინებით დაამთავრა სანკტ-პეტერბურგის უნივერსიტეტის ფიზიკა-მათემატიკის ფაკულტეტი მათემატიკის სპეციალობით და პროფესორობისთვის მოსამზადებლად დატოვეს ამავე უნივერსიტეტის თეორიული მექანიკის კათედრაზე.

1915 წელს სანკტ-პეტერბურგის ელექტროტექნიკური საიმპერატორო ინსტიტუტის შრომებში თავის მეცნიერ-ხელმძღვანელ პროფესორ გური კოლოსოვთან თანაავტორობით ნ. მუსხელიშვილმა გამოაქვეყნა თავისი პირველი სამეცნიერო ნაშრომი: "ძაბვათა გავლენის შედეგად დრეკად მრგვალ დისკოთა წონასწორობა, როდესაც ძაბვები მიღებულია დისკოთა სიბრტყეში" (ელექტროტექნიკური ინსტიტუტის მოამბე, 1915, ტ. XII, გვ. 39-55). ნაშრომი

დრეკადობის თეორიის ერთ კონკრეტულ ამოცანას ეხებოდა. მაშინაც და შემდგომშიც ბატონი ნიკოს მეცნიერული ინტერესების სფეროში ძირითადად დრეკადობის თეორიის, უფრო ზოგადად კი მექანიკისა და მათემატიკური ფიზიკის საკითხები იყო.

1916-1919 წლებში ნ. მუსხელიშვილმა გამოაქვეყნა სამი შრომა. 1917 წლის 2 მარტიდან 1919 წლის 2 ივნისამდე მან წარჩინებით ჩააბარა ყველა სამაგისტრო გამოცდა, ამავე დროს ეწეოდა ინტენსიურ პედაგოგიურ საქმიანობას.

1920 წელს ნ. მუსხელიშვილი თბილისში დაბრუნდა და მოღვაწეობა დაიწყო თბილისის სახელმწიფო უნივერსიტეტში.

1920 წლის 1 სექტემბერს თბილისის უნივერსიტეტის სამათემატიკო-საბუნებისმეტყველო ფაკულტეტის სამეცნიერო საბჭომ ნ. მუსხელიშვილი აირჩია მექანიკის კათედრის გამგედ, ხოლო 29 ოქტომბერს უნივერსიტეტის პროფესორთა საბჭომ – პროფესორის თანამდებობაზე.

1926-28 წლებში ნ. მუსხელიშვილი თბილისის უნივერსიტეტში იყო პოლიტექნიკური ფაკულტეტის დეკანი. 1928 წლის 1 ოქტომბერს უნივერსიტეტის პოლიტექნიკური ფაკულტეტის ბაზაზე შეიქმნა საქართველოს პოლიტექნიკური ინსტიტუტი, სადაც ნ. მუსხელიშვილი 1928-30 წლებში იყო პრორექტორი სასწავლო ნაწილში, ხოლო 1928-38 წლებში – თეორიული მექანიკის კათედრის გამგე.

1933 წელს ნ. მუსხელიშვილი სსრკ მეცნიერებათა აკადემიის წევრ-კორესპონდენტად აირჩიეს. ამავე წელს მისი თაოსნობით თბილისის სახელმწიფო უნივერსიტეტში ჩამოყალიბდა მათემატიკის და ფიზიკის სამეცნიერო-კვლევითი ინსტიტუტი. 1935 წელს ცალკე გამოიყო მათემატიკის ინსტიტუტი, რომელიც 1937 წლიდან სსრკ მეცნი-ერებათა აკადემიის საქართველოს ფილიალის სისტემაში, ხოლო 1941 წლიდან – საქართველოს სსრ მეცნიერებათა აკადემიის სისტემაში გადავიდა.

1939 წელს ნ. მუსხელიშვილი აირჩიეს სსრკ მეცნიერებათა აკადემიის ნამდვილ წევრად. 1942-53 და 1957-72 წლებში ის სსრკ მეცნიერებათა აკადემიის პრეზიდიუმის წევრი იყო.

1920-62 წლებში ნ. მუსხელიშვილი იყო თსუ თეორიული მექანიკის კათედრის გამგე, ხოლო 1962-71 წლებში – თსუ უწყვეტი გარემოს მექანიკის კათედრის გამგე.

1941 წელს შეიქმნა საქართველოს სსრ მეცნიერებათა აკადემია, რომლის პრეზიდენტი 1972 წლამდე (ხოლო 1972-76 წლებში საპატიო პრეზიდენტი) იყო ნიკოლოზ მუსხელიშვილი. 1941 წლიდან გარდაცვალებამდე იგი იყო საქართველოს მეცნიერებათა აკადემიის ა. რაზმაძის სახელობის მათემატიკის ინსტიტუტის დირექტორი.

1941 წელს ნ. მუსხელიშვილს მონოგრაფიისთვის "დრეკადობის მათემატიკური თეორიის ზოგიერთი ძირითადი ამოცანა" (რუსულ ენაზე, 1939 წ.), რომელიც 1933 წელს გამოსცა სსრკ მეცნიერებათა აკადემიამ, პირველი ხარისხის სტალინური პრემია მიენიჭა. ეს მონოგრაფია ხუთჯერ გამოიცა და მრავალ ენაზეა თარგმნილი.

1945 წელს ნ. მუსხელიშვილს მიენიჭა სოციალისტური შრომის გმირის წოდება.

1946 წელს გამოვიდა ნ. მუსხელიშვილის მეორე მონოგრაფია "სინგულარული ინტეგრალური განტოლებები" (რუსულ ენაზე), რომლისთვისაც მას სტალინური პრემია მიენიჭა. 1957-76 წლებში ის იყო სსრკ თეორიული და გამოყენებითი მექანიკის ეროვნული კომიტეტის თავმჯდომარე.

1952 წელს ნ. მუსხელიშვილი აირჩიეს ბულგარეთის მეცნიერებათა აკადემიის წევრად, 1960 წელს – პოლონეთის მეცნიერებათა აკადემიის წევრად, 1961 წელს – სომხეთის სსრ მეცნიერებათა აკადემიის წევრად, 1967 წელს – გერმანიის დემოკრატიული რესპუბლიკის (ბერლინის) მეცნიერებათა აკადემიის უცხოელ წევრად, 1972 წელს – აზერბაიჯანის სსრ მეცნიერებათა აკადემიის წევრად.

1969 წელს ნ. მუსხელიშვილს მიენიჭა ტურინის (იტალია) აკადემიის საერთაშორისო პრემია – "მოდესტო პანეტი"; 1970 წელს დააჯილდოვეს სლოვაკიის მეცნიერებათა აკადემიის ოქროს მედლით, 1972 წელს – სსრკ მეცნიერებათა აკადემიის უმაღლესი ჯილდოთი – მ. ლომონოსოვის სახელობის ოქროს მედლით.

ნ. მუსხელიშვილი გარდაიცვალა 1976 წლის 15 ივლისს. დაკრძალულია თბილისის მამა-დავითის ეკლესიის ქართველ მწერალთა და საზოგადო მოღვაწეთა პანთეონში.

- აკადემიკოს ნიკოლოზ მუსხელიშვილის სახელი მისი გარდაცვალების შემდეგ მიენიჭა: საქართველოს მეცნიერებათა აკადემიის გამოთვლითი მათემატიკის ინსტიტუტს, ქუთაისის პოლიტექნიკურ ინსტიტუტს, თბილისის 55-ე საჯარო სკოლას, მანგლისის საჯარო სკოლას.

- დაარსდა საქართველოს მეცნიერებათა აკადემიის ნ. მუსხელიშვილის სახელობის პრემია მათემატიკის, მექანიკის, ფიზიკის დარგში გამოკვლევებისთვის (1977 წ.)

- დაწესდა ნ. მუსხელიშვილის სახელობის სტიპენდიები ასპირანტებისა და სტუდენტებისთვის.
- თბილისის სახელმწიფო უნივერსიტეტში დაიდგა ნ. მუსხელიშვილის ბიუსტი.
- ნ. მუსხელიშვილის ბინაში გაიხსნა სახლ-მუზეუმი.
- თბილისში, ი. ჭავჭავაძის გამზირზე დაიდგა ნ. მუსხელიშვილის ძეგლი.

ნ. მუსხელიშვილის სამეცნიერო შრომები მიძღვნილია მექანიკისა და მათემატიკის 4 ძირითადი პრობლემისადმი:

1. დრეკადობის თეორიის ბრტყელი ამოცანა.
2. ერთგვაროვანი და შედგენილი ძელების გრეხვა და ღუნვა.
3. ჰარმონიულ და ბიჰარმონიულ განტოლებათა სასაზღვრო ამოცანები.
4. სინგულარული ინტეგრალური განტოლებები და ანალიზურ ფუნქციათა თეორიის სასაზღვრო ამოცანები.

ამ პრობლემების დამუშავებამ დიდი გავლენა მოახდინა მათემატიკისა და მექანიკის რიგი დარგების შემდგომ განვითარებაზე.

ნ. მუსხელიშვილის მეთოდებმა დრეკადობის ბრტყელ თეორიაში გამოყენება და შემდგომი განვითარება ჰპოვეს ს. მიხლინის, დ. შერმანის და სხვათა ნაშრომებში. ამ მეთოდების საშუალებით გ. სავინის, დ. ვაინბერგის და სხვათა ნაშრომებში ამოხსნილია მრავალი ამოცანა, რომელიც ტექნიკაში გვხვდება. ნ. მუსხელიშვილის შედეგებმა შემდგომი გამოყენება და განვითარება ჰპოვეს საკონტაქტო ამოცანების თეორიაში ლ. გალინის, ა. კალანდიას, ი. ქარცივაძის, ი. შტაერმანის და სხვათა ნაშრომებში.

გამოკვლევები ძელების გრეხვისა და ღუნვის ამოცანებში სხვადასხვა მიმართულებით გავრცელდა ა. გორგიძის, ა. რუხაძისა და სხვათა ნაშრომებში.

ნ. მუსხელიშვილის იდეებმა დიდი გავლენა მოახდინეს საბჭოთა კავშირში ანალიზურ ფუნქციათა თეორიის სასაზღვრო ამოცანათა და სინგულარულ ინტეგრალურ განტოლებათა თეორიის პრობლემატიკის დამუშავებაზე (თ. გახოვის, ი. ვეკუას, ნ. ვეკუას, ა. ბიწაძის, დ. კვესელავას, ბ. ხვედელიძის, ლ. მაღნარაძის, გ. მანჯავიძის და სხვათა ნაშრომები). იმავე იდეებმა მტკიცედ მოიკიდეს ფეხი ელიფსური ტიპის კერძო წარმოებულებიან დიფერენციალურ განტოლებათა ზოგად თეორიაში (ი. ვეკუას, ბ. ხალილოვის და სხვათა ნაშრომები). კერძოდ მათ არსებითი გამოყენება ჰპოვეს გარსთა თეორიის საკითხებში. ნ. მუსხელიშვილის ნაშრომები დიდი პოპულარობით სარგებლობენ საზღვარგარეთის სპეციალისტთა ფართო წრეებში. ა. გრინისა და ვ. ზერნას (ინგლისი), ი. სოკოლნიკოვის (აშშ), ი. ბაბუშკას, კ. რექტორისის, ფ. ვიჩიხლოს (ჩეხეთი, სლოვაკეთი) და სხვათა მონოგრაფიებში დიდი ადგილი ეთმობა ნ. მუსხელიშვილის მეთოდებისა და შედეგების საფუძვლიან გადმოცემას.

ჯონდო შარიქაძე

Nikoloz Muskhelishvili
Scientist and Public Figure

“Tbilisi University?! This is just a mirage! It is virtually impossible to express even the simplest terminology of sciences like mathematics, chemistry and biology in Georgian that has no appropriate tradition!” – argued numerous opponents of the foundation of the University in Tbilisi.

Ivane Javakhishvili and his associates refuted this skepticism, and as early as November 1918 Professor Andrea Razmadze delivered his first lecture in Mathematical Analysis in perfect Georgian. Later, a young graduate of Moscow University Archil Kharadze communicated mathematics in Georgian in the same elegant way. General Andrea Benashvili, still wearing his uniform, did likewise in Astronomy. Soon they were joined by Giorgi Nickoladze and Nikoloz Muskhelishvili.

Shalva Nutsubidze recalls: “Andrea Razmadze, then the Dean of the Physics and Mathematics Faculty, entered my room followed by an energetic-looking young man. - He has just come from Petrograd where he had been engaged in scientific work. I need your consent to offer him a position. He seems talented and energetic. Andrea Razmadze’s request was of course granted. The young scientist fully justified Andrea Razmadze’s faith in him. That man was Nickoloz Muskhelishvili whom we all know now.”

. . .

Andrea Razmadze, Giorgi Nikoladze, Nikoloz Muskhelishvli and Achil Kharadze - “the Great Four” - were the founders of the Georgian Mathematics School. It is difficult to imagine the amount of work done by “the four” in a decade. On top of the intensive pedagogical work, they had to establish and refine scientific mathematical terminology in Georgian, to write and publish the first original textbooks in their native language and to form foundations of scientific research in various branches of mathematics.

. . .

In 1929, Andrea Razmadze passed away unexpectedly. Giorgi Nikoladze passed away in 1931. The whole burden fell upon Nikoloz Muskhelishvili, Archil Kharadze and upon some young mathematicians who had graduated from Tbilisi University. The latter were shown great examples of devotion and service to their country by their senior colleagues who never stopped their research and pedagogical work at Tbilisi State University and Georgian Polytechnic Institute.

. . .

During his career, Niko Muskhelishvili has worked as the Dean of the University Polytechnic Faculty, the Dean of the Physics and Mathematics Faculty, as the Pro-Rector of the Georgian Polytechnic Institute, the Chair in Theoretical Mechanics, as well as the Head of the Physics, Mathematics and Mechanics Institute, which had been founded at the University by his initiative. With his usual energy and enthusiasm, he also continued his pedagogical work. The older generation still remembers his lectures in analytic geometry, theoretical mechanics, and the theory of differential equations...

Niko Muskhelishvili wrote an original textbook in analytic geometry which was published several times and was widely regarded as one of the main University textbooks. Originality is also a distinctive feature of his “Course in Theoretical Mechanics” which was published in two parts “Statics” and “Kinematics” in 1926 and 1928 respectively and which later appeared in a second edition. Niko Muskhelishvili started to work on mathematical terminology soon after returning to his country. In the preface of “Mathematical Terminology” (1944, Russian-Georgian part), its editor professor Vukol Beridze wrote: “A particularly great contribution to the terminology is due to Academician N. Muskhelishvili who checked each word and tried to achieve the maximum accuracy and conformity between a mathematical notion and the term that describes it." It should be noted here, that the universally used term "toloba" (equality) and the naturally derived from it "utoloba" (inequality) and "gantoleba" (equation) were introduced by Niko Muskhelishvili in the early twenties.

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In 1922, Niko Muskhelishvili's book "Applications des intégrates analogues à celles de Cauchy à quelques problémes de la physique mathématique" was published in French in Tbilisi. This was in a sense a predecessor of his fundamental monograph "Some Basic Problems of the Mathematical Theory of Elasticity" (1933) which was based on the lectures delivered by the author in 1931-32 for the staff of the Leningrad Seismologic Institute and for PhD students of the Physics and Mathematics Institute as well as the Mathematics and Mechanics Institute of the Leningrad University.

The monograph soon gained popularity and its author became recognised as a prominent expert in elasticity theory. The same year,1933, Muskhelishvili was elected a Corresponding Member of the Academy of Sciences of the USSR, and in 1939 he became a Full Member of the Academy. At the same time, he served as the Chairman of the Georgian Branch of the Academy of Sciences of the USSR.

When the Academy of Sciences was established in Georgia in 1941, Muskhelishvili was unanimously elected its President. At the very first meeting of the Academy on 27 February 1941, Muskhelishvili ended his speech as follows: "Unfortunately today's festive mood is spoiled by the feeling that the scientist who had been looking forward to this great day with an utmost admiration is no longer among us. There is no doubt that had Ivane Javakhishvili been alive, he would have taken the high position that I am honoured to take now."

An extended second edition of "Some Basic Problems of the Mathematical Theory of Elasticity" was published in 1935, and its author was awarded a Stalin Prize in 1941. Muskhelishvili received the same Prize in 1946 for his other well known monograph "Singular Integral Equations". Before that, in 1945 Academician Nikoloz Muskhelishvili was awarded the title of a "Hero of Socialist Labour".

...

Both monographs have been translated and published abroad in many languages. Many complimentary reviews have been written about them.

...

Muskhelishvili's scientific work was recognized by dozens of prizes and awards. In particular, the Turin Academy of Sciences awarded him in 1969 its international prize and gold medal "Modesto Paneti". Our fellow countryman was the first Soviet and the sixth world scientist whose scientific achievements were marked with this high award. "This prize was absolutely unexpected – said the scientist to the correspondent of the newspaper "Komunisti",– I am delighted to receive this high recognition. The Turin Academy is one of the oldest Academies in Italy. Many of my works are connected with the works of Italian mathematicians. Italy was the first foreign country where my work was published upon presentation by the great Italian mathematician Vito Voltera".

...

Muskhelishvili was buried on Mount Mtatsminda - the burial place of Georgia's most revered sons and daughters. A prize bearing Muskhelishvili's name was created in his memoriam, the Institute of Computational Mathematics was named after him, and his monument was erected on Chavchavadze Avenue near the house where Muskhelishvili lived from 1941 to 1976 till the last day of his life. A memorial plaque with a bas-relief image of the First President of the Georgian Academy of Sciences marks the house.

Nikoloz Muskhelishvili's son, Doctor of Technical Sciences, Professor Guram Muskhelishvili never neglected the room where his father used to work. This room is also like a memorial with photographs on the walls, with medals and awards on special stands and books on the shelves ... Guram Muskhelishvili and his daughters Olga and Marina are looking with great care after their father's and grandfather's things and after the books he used ...

...

Here is how Guram Muskhelishvili recalls his renowned father: " I'll start from a distant. While living in St Petersburg, my father used to spell his name as Muskhelov, and the books bought there and his published articles were signed as Muskhelov. Many Georgian scientists and public figures wrote their names in a Russian way in those days. Soon after coming back to Georgia, father asked Ivane Javakhishvili about changing his name back to Georgian. Javakhishvili replied in his usual polite way: "choosing your name is entirely up to you, but I have to say that Muskhelishvili is better, this is a Georgian name and it is second to none". By the way, our ancestors were called Muskheli, but under the influence of Ivane Javakhishvili's authority my father chose the name Muskhelishvili.

My father liked working at night. I remember, once he sat at his desk in the evening, before going to school in the morning I saw him still sitting at the table and working, and when I returned home from school I saw the same scene...

Still being a student, my father developed a habit of making not only a list of the scientific works that were of interest to him, but also of writing their short summaries in a special notebook. He never changed this habit. The purpose of his first work trip abroad was to acquaint himself with and to acquire scientific literature. He brought a lot of books from Germany and France. Back then, I mean in the twenties, we were very poorly supplied with specialised literature.

I am often asked why I did not follow in my father's footsteps, why I chose to become a physicist. I had a great desire to become a radio engineer, but it was wartime and I could not go to Leningrad, so I enrolled in the Physics and Mathematics Faculty of Tbilisi University. Initially, I liked mathematics and was going to carry on studies in this field, but my father told me "you've got good hands and it would be better to become a physicist". I followed his advice. By the way, he loved physics himself and he enjoyed reading classical works. Here are Einstein's "Relativitätstheorie" in German and Newton's "Philosophiæ Naturalis Principia Mathematica" also in German. The latter was bought in 1921 by Niko Ketskhoveli who later presented it to my father. Look how laconically it is signed: "to Niko from Niko. 1946."

I graduated from the University in the field of Physics. Don't assume that being Niko's son meant any preferential treatment… To tell the truth, my teachers never spared me and always had high expectations for me. I remember how strict my father's student and his good friend Ilia Vekua was at the examination; he often visited us and naturally he knew me quite well. Others behaved in much the same way and sometimes tested me for hours. Later I learned that my father asked them to test me as strictly as possible.

My father himself taught me two subjects, analytic geometry and differential equations. In the first one I got the top mark. I derived one of the formulae in an original way and he liked that. As for the other subject… A friend of mine and I were taking the exam together in this very room. My friend passed it in five minutes and got the top mark, while I got a "fail" also in five minutes and in a very peculiar way too: when my father heard my answer, he stood up and left the room, which meant that the exam was over.

My father treated books with great care, and he taught me and my children to behave in the same way. You have probably noticed that he got many of his books rebound. Quite a few of these books are here too.

My father loved poetry immensely and it is natural that he often read verses and poems. He loved Rustaveli, Barathashvili, Pushkin, but I think his favourite poet was Barathashvili.

Father read Dostoevsky and Leskov with great enthusiasm, and he enjoyed reading Pushkin and Gogol to his grandchildren.

I would like to add that my father read a lot in French, especially Anatole France, Alphonse Daudet and French translations of Conan Doyle. He loved other writers too, and it is difficult to single out someone, but I remember he particularly enjoyed reading Charles Dickens, and he liked "Napoleon" by Tarle.

My father was not indifferent to Georgian folklore, especially to proverbs and shairi (a short form of a witty verse like a pun). He often made a pun with his friends Niko Keckhoveli and Mikheil Chiaureli, and I was asked to leave the room of course. As for the proverbs, he knew quite a few, but his favourite one was: "If an aubergine had wings it would have been a swallow".

My father loved hunting, especially on quails around Manglisi and around his native Matsevani. His other passion or as they say today, hobby was carpentry. We used to have lots of things made by him, we still have some of them..."

...

Here is a prediction of a great Russian applied mathematician, Academician Aleksey Krylov for the Georgian mathematical school in 1939:

"My dear friends! Kupradze, Mikeladze, Gorgidze, Nodia!

In Moscow, I met N. Muskhelishvili who had been unanimously elected a Full Member of the Academy of Sciences of the USSR. Nikolai Ivanovich is the founder of the brilliant Georgian mathematical school, and you are his first and closest colleagues – the pioneers in this field.

I had a pleasure of rising a glass with Nikolai Ivanovich to your health and the prosperity of the Georgian mathematical society, and of expressing my deep belief that like the life-giving elixir of the vines of the Tsinandali and Mukuzani vineyards surpasses in its natural qualities the produce of the vines of Bordeaux and Sauternes, so the output of the Georgian mathematical school established by the genius N. Muskhelishvili and his colleagues – Kupradze, Vekua, Mikeladze, Rukhadze, Gorgidze, Nodia – will develop rapidly and its scientific merits will become comparable to those of the schools of Lagrange and Cauchy."

Academician A. Ishlinski wrote in 1997: "The Georgian people should rightly be proud of the world-wide recognition of the achievements of the Georgian school of Mathematics and Mechanics established in this century, the founder of which is N. Muskhelishvili. There are many celebrated names among the representatives of this school: I. Vekua, V. Kupradze, K. Marjanishvili, A. Gorgidze, and others.

A brief chronology of Nikoloz Muskhelishvili's life and work

Nikoloz (Niko) Muskhelishvili was born on 16 February 1891 in Tbilisi in the family of a military engineer General Ivane Muskhelishvili and Daria Saginashvili. He spent most of his childhood in the village Matsevani of Tetrickharo region where his maternal grandfather Alexander Saginashvili lived.

In 1909, Muskhelishvili finished the Second Classical Gymnasium in Tbilisi. The same year, he enrolled in the Physics and Mathematics Faculty of St Petersburg University.

In 1914, he graduated with Distinction from the Physics and Mathematics Faculty of St Petersburg University specializing in Mathematics, and he was invited to continue his postgraduate studies in Theoretical Mechanics at the same University.

In 1915, Muskhelishvili, jointly with his scientific supervisor professor Guri Kolosov, published his first scientific work in the proceedings of the Imperial Electrotechnical Institute: "On the equilibrium of elastic circular disks under the influence of stresses applied at the points of their encirclement and acting in their domains" (Russian), Izv. Electrotekhnich. Inst., Petrograd, 12 (1915), 39-55 (jointly with G.V. Kolosov). The work was about a particular problem in elasticity theory. Then and later, Muskhelishvili's research interests were mostly in the field of elasticity theory and, more generally, in the field of mechanics and mathematical physics.

In 1916-1919, Muskhelishvili published three works. From 2 March to 2 June 1919, he passed with Distinction all his Magister exams, while being heavily involved in teaching at the same time.

In 1920, Muskhelishvili returned to Tbilisi and started working at Tbilisi State University.

On 1 September 1920, the Scientific Board of the Faculty of Mathematical and Natural Sciences elected Muskhelishvili as the Chair in Mechanics, while on 29 October, the Board of Professors elected him as a Professor.

In 1926-28, Muskhelishvili was the Dean of the Polytechnic Faculty of Tbilisi University. In 1928, the Georgian Polytechnic Institute was established on the basis of the Faculty, where Muskhelishvili was the Pro-Rector in Education in 1928-1930, and the Chair in Theoretical Mechanics in 1928-38.

In 1933, Muskhelishvili was elected a Corresponding Member of the Academy of Sciences of the USSR. The same year a research Institute of Mathematics and Physics was established under his leadership at Tbilisi University. In 1935, a separate Institute of Mathematics was created, and in 1937 it first passed into the system of the Georgian Branch of the Academy of Sciences of the USSR and then in 1941 – into the system of the Academy of Sciences of Georgia.

In 1939, Muskhelishvili was elected a Full Member of the Academy of Sciences of the USSR, and in 1942-53 and in 1957-72 he was a member of the Presidium of the Academy of Sciences of the USSR.

In 1920-62, Muskhelishvili was the Chair in Theoretical Mechanic, and in 1962 -71 the Chair in Continuum Mechanics at Tbilisi University.

In 1941, the Academy of Sciences of Georgia was established, and Nikoloz Muskhelishvili was its President until 1972 and the Honorary President from 1972 to 1976. From 1945 to the end of his life he was the Director of A. Razmadze Institute of Mathematics of the Academy of Sciences of Georgia.

In 1941, a Stalin Prize of the First Degree was awarded to Muskhelishvili's monograph "Some Basic Problems of the Mathematical Theory of Elasticity" (Russian, 1939), an earlier edition of which was published by the Academy of Sciences of the USSR in 1933. The monograph has been published five times and translated into many languages.

In 1945, Muskhelishvili was awarded the title of a "Hero of Socialist Labour".

In 1946, Muskhelishvili's second monograph "Singular Integral Equations" (Russian) was published and he was awarded a Stalin Prize for it.

In 1957-76, he was the Chairman of the USSR National Committee for Theoretical and Applied Mechanics.

In 1952, Muskhelishvili was elected a Member of the Bulgarian Academy of Sciences, in 1960 – a Member of the Academy of Sciences of Poland, in 1967 – a Foreign Member of the Academy of Sciences of German Democratic Republic (Berlin), in 1961- a Member of the Academy of Sciences of Armenia, in 1972 – a Member of the Academy of Science of Azerbaijan.

In 1969, Turin Academy of Sciences awarded Muskhelishvili its international prize "Modesto Paneti"; in 1970 he was awarded a Gold Medal of the Slovak Academy of Sciences, and in 1972 – the highest award of the Academy of Sciences of the USSR, the M. Lomonosov Gold Medal.

Nikoloz Muskhelishvili passed away on 15 July 1976. He is buried at the Pantheon of Georgian writers and public figures at Mama David church on Mount Mtatsminda.

- The Institute of Computational Mathematics of the Academy of Sciences of Georgia, the Kutaisi Polytechnic Institute, Tbilisi state school No. 55 and Manglisi state school have been named after Muskhelishvili.

- The Academy of Sciences of Georgia introduced the Muskhelishvili Prize for research in Mathematics, Mechanics and Physics in 1977.

- Muskhelishvili Scholarships were established for undergraduate and postgraduate students.

- Muskhelishvili's bust was put up in Tbilisi University.

- His museum was opened in his flat.
- His monument was erected on Chavchavadze Avenue.

Muskhelishvili's works were devoted to the following four basic problems of mechanics and mathematics:

1. The plane problems of elasticity theory.
2. Torsion and bending of homogeneous and composite beams.
3. Boundary value problems for the harmonic and biharmonic equations.
4. Singular integral equations and boundary value problems of the theory of analytic functions.

The study of these problems has had a major influence on the further development of several branches of mathematics and mechanics.

Muskhelishvil's methods in plane elasticity theory were applied and further developed in the works of S. Mikhlin, D. Sherman, and others. With the help of these methods, many problems that arise in industry were solved in the works G. Savin, D. Vainberg, and others. Muskhelishvili's results were applied and further developed in the theory of contact problems by L.Galin, A. Kalandia, I. Karcivadze, I. Shtaerman, and others. Applications to problems of torsion and bending of beams developed in various directions in the works of A. Gorgadze, A. Rukhadze, and others. Muskhelishvili's ideas have had a major impact on the work on boundary value problems of the theory of analytic functions and singular integral equations carried out in the Soviet Union (by T. Gakhov, I. Vekua, N. Vekua, A. Bitsadze, D. Kveselava, B. Khvedelidze, L. Magnaradze, G. Manjavidze, and others).

The same ideas have firmly established themselves in the general theory of elliptic partial differential equations (works of I. Vekua, B. Khalilov, and others). In particular, they have found important applications in shell theory.

Muskhelishvili's works enjoy wide popularity among a large number of foreign experts. Large parts of monographs by A. Green and W. Zerna (England), I. Sokolnikoff (USA), I. Babushka, K. Rektoris (Czech Republic, Slovakia) and others are devoted to a detailed exposition of Muskhelishvili's methods and results.

Jondo Sharikadze

Plenary Talks

A New Trend in Real Analysis Interlacing with Different Branches of Pure and Applied Mathematics

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In this lecture we present a new trend in real analysis interlacing with rather different fields (Hilbert problem 16 in real algebraic geometry, Nevanlinna theory and Gamma-lines theory in complex analysis, integral geometry) and admitting interpretations in many applied topics (hydro-aero dynamics, meteorology, wave processes etc.).

We study the geometry of level sets of real functions: the length, integral curvature of the level sets. Also we study the number of connected components of level sets of real functions which, in particular case of polynomials, was widely studied in the frame of Hilbert problem 16.

The results of this new trend strength and generalize the key result in all above mentioned fields. This development unfolds as follows. The new results: (a) imply the key conclusions in Gamma-lines theory [1] which, in turn, contains so called proximity property, which, in turn, strengths the key results in Nevanlinna theory; (b) imply estimates of the cardinalities of level sets of real functions which, in particular case of polynomials, strength the key result in real algebraic geometry; (c) imply some new formulas in integral geometry which, in turn, generalize the key Crofton's identity in integral geometry.

The geometry of level sets was studied earlier in the frame of Gamma-lines theory dealing with some classes of real functions determined by complex functions. In fact the obtained results constitute a far going generalization of Gamma-lines theory which now is valid for any "reasonably smooth" real function.

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Projection Methods and Generalized Solutions for a Class of Singular Integral Equations with Carleman Shift

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We will present a polynomial collocation method which will be applied to a class of singular integral equations with Carleman shift. The main goal is to obtain information about the approximation numbers and the Moore–Penrose invertibility of the corresponding singular integral operators with shift. Namely, a relation between the so-called k-splitting property and the kernel dimension of certain auxiliary operators will be presented. To this end, some projection methods and an algebraization of stability will be applied to the equations in study. The talk is based in part on a joint work with E. M. Rojas.

The L^p-Dissipativity of Partial Differential Operators

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In this talk I will present some results obtained jointly with Vladimir Maz'ya.

They concern the dissipativity of partial differential operators with respect to the L^p norm $(1 < p < \infty)$. In particular we have proved that the algebraic condition

$$|p-2|\,|\langle \mathscr{I}m\,\mathscr{A}\xi,\xi\rangle| \leq 2\sqrt{p-1}\,\langle \mathscr{R}e\,\mathscr{A}\xi,\xi\rangle$$

(for any $\xi \in \mathbb{R}^n$) is necessary and sufficient for the L^p-dissipativity of the Dirichlet problem for the differential operator $\nabla^t(\mathscr{A}\nabla)$, where $\mathscr{A}$ is a matrix whose entries are complex measures and whose imaginary part is symmetric. This result is new even for smooth coefficients, when it implies a criterion for the L^p-contractivity of the corresponding semigroup.

This condition characterizes the L^p-dissipativity individually, for each p, while usually the results in the literature concern the L^p-dissipativity for all p's simultaneously.

I will discuss also the L^p-dissipativity for some other operators, in particular for

- the operator $\nabla^t(\mathscr{A}\nabla) + \mathbf{b}\nabla + a$ with constant coefficients;
- systems of partial differential operators of the form $\partial_h(\mathscr{A}^{hk}(x)\partial_k)$, were $\mathscr{A}^{hk}(x)$ are $m \times m$ matrices;
- the two-dimensional elasticity operator;
- the operator $\partial_h(\mathscr{A}^h(x)\partial_h)$, where $\mathscr{A}^h(x)$ are $m \times m$ matrices with complex L^1_{loc} entries.

Moreover I will show how our conditions can completely determine the angle of dissipativity of the considered operators.

Effective Field Theories

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It is widely believed that quantum chromodynamics (QCD) is the fundamental theory of strong interactions. Quarks and gluons are the dynamical degrees of freedom of QCD. It is an asymptotically free theory. This means that its effective coupling constant decreases at high energies and quarks behave as almost free particles. Due to this property of QCD the physical quantities at high energies can be reliably calculated using perturbation theory.

Although not yet proven, it is believed that quarks and gluons are confined in QCD. I.e. they never appear as free particles. Instead the eigenstates of the QCD Hamiltonian are the states observed in nature, i.e. hadrons (mesons and baryons). The confinement picture is supported by the behavior of the effective coupling at low energies, it becomes large and the perturbation theory cannot be applied.

Effective field theory provides with a solutions to the above problem with large coupling of the QCD at low energies. It incorporates all symmetries of QCD and reproduces the Green's functions of QCD as an expansion in powers of small masses and energy. Dynamical degrees of freedom of effective field theory are the mesons and baryons.

I will review some well-established as well as new results of the effective field theory of strong interactions.

Regularization Parameter Dependence in NJL Model

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The fundamental theory to describe phenomena of quarks and gluons is the quantum chromo dynamics (QCD). As is well-known, the QCD coupling blows up at the QCD scale. A usual perturbative procedure loses its validity and a non-perturbative effect is essential for a study of low energy phenomena. Thus we often use a phenomenological effective model to evaluate phenomena below the QCD scale. Nambu–Jona-Lasinio (NJL) model is one of models which well describes the low energy (pseudo) scalar meson phenomena [1]. NJL model contains non-renormalizable higher dimensional operators in four space-time dimensions. It is ordinary considered that the physical results depend on a regularization parameter. It seems to generate some ambiguities. Thus it is important to understand the regularization parameter dependence of the results.

The model parameters are fixed to exactly reproduce some of observed quantities. Regularization parameter dependence is introduced in fixing the model parameters. For the dimensional regularization the regularization parameter is the space-time dimension for a fermion loop integral. We evaluate the regularization parameter dependence for some of physical observables in NJL model with the dimensional regularization [2]. It is shown that a four dimensional limit is well-defined for some physical observables. A possibility of the regularization parameter independent analysis is discussed for some physical observables.

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On the Cobar Construction of a Bialgebra

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The cobar construction ΩC of a DG-coalgebra $(C, d: C \to C, \Delta: C \to C \otimes C)$ is a DG-algebra. If C additionally is equipped with a multiplication $\mu: C \otimes C \to C$ turning it into a DG-bialgebra, how this structure reflects on the cobar construction ΩC?

By the classical result of Adams in this case μ produces $\smile_1$ product which measures the deviation from the commutativity of ΩC.

In this talk we show that μ induces a richer structure, namely a sequence of operations

$$E_{1,k}: \Omega C \otimes (\Omega C)^{\otimes k} \to \Omega C, \ \ k = 1, 2, 3, \dots, \ \ E_{1,1} = \smile_1,$$

which turns the cobar construction ΩC into a *homotopy G-algebra*. This particularly implies the construction of well known Lie bracket on homology of double loop space $H_*(\Omega^2 X)$.

On the Anisotropic Maxwell's Equations in the Screen Configuration

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In the paper M. Sh. Birman and M. Z. Solomyak proved that the main singularity of an electromagnetic fields within an ideally conducting isotropic resonator with smooth walls, inside of which are a number of screens is associated with the gradient of a weak solution to a scalar Dirichlet problem. The treatment is abstract and does not species the partial differential equation responsible for the main singularity.

The talk is based upon joint work with R. Duduchava and O. Chkadua, where it is shown that the solutions to the Neumann type "magnetic" boundary value problems (BVPs) for a Maxwell's system in a finite or infinite anisotropic media outside a smooth hypersurface with the smooth boundary is represented as a sum $E = \operatorname{grad} \psi + E_0$. Here E_0 is a solutions to an elliptic BVP in the subspace of Bessel potential space of vector-functions, orthogonal to a certain vector field on the boundary, while ψ is a solutions to the scalar elliptic BVP in the Bessel potential space of functions. The elliptic BVPs, responsible for both summands, are written explicitly. Using potential method and investigating pseudodifferential boundary operators the unique solvability and regularity results for the auxiliary BVPs are proved when the permeability and the permittivity matrix coefficients of the anisotropic Maxwell's equations are real valued, constant, positive definite and symmetric. Moreover, the precise asymptotic behavior of a solution near the edge of the "screen" is established. The obtained results describe the asymptotic of a solution to the original problem.

Asymptotic Analysis of Initial-Value Problems for thin Plates

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The initial-value problems for thin elastic plates are formulated within the framework of 3D linear elasticity. The plate thickness is assumed to be small in comparison with a typical scale of the initial data along plate faces. At the same time arbitrary variation of the initial data through the plate thickness is taken in consideration. Composite asymptotic expansions are derived starting from both low-frequency and high-frequency long-wave plate models. For each of the latter asymptotic initial conditions are derived including in particular the refined initial conditions in the classical theory of plate bending.

მათემატიკის სასკოლო კურსის "მათემატიკური სიმკაცრის" შესახებ

ეკატერინე კორძაძე

ეროვნული სასწავლო გეგმებისა და შეფასების ცენტრი

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სტატია ეხება მათემატიკის სასკოლო კურსის აგების მთავარ პრობლემას: შესაძლებელია თუ არა და რამდენად აუცილებელია მათემატიკური სიზუსტის დაცვა თეორემების დამტკიცებისას სასკოლო სახელმძღვანელოებში, რა განსხვავებაა აღწერით და კონსტრუქციულ აქსიომატიზაციას შორის და როგორ შეიძლება იქნას ეს გამოყენებული მათემატიკის სწავლებისას.

Generalized Co-Normal Derivatives and Boundary Value Problems for Elliptic PDEs

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Elliptic PDE systems of the second order on a Lipschitz domain are considered in this contribution. For functions from the Sobolev space $H^s(\Omega)$, $\frac{1}{2} < s < \frac{3}{2}$, definitions of non-unique generalized and unique canonical co-normal derivative are considered, which are related to possible extensions of a partial differential operator and PDE right hand side from the domain

Ω to its boundary. It is proved that the canonical co-normal derivative coincides with the classical one when the both exist, while the generalized co-normal derivative is not unique. A generalization of the boundary value problem variational settings, which makes them insensitive to the co-normal derivative inherent non-uniqueness is given.

In addition to the case of the infinitely smooth coefficients, we consider also the cases with coefficients from L_∞ or Hölder–Lipschitz spaces. Continuity of the partial differential operators in corresponding Sobolev spaces is stated and the internal (local) solution regularity theorems are generalized, which allows to extend the notions of the generalized and canonical co-normal derivatives and BVP settings also to the non-smooth coefficients.

The talk is related to the papers [1, 2].

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Modeling Phase Segregation by Atomic Re-Arrangement

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In crystalline matter, phase segregation may occur with a without diffusion, under essentially isothermal condition or not. The standard nondiffusional model is the nonlinear parabolic equation associated with the names of Allen and Cahn; when diffusion is important, the standard equation is the nonlinear, fourth-order parabolic equation due to Cahn and Hilliard; in both cases, thermal effects are not accounted for. I shall discuss an alternative modeling approach [1–4].

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Almost Commuting Operators

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The talk will discuss the following naive question: if the commutator $[X, Y]$ of two bounded operators X and Y is small in an appropriate sense, is there a pair of commuting operators $\tilde{X}$ and $\tilde{Y}$ which are close to X and Y? Examples show that for general operators it is not necessarily true. For self-adjoint operators, taking $A := X + iY$, one can reformulate the question as follows: if the self-commutator $[A^*, A]$ is small, is there a normal operator $\tilde{A}$ close to A? There are two known positive results on this problem.

The Brown–Douglas–Fillmore theorem. *If $[A^*, A]$ is compact and the corresponding to A element of the Calkin algebra has trivial index function then there is a compact operator K such that $A + K$ is normal.*

Huaxin Lin's theorem. *There exists a nondecreasing function F vanishing at the origin such that the distance from A to the set of normal operators is estimated by $F(\|[A^*, A]\|)$ for all finite rank operators A.*

Let L be a C^*-algebra of bounded operator on a Hilbert space. One says that L has real rank zero if the set of invertible self-adjoint elements of L is norm dense in the set of all self-adjoint elements. Our main result is the following theorem.

Theorem. *Let L have real rank zero, and let $A \in L$ satisfy the following condition*

(C) *the operators $A - \lambda I$ belong to the closure of the connected component of unity in the set of invertible elements of L for all $\lambda \in \mathbf{C}$.*

Then, for each $\epsilon > 0$, the operator A can be represented in the form $A = A_1 + h(\epsilon)A_2 + \epsilon A_3$, where h is a 'universal' (that is, independent of A and L) nonnegative function on $(0, \infty)$ and A_j are operators from L such that

- $\|A_1\| \le \|A\|$, $\|A_3\| \le \|A\|$ *and* $\|A_2\| \le \|[A^*, A]\|$;
- A_1 *is normal and* A_2 *is self-adjoint;*
- A_2 *is a finite convex combination of operators of the form $U[A^*, A]V$, where U and V are unitary elements of L.*

The theorem implies the above results and allows one to extend Huaxin Lin's theorem to operators of infinite rank and other norms. Note that in the separable case **(C)** is equivalent to the index condition in the Brown–Douglas–Fillmore theorem.

Now the main challenge in this field is to obtain explict estimates for the functions F and h. Some conjectures and recent results in this direction will be mentioned in the end of the talk.

Operator Factorization and Applications to the Solution of Boundary Value Problems

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Boundary value problems in Mathematical Physics are often proved to be Fredholm or even well-posed, sometimes explicitly solvable by formulas in closed analytical form. Many of these conclusions can be formulated in terms of operator factorization methods (in Banach spaces as central case). We explain various principles and their realization in terms of applications. The advantages are (1) very clear and compact formulations, e.g., of “equivalent reduction” of operators associated to boundary value problems to “more convenient operators” (like boundary pseudodifferential operators), (2) explicit inversion by operator factorization methods, (3) some related concepts such as reduction to semi-homogeneous problems, normalization and others. Several examples will underline the ease and usefulness of operator factorization.

The talk is based upon joint work with E. Meister, R. Duduchava, F. S. Teixeira, L. P. Castro and A. Moura Santos.

Probability Investigations in Niko Muskhelishvili Institute of Computational Mathematics

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The Computing Center of the Academy of Sciences of Georgian SSR was founded in 1956.

Since 1977 it carries the name of N. Muskhelishvili.

In 1983 the Center was renamed as “Niko Muskhelishvili Institute of Computational Mathematics of the Academy of Sciences of Georgian SSR”.

Since 1991, naturally, the Institute is named as “Niko Muskhelishvili Institute of Computational Mathematics of Georgian Academy of Sciences”.

During 2006 – 2010 the Institute existed as a “LEPL (*Legal Entity of Public Law*) Niko Muskhelishvili Institute of Computational Mathematics”.

Since the beginning of 2011 the Institute is *an independent structural unit* of the Georgian Technical university.

During more than 55 years of existence in the frames of the Institute, among other directions, the probabilistic and statistical investigations were playing an important role.

In the talk these investigations and their impact are surveyed.

Acknowledgements

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Real Analysis

Otar Tsereteli – a Scientist and a Maître

(To the 85th Birthday Anniversary)

In 2011 Doctor of Physical and Mathematical Sciences, Professor, Corresponding Member of the Georgian Academy of Sciences Otar Tsereteli would have been 85 and marked 60 years of his scientific activities. Otar Tsereteli was a brilliant Georgian mathematician, teacher and organizer of science. His scientific works produced an essential impact on the development of the theory of harmonic analysis as well as on some other areas of mathematical analysis.

O. Tsereteli was born on November 22, 1926 in the town of Akhaltsikhe (Georgia) and died on April 17, 1991 at the age of 65. But his ideas and results still remain topical and the directions he set up in mathematical science continue to develop.

O. Tsereteli was profoundly respected as a highly skilled mathematician and his results were highly appreciated by outstanding scientists of modern times Antoni Zygmund, Elias Stein, Pyotr Ulyanov, Dimitri Menshov, Sergei Stechkin, Niko Muskhelishvili, Ilia Vekua and others.

O. Tsereteli graduated from the physical-and-mathematical faculty of Ivane Javalhishvili Tbilisi State University in 1948. In 1952 he finished a post-graduate course and took up the work at the State Industrial Institute. After defending his Ph.D. thesis in 1956, he started working at A. Razmadze Tbilisi Mathematical Institute of the Georgian Academy of Sciences, where to the last day of his life he was in charge of the function theory and functional analysis department. Concurrently, he was engaged in the pedagogical activity at I. Javakhishvili Tbilisi State University, was a professor and held the chair of improvement of qualifications in mathematics of teachers working at higher educational institutions.

The scientific activities of O. Tsereteli are related to the theory of functions of real and complex variables, and also to the metric theory of functions. His extraordinary intellect, creativity and rich knowledge had an imprint both on his choice of scientific studies and on his teaching work. He obtained first-rate results in the theory of Fourier series, the metric theory of conjugate functions and the analytic function theory. He always tried to grasp the gist of the matter and speak about it in a simple, easily understood way. Owing to the refined manner of exposition, clearness and convincingness of argumentation his published works used to produce and continue to produce nowadays a great impression on readers.

The early papers of O. Tsereteli were dedicated to the theory of integrals and the metric theory of functions of bounded variation. He introduced the notions of contracting and dissipative functions and constructed by means of these notions the metric theory of functions of bounded variation. He furthermore introduced the notion of variation of a mapping on the ring and the notion of mapping multiplicity with respect to the ring, and constructed the Banach theory of mappings of arbitrary spaces with measure.

O. Tsereteli studied the ergodic properties of inner analytic functions and boundary values of the Schwartz integral of a Borel measure. In particular he established that the boundary values of an inner function that differs from rotation and vanishes at the point $z = 0$ are a strongly mixing transformation; the ergodic means of any conjugate function with respect to measure-preserving transformations generated by inner functions, converge in measure (but, generally speaking, do not converge to zero a.e.).

He also studied the uniqueness properties of inner analytic functions. For instance, he proved that a singular nonnegative measure is completely defined by its variation and some Lebesgue set of its conjugate function.

In problems of "correction" of a function on a set of small measure, O. Tsereteli proposed his own "Tsereteli version" in order to obtain a function with that property or another. Namely, the function f is corrected on a set of small measure not in an arbitrary fashion like in the classical theorems of N. Luzin and D. Menshov, but only by the rearrangement of its values or multiplication by –1 (in that case, the metric class of a function does not change). The following result of O. Tsereteli is impressive: the values of an integrable function can be rearranged on a set of an arbitrarily small measure so that the trigonometric Fourier series of the obtained function will converge almost everywhere. He furthermore showed that the sign of an arbitrary integrable function can be changed on some set of an arbitrarily small measure so that the conjugate of the obtained function will be integrable. The latter means that the integrability of the conjugate function does not impose any restrictions on the modulus of an integrable function.

The construction of the metric theory of conjugate functions proposed by O. Tsereteli can be extended nearly without any modification to the case of general functional Dirichlet algebras. In particular he obtained a generalization of P. Ulyanov's theorem on the Riesz equality to the case of conjugate functions arising in the theory of uniform Dirichlet algebras.

In the theory of Fourier series, O. Tsereteli obtained a conceptual result on general orthogonal systems. He proved that the values of any nonconstant (nonzero) function from the space L^2 can be rearranged (multiplied by-1) on the set of an arbitrarily small measure so that the Fourier series of the obtained function with respect to a given complete orthonormal system may – after some rearrangement – diverge a.e. This means that there exists no criterion imposing some restrictions on a distribution function and on the modulus of an integrable function and providing the absolute convergence a.e. of Fourier series of this function with respect to a given complete orthonormal system.

Most of the results of O. Tsereteli and his followers related to the "correction" of functions were obtained while working on the solution of the following general problem posed by O. Tsereteli: given an equivalence relation R on the set X, characterize a set E from X in terms of R, i.e. define explicitly the largest R-set $\underline{R}(E)$(i.e. the set which is a union of R-equivalence classes) contained in E and the smallest R-set $\overline{R}(E)$ containing E.

The problem of characterization of a set with respect to a given equivalence relation as formulated by O. Tsereteli is a powerful source of new interesting problems and its application in concrete cases leads to concrete results. One of the remarkable statements of O. Tsereteli concerns the metric characterization of a set of integrable functions whose conjugates are integrable: if $X = L^1,\ E = \mathrm{Re}H^1$ (where $H^1$is the Hardy class) and fRg means that f and g are equimeasurable (or $|f| = |g|$ a.e.), then $\underline{R}(E) = Lg^+L$, and $\overline{R}(E) = Z_1$ ($\overline{R}(E) = L_1$), where the class Z_p, $p > 0$, introduced by O. Tsereteli is defined as follows: if $f \in L^{\mathbf{1}}$ and $F(t),\ t > 0$, is an integral of f on $\{x : |f(x)| > t\}$, then $f \in Z_p$ if and only if the function $|F|^p$ is integrable on $(1,\ \infty)$ over the measure $t^{-1}dt$. Analogous problems were solved by him for maximal Hardy-Littlewood functions as well.

O. Tsereteli established that a set of A-integrable functions is a metrically invariant set containing a set of all conjugate functions $\tilde{L}$, but is not a minimal metric set containing $\tilde{L}$. More precisely, he constructed an example of an A-integrable function f such that none of functions g equimeasurable with f on T could be represented as the conjugate of some integrable function φ, which means that the equality $g = \tilde{\varphi}$, where $\varphi \in L(T)$, is impossible.

O. Tsereteli proved that if a 2π-periodic function is integrable on $(0,\ 2\pi)$ and monotone on an open interval $(0,\ 2\pi)$, then $f \in \mathrm{Re}H^1$ if and only if$f \in Z_1$.

His study of the A-integral actually completed the previous studies of the theory of this integral and its applications carried out by Georgian and foreign scientists. O. Tsereteli proved that A-integrability is the property not only of a conjugate function, but also of all operators, continuous with respect to a measure and commutative with shear. In particular he established that any trigonometric series is a Fourier (A) series of some nonzero A-integrable function. He obtained generalizations of Titchmarsh's theorem on the A-integrability of conjugate (in the sense of Luzin) functions and showed that the values of any linear operator are A-integrable when it is continuous with respect to a measure, is given on the Lebesgue space of Borel functions defined on a compact group with Haar measure, and is permutable with shears.

O. Tsereteli combined research with active teaching work at I. Javakhishvili Tbilisi State University, where he delivered lectures at the philosophy and psychology faculty and also read a special course for mathematicians who worked at higher education institutions and wished to improve their qualifications. In the autumn of 1969, O. Tsereteli began reading lectures on harmonic analysis at the mechanical and mathematical faculty. The author of these reminiscences had the honor to attend together with other students those lectures and, in senior years, to listen to his lectures on the theories of measures, Fourier series, holomorphic functions, metric spaces and so on.

O. Tsereteli had an original manner of delivering lectures. They were invariably based on the latest achievements in mathematical science. He tried to explain even the most difficult mathematical problems so that they could be easily understood by the audience, focusing attention on the main ideas and comparing them with a multitude of other mathematical facts and theories. It was surprising how he managed to arouse interest in mathematics, to make every discussed problem attractive and to speak of solutions intuitively, fully preserving the logic of argumentation and the clarity of exposition.

Yet another merit of O. Tsereteli was that in 1966 he founded a weekly scientific seminar on the function theory at A. Razmadze Mathematical Institute and became its permanent leader. By the rules of the seminar set up by O. Tsereteli a speaker was strictly obliged to meet a high standard of material presentation. Productive discussions of questions arising at seminar sessions and the statement of new topical problems made the seminar an excellent school for young researchers. From the very start the seminar became quite an event and was commonly recognized as one of the most popular seminars on the function theory in the former Soviet Union – not only Georgian mathematicians but also specialists in the function theory from other Soviet republics and foreign countries used to come to Tbilisi to take part in its work. O. Tsereteli was convinced that science was a sacred matter and deserved disinterested service.

There are several generations of scientists who grew up under the direct guidance of O. Tsereteli. His scientific results stimulated the formation of the mathematical school on the function theory in Georgia. He will always remain the deeply honored bright personality for his colleagues and the wise teacher for his former students and participants of the seminar who are now working as researchers and teachers at research institutions and universities of Georgia and in the leading research centers of various countries of the world.

Shadiman Kheladze, Grigor Barsegyan, Leri Gogoladze,

Merab Gvaradze, Aleksandr Talalyan, Nicholas Vakhania

Sergo Topuria

(To the 80th Birthday Anniversary)

This year Professor Sergo Topuria, Honored Scientist, Doctor of Physical and Mathematical Sciences, would have been 80 years old and marked 55 years of his scientific and pedagogical activities. He was a prominent Georgian mathematician, a remarkable representative of the Georgian mathematical school, an excellent teacher and educator. He was one of those Georgian mathematicians who in the 60s of the past century made the first bold steps in mathematical research and thereby won general recognition and respect.

Sergo Topuria was born on December 27, 1931 and died this year, on March 15th. He was a person with lofty ideals and adhered to high moral and civic principles. His path in life, high professional competence and general public recognition are an evidence of his faithful service to the country and people.

The results obtained by S. Topuria reflect the onward development of the studies carried out by such famous mathematicians as B. Luzin, I. Privalov, A. Zygmund, G. Hardy, I. Marcinkiewisz, I. Stein, V. Shapiro, E. Gobson and others. He was deeply respected and his scientific works were highly appreciated by modern well-known mathematicians S. Nikolski, P. Ulyanov, S. Stechkin, N. Muskhelishvili, I. Vekua and others.

In 1953, Sergo Topuria graduated with honors the physical and mathematical faculty of Sukhumi Pedagogical Institute and continued his education as a post-graduate student under the supervision of well-known Georgian mathematician, Corresponding Member of the Georgian Academy of Sciences, Professor Vladimer Chelidze. In 1959, he defended his Master's thesis "On Some Tauber Type Theorems for Multiple Series and Multiple Integrals" at A. Razmadze Mathematical Institute of the Georgian Academy of Sciences.

In 1960, Sergo Topuria was elected head of the higher mathematics and theoretical mechanics chair of the Georgian Subtropical Agriculture Institute in the city of Sukhumi. In 1966, he moved to Tbilisi and took up work as docent at higher mathematics chair no. 3 at Georgian Polytechnical Institute. From 1967 to the last day of his life he headed higher mathematics chair no. 63 at the above-mentioned institute, which later was reorganized into Georgian Technical University. Due to his outstanding organizational capacity and strenuous efforts, in the course of many years this chair had been one of the leading chairs of Georgian Technical University. Concurrently, for many years he was delivering a special course of lectures for students of the mechanical-and-mathematical faculty of I. Javakhishvili Tbilisi State University. Along with teaching, organizational and social activities, he carried out scientific research with enthusiasm typical of him and, in 1973, he defended his Ph.D. thesis on the topic "Some Problems of the Boundary Properties of Harmonic Functions, the Theory of Fourier-Laplace and Fourier multiple trigonometric series".

In 1975, the title of professor was conferred on Sergo Topuria and, in 1978, the title of an Honored Scientist.

Sergo Topuria was known as a highly skilled specialist in the function theory. Comprehensive studies were carried out by him in multidimensional harmonic analysis. His scientific results are related to the following main directions: summability of multiple trigonometric series for various types of convergence; Tauber type theorems for multiple series and integrals; summation of Fourier-Laplace and differentiated Fourier-Laplace series by the linear method; representation of various measurable and almost everywhere finite functions of many variables by multiple trigonometric series and Laplace series; the boundary properties of harmonic functions in multidimensional domains.

Sergo Topuria established an analogue of S. Bernstein's inequality for a spherical polynomial in the space $L_p(S^3)$, $1 < p < \infty$.

Sergo Topuria carried out a detailed study of the question of summability of Fourier-Laplace series and their differentiated series (in terms of various types of convergence). In particular, he proved analogues of the theorems of P. Siolini, I. Stein and G. Sunuochi on almost everywhere convergence of the Cesaro means (C,α) of Fourier-Laplace series for a critical exponent.

Furthermore, he proved the theorems on the summability in the sense of the Abel and (C,α) methods of Fourier-Laplace series and their differentiated series in the case where the angular part of the Laplace operator written in polar coordinates is used as a differentiation operator. He also obtained the analogues of the theorems of G. Riesz and I. Stein on convergence the metric of a space of Cesaro means for a critical exponent when $1 < p < \infty$. He also studied the question of strong summability of Fourier-Laplace series. Here he obtained the theorems which are specific analogues of the Hardy-Littlewood , Marcinkiewicz and Stein theorems.

S. Topuria established that if $f(x)$ is a measurable and a.e.finite function defined on the spherical surface, then there exists a Laplace series which is summable almost everywhere to a function $f(x)$ by the A* method as well as by the Rudin-Riemann method (this is an analogue of Luzin's theorem).

S. Topuria obtained quite a number of results related to the boundary properties of the differentiated Poisson integral for various domains (circle, ball, half-plane, half-space, bicylinder) and its application. He solved the Dirichlet problem for the ball and the half-space in the case where a boundary function is measurable and finite a.e., i.e. in a completely general case, and also he proved the existence of an angular boundary value of a harmonic function with the so-called B property in the ball.

S. Topuria was the author of over 100 scientific works, including 3 monographs. Over 30 manuals and hand-books were written by him and published under his supervision, of which the higher mathematics manual in 5 volumes is especially noteworthy. These volumes make up a complete course on higher mathematics (the theory and a collection of problems). Mention should also be made of the manual in 2 volumes for university entrants. This two-volume manual has already run through 5 editions, has remained very popular for nearly 30 years and is successfully used in senior classes in secondary schools.

S. Topuria's scientific papers and manuals are distinguished by a simple and clear presentation of facts and ideas, refined argumentation, a multitude of original examples and counter-examples – these qualities produce a favorable impression on readers.

Vladimir Khocholava

On the Summability of Fourier Series by the Generalized Cesáro (C,α_n)-Means

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One of the most general test of convergence at a point of Fourier Series was given by Lebesgue [1] in 1905. In 1930 Gergen [2] improved the Lebesgue test. Later Zhizhiashvili [3] (see, also, [4]) proved analogous of Lebesgue theorem for Cesaŕo (C, α_n)-means. In the present research analogous of Lebesgue-Gergen convergence test for generalized Cesaŕo (C, α_n)-means $(-1 < \alpha_n < 0)$ of Fourier trigonometric series is given.

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On Fourier Trigonometric Series

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Let us imply $T = [-\pi; \pi]$ and the functions $f : R {\to} R$ are 2π-periodics. If the function $f{\in}L(T)$, then, as a rule, by symbol $\sigma[f]$ and $\overline{\sigma}(f)$ we denote correspondingly Fourier trigonometric series of function f and conjugate series. By $\sigma_n^\alpha(x, f)$ and $t_n^\alpha(x, f)$ we denote Cesaro means of $\sigma[f]$ and $\overline{\sigma}(f)$ of the order $\alpha > -1$.

We imply that

$$\overline{f}_n(x) = -\frac{1}{2\pi} \int_{\frac{\pi}{n}}^{\pi} [f(x+t) - f(x-t)] \operatorname{ctg} \frac{t}{2} \, dt, \quad n > 1.$$

Let $p \in [1; +\infty]$ be a certain number. If $f \in L^p(T)$, $(L^\infty(T) = c(T)$, $\|f\|_c = \|f\|_\infty = \sup\limits_{x \in T} |f(x)|)$ then by $\omega(\delta, f)_p$, $0 < \delta \leq \pi$ we denote the L^p-modulus of f continuity.

In future we will imply that

$$g(n,f) \equiv \frac{1}{n} \int_{\frac{1}{n}}^{\pi} \frac{\omega(t,f)_c}{t^2}\, dt.$$

The following Theorems are true.

Theorem 1. a) *Let $\alpha \in (0;1)$ is any number, $p \in (1;+\infty)$ and $\alpha p > 1$. If the function $f \in C(T)$, then*

$$\|\sigma_n^{-\alpha}(f) - f\|_c \le A(p) n^{\alpha} \omega\left(\frac{1}{n}, f\right)_p + A(\alpha) g(n,f).$$

b) *If $p \in (1;+\infty)$ and $f \in C(T)$, then*

$$\|\sigma^{-\frac{1}{p}}(f) - f\|_c \le A(p) \left[n^{\frac{1}{p}} (\ln n)^{1-\frac{1}{p}} \omega\left(\frac{1}{n}, f\right)_p + g(n,f) \right].$$

Theorem 2. a) *Let $\alpha \in (0;1)$ is any number. If the $f \in C(T)$ and $\alpha p > 1$, then*

$$\|t_n^{-\alpha}(f) - \overline{f}_n\|_c \le A(p,\alpha) n^{\alpha} \omega\left(\frac{1}{n}, f\right)_p + A(\alpha) g(n,f).$$

b) *If $p \in (1;\infty)$ is any number and $f \in C(T)$, then*

$$\|t_n^{-\frac{1}{p}}(f) - \overline{f}_n\|_c \le A(p) \left[n^{\frac{1}{p}} (\ln n)^{1-\frac{1}{p}} \omega\left(\frac{1}{n}, f\right)_p + g(n,f) \right].$$

Unconditional Convergence of Wavelet Expansion on the Cantor Dyadic Group

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The MRA theory for Cantor dyadic group was developed by W. Lang and later by Yu. Farkov and V. Protassov, who interpreted it as theory of wavelets on the half-line with the dyadic addition. We prove that wavelet expansions on the Cantor dyadic group G converge unconditianally in the dyadic Hardy space $H_1(G)$. We will do it for wavelets satisfying the regularity condition of Hölder–Lipshitz type.

Continuous Functions and their Fourier Coefficients with Respect to General Orthonormal Systems

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It is well known that, for any continuous function there exists the orthonormal system(ONS), such that its coefficients with respect to these ONS have arbitrarily slow order of tending to zero.

In the paper the conditions will be given, which should be satisfied by ONS so that the Fourier coefficients of continuous functions can be bounded from above by the modulus of continuity or modulus of smoothness of these functions.

ლაპლასის გარდაქმნების შესახებ გაფართოებულ ხუთგანზომილებიან ევკლიდურ სივრცეში

რაჟდენ ხაბურძანია

აკაკი წერეთლის სახელმწიფო უნივერსიტეტი

ქუთაისი, საქართველო

განიხილება ოთხგანზომილებიანი ზედაპირი V_4 გაფართოებულ ხუთგანზომილებიან ევკლიდურ $\overline{E}_5 = E_5 \cup E_4^*$ სივრცეში, სადაც E_4^*-ელიფსური S_4 სივრცის სტრუქტურის მქონე არასაკუთრივი ჰიპერსიბრტყეა.

V_4 ზედაპირს მიუერთდება მოძრავი რეპერი $R = \{A, A_i, A_5\}$ $(i, j = 1, 2, 3, 4)$, $A \in V_4$, $A_i \in T_4(A)$ ($T_4(A)$-მხები 4-სიბრტყეა V_4 ზედაპირისადმი A წერტილში), $\{A_i, A_5\} \subset E_4^*$.

ვთქვათ, V_4 ზედაპირზე მოცემულია $(\omega^1, \omega^2, \omega^3, \omega^4)$ წირთა შეუღლებული ბადე, ხოლო R რეპერი აგებულია ამ ბადის წირებისადმი მხებებზე. როცა A წერტილი აღწერს V_4 ზედაპირს, მაშინ A_i, A_5 წერტილები E_4^* სივრცეში, ზოგად შემთხვევაში, აღწერენ 4 განზომილებიან (A_i), (A_5) ზედაპირებს, რომლებზეც ბუნებრივად აღმოცენდებიან ოთხგანზომილებიანი $(\omega^1, \omega^2, \omega^3, \omega^4)$ წირთა ბრტყელი ბადეები.

ამ ბადის წირებისადმი თითოეულ (AA_i) მხებზე არსებობს სამი ისეთი წერტილი F_i^j $(i \neq j)$, რომ როცა A წერტილი გადაადგილდება (AA_j) მიმართულებით $d\vec{F_i}$ ეკუთვნის $(AA_1 \cdots A_{j-1} A_{j+1} \cdots A_5)$ ჰიპერსიბრტყეს. A წერტილის მიერ V_4 ზედაპირის აღწერისას, თითოეული F_i^j წერტილი აღწერს 4-ზედაპირს (F_i^j) -ს, რომლებზეც ასევე აღმოცენდე-ბიან წირთა $(\omega^1, \omega^2, \omega^3, \omega^4)$ ბადეები, ამასთან (AA_i) წრფე თითოეულ (F_i^j) ზედაპირს ეხება F_i^j წერტილში.

V_4 ზედაპირიდან (F_i^j) ზედაპირზე გადასვლას ეწოდებათ ლაპლასის გარდაქმნები, ანუ შესაბამის შეუღლებულ ბადეთა ლაპლასის გარდაქმნები.

დამტკიცებულია, რომ (A_i) ზედაპირებზე $(\omega^1, \omega^2, \omega^3, \omega^4)$ წირთა ბადეები წარმოადგენენ ლაპლასის გარდაქმნებს ერთიმეორეში.

On the everywhere Divergence of Double Fourier–Walsh Series

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Let f be a measurable function of two variables, which is integrable on $T^2 = [0,1)^2$ and periodic (with period 1) with respect to each variable;

$$\sum_{k,j=0}^{\infty} \widehat{f}(k,j)\omega_k(x)\omega_j(y) \tag{1}$$

be a double Fourier–Walsh–Paley series of the function f and

$$S_{m,n}(f)(x,y) = \sum_{k=0}^{m}\sum_{j=0}^{n} \widehat{f}(k,j)\omega_k(x)\omega_j(y),$$

$(m,n) \in \mathbb{N}_0^2$ ($\mathbb{N}_0 = \{0,1,2,\dots\}$), be a right partial sum of series (1).

Denote by $\mathbb{B}$ the family of sets B from $\mathbb{N}_0^2$ satisfying the condition: for any $k>0$ there exist natural numbers m and n such that $\big([0,k]^2 + (m,n)\big) \cap \mathbb{N}_0^2 \subset B$.

Definition. We say that the double series $\sum\limits_{k,j=0}^{\infty} a_{k,j}$ converges in the sense of B if there exists the limit

$$\lim_{\substack{m,n\to\infty \\ (m,n)\in B}} \sum_{\substack{0\le k\le m \\ 0\le j\le n}} a_{k,j}$$

and it is finite. Otherwise we say that the series diverges in the sense of B.

Theorem. *For any $B \in \mathbb{B}$ there exists a bounded function $f \in L^\infty(T^2)$ such that $\widehat{f}(m,n) = 0$ for $(m,n) \notin B$ and the double Fourier–Walsh–Paley series unboundedly diverges everywhere in the sense of B.*

Localization Principle for the Summation of Fourier–Laplace Series by the (C,α)-Method

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Let R^k be a k-dimensional Euclidean space, $S^{k-1}=\{x:\ x\in R^k;\ |x|=1\}$ be the unit sphere.

If $f\in L(S^{k-1})$, $k\geq 3$, then the series $S(f;x)=\sum\limits_{n=0}^{\infty}Y_n^{\lambda}(f;x)$ is called the Fourier Laplace series of f; here $Y_n^{\lambda}(f;x)$ is a hyperspherical harmonic of f of order n, $\lambda=\frac{k-2}{2}$ is a critical exponent. The Cesáro (C,α)-means of the series $S(f;x)$ are defined as follows

$$\sigma_{\nu}^{\lambda,\alpha}(f;x)=\frac{1}{A_n^{\alpha}}\sum_{\nu=0}^{n}A_{n-\nu}^{\alpha-1}S_{\nu}^{\lambda}(f;x),$$

where $S_{\nu}^{\lambda}(f;x)$ is a partial sum of the series $S(f;x)$.

It is well known that if $\alpha\geq 2\lambda$, then the localization principle (in the usual sense) holds for the (C,α)-means of a Fourier–Laplace series.

If for $f\in L(S^{k-1})$ the condition $f(x)=0$ implies that $\lim\limits_{n\to\infty}\sigma_n^{\lambda,\alpha}(f;x_0)=0$ in some neighborhood of the point x_0 and its diametrically opposite point, then we say that the localization principle holds in a weak sense.

It is well-known that if $\lambda<\alpha<2\lambda$, then for the (C,α)-means of a Fourier-Laplace series the localization principle holds in a weak sense, but does not hold in the usual sense.

The following is proved: for the Cesáro (C,α)-means of a Fourier–Laplace series the localization principle holds in a weak sense for the critical exponent $\alpha=\lambda$.

On Measurability of Functions and Extensions of Measures

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Let E_1 be a basic set, (E_2,S_2) be a measurable space and let M be a class of measures on E_1(we assume, that the domains of measures from M are various sigma-algebras of subsets of E_1). We shall say that a function $f:E_1\to(E_2,S_2)$ is relatively measurable with respect to M if there exists at least one measure $\mu\in M$ such that f is measurable with respect to μ (see [1, 2]).

Let (E_1,S_1,μ_1) and (E_2,S_2,μ_2) be measurable spaces equipped with sigma-finite measures. We Recall that a graph $\Gamma\subset E_1\times E_2$ is $(\mu_1\times\mu_2)$-thick in $E_1\times E_2$ if for each $(\mu_1\times\mu_2)$-measurable set $Z\subset E_1\times E_2$ with $(\mu_1\times\mu_2)(Z)>0$, we have $\Gamma\cap Z\neq 0$ (see, [2, 3]]).

Theorem 1. *Let E_1 be a set equipped with a sigma-finite measure μ and let $f:E_1\to E_2$ be a function satisfying the following condition: there exists a probability measure μ_2 on $ran(f)$*

such that the graph of f is a ($\mu_1 \times \mu_2$-thick of the product set $E_1 \times ran(f)$. Then there exists the measure μ' such that:

1) *μ' is measure extending μ_1;*
2) *f is measurable with respect to μ'.*

Theorem 2. *Let (E_1, S_1, μ_1) and (E_2, S_2, μ_2) be two uncountable sets equipped with sigma-finite measures and $card(E_1) = card(E_2) = \alpha$. Suppose that there exists a family $\{Z_\xi : \xi < \alpha\}$ of subsets of $E_1 \times E_2$ satisfying the following conditions:*

(1) *for any $(\mu_1 \times \mu_2)$-measurable set $Z \subset E_1 \times E_2$, with $(\mu_1 \times \mu_2)(Z) > 0$, there is an index $\xi < \alpha$ such that $Z_\xi \subset Z$;*

(2) *for any set $X \subset E_1$ with $card(X) < \alpha$ and any Z_ξ $(\xi < \alpha)$, we have $Z_\xi \setminus (X \times E_2) \neq \emptyset$.*

Then there exists a function $f : E_1 \to E_2$ whose graph is $(\mu_1 \times \mu_2)$-thick in $E_1 \times E_2$. Consequently, f is relatively measurable with respect to the class $M(\mu_1)$.

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Lebesgue Constants of the Fourier Series with Respect to the System of Generalized Spherical Functions

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Lebesgue constant of (C, α) means of the Fourier series with respect to the system of generalized spherical functions is denoted by L_n^α for $n \in N$, $\alpha \in R$.

The following estimations are valid:

(i) $C_1 n^{\frac{1}{2}-\alpha} < L_n^\alpha < C_2 n^{\frac{1}{2}-\alpha}$ for $\alpha < \frac{1}{2}$.

(ii) $C_1 \ln n < L_n^\alpha < C_2 \ln n$ for $\alpha = \frac{1}{2}$.

(iii) $0 < L_n^\alpha < C$, for $\alpha > \frac{1}{2}$.

Approximate Properties of the Cezaro Means of Trigonometric Fourier Series of the Metric of Space L^p

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Let us suppose, that $T = [-\pi;\pi]$, $[a;b] \subset T$, $b-a>0$, and the functions $f: R \to R$ are periodic with phase 2π, where $R=(-\infty;\infty)$. As a rule, if $f \in L(T)$, then the symbol $\sigma[f]$ designates trigonometric Fourier Series of function f.

The symbol $\sigma_n^\alpha(x,f)$ designates the mean order α Cezaro of series $\sigma[f]$. Let's take advantage of the known labels

$$\phi(x,t) = f(x+t) + f(x-t) - 2f(x).$$

As to entry $\omega \in \Phi$ (see [1]), it designates that the function ω is defined on a segment $[0;\pi]$ and has following properties:

1. It is continuous on $[0;\pi]$;
2. $\omega \uparrow$, $t\uparrow$;
3. $\omega(0) = 0$;
4. $\omega(t) > 0$, $0 < t \le \pi$.

Below $A(f,p), A(f,p,\alpha,\eta),\ldots$ are positive, final values of which depend only on the indicated parameters. We present approximate properties of the Cezaro means of series $\sigma[f]$ from the view point of the metric of space $L^p([a;b])$, $1 \le p < +\infty$.

Theorem. L*et* $p \in [1;+\infty)$, $\omega \in \Phi$ *and* $\alpha > 1$. *If*

$$\left\{ \int_a^b \left[\int_0^t |\phi(x,s)|\,ds \right]^p dx \right\}^{\frac{1}{p}} \le A(f,p)t\omega(t), \quad 0 < t \le \eta \le \pi,$$

then

$$\left\{ \int_a^b \left| \sigma_n^\alpha(x,f) - f(x) \right|^p dx \right\}^{\frac{1}{p}} \le A(f,p,\alpha,\eta)\,\frac{1}{n} \int_{\frac{1}{n}}^{\eta} \frac{\omega(t)}{t^2}\,dt.$$

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On the Rearranged Block-Orthonormal and Block-Independent Systems

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Block-orthonormal systems were considered first by V. F. Gaposhkin.

Definition. Let $\{N_k\}$ be an increasing sequence of natural numbers, $\Delta_k = (N_k, N_{k+1}]$, $(k = 1, 2, \dots)$ and $\{\varphi_n\}$ be a system of functions from $L^2(0,1)$. The system $\{\varphi_n\}$ will be called a Δ_k-orthonormal system if:

1) $\|\varphi_n\|_2 = 1$, $n = 1, 2, \dots$;

2) $(\varphi_i, \varphi_j) = 0$ for $i, j \in \Delta_k$, $i \neq j$, $k \geq 1$.

The system $\{\varphi_n\}$ will be called a Δ_k-independent system, if for each $k = 1, 2, \dots$ the functions $\{\varphi_n\}_{n \in \Delta_k}$ are independent.

Gaposhkin proved theorem on almost everywhere convergence of series with respect to block-orthonormal and block-independent systems. Also he proved that the strong law of large numbers are valid for block-orthonormal systems in certain conditions.

We obtained analogous results for rearranged systems. In particular, we established that if for the sequence $\{N_k\}$ the condition $\overline{\lim\limits_{k\to\infty}}(N_{k+1} - N_k) = \infty$ is fulfilled and $\Delta_k = (N_k, N_{k+1}]$, then there exists a rearrangement of functions of Δ_k-orthonormal Δ_k independent system $\{\varphi_n\}$ such that the condition $\sum\limits_{n=1}^{\infty} a_n^2 w(n) < \infty$, $(w(n) \uparrow \infty)$ guarantees the convergence almost everywhere of series with respect to rearranged system $\{\varphi_{\nu_n}\}$. And if $\{\varphi_n\}$ is a Δ_k-orthonormal system, then there exists a rearrangement of $\{\varphi_n\}$, such that for rearranged system $\{\varphi_{\nu_n}\}$ the strong law of large numbers holds true, that is

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \varphi_{\nu_k}(x) = 0$$

almost everywhere.

Maximal Operators in Variable Lebesgue Spaces

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We study the questions on the boundedness and the validity of modular inequality in variable Lebesgue spaces for maximal operators corresponding to homothety invariant and translation invariant bases. The obtained results extend the previous ones by T. Kopaliani and A. Lerner. The results concerning the boundedness question are joint with T. Kopaliani.

On Representation of Functions

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Let consider a trigonometric series

$$\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx, \tag{1}$$

such that $\sum\limits_{n=1}^{\infty} a_n^2+b_n^2>0$.

Partial sums of series (1) we denote by $S_m(x)$.

Suppose that

$$\{g_k(t)\} \tag{2}$$

is a sequence of functions acting from N to N, where N denotes the set of all natural numbers. So

$$g_k : N \to N \quad (k=1,2,3,\ldots).$$

Then the following statements hold.

Theorem 1. *For any continuous on $(-\pi,\pi)$ function $f(x)$ and for any sequence (2) there exist a series (1) and a sequence of natural numbers $\{m_k\}$, such that the conjunction of the following two assertions is satisfied:*

1) $\lim\limits_{k\to\infty} S_{m_k}(x)=f(x)$ *for any $\delta>0$ uniformly on $[-\pi+\delta,\pi-\delta]$;*

2) $a_n=b_n=0$ *if $m_k<n\le m_k+g_k(m_k)$ $(k=1,2,3,\ldots)$.*

Theorem 2. *For any Lebesgue measurable on $(-\pi,\pi)$ function $f(x)$ and for any sequence (2) there exist a series (1) and a sequence of natural numbers $\{m_k\}$, such that the conjunction of the following two assertions is satisfied:*

1) $\lim\limits_{k\to\infty} S_{m_k}(x)=f(x)$ *almost everywhere on $(-\pi,\pi)$;*

2) $a_n=b_n=0$, *if $m_k<n\le m_k+g_k(m_k)$ $(k=1,2,\ldots)$.*

Different Classes of Functions and Fourier Series According to Generalized Spherical Functions

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The conditions have been established which must be satisfied by multipliers of Fourier's coefficients so that as a result of the corresponding changes Fourier's series of the function of one class would be transformed into fourier's series of the function of another class.

Hardy Operator in Grand Lebesgue Spaces

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Let $\Omega \subseteq R^n$, $w(x)$ weight on Ω, $1 < p < \infty$, $\theta > 0$. Weighted grand Lebesque space $\mathscr{L}_\alpha^{p),\theta}(\Omega, w)$ is defined as the set of functions f with the finite norm

$$\|f\|_{\mathscr{L}_\alpha^{p),\theta}(\Omega,w)} = \sup_{0<\varepsilon<p-1} \bigg(\varepsilon^\theta \int\limits_\Omega |f(x)|^{p-\varepsilon} w(x)^{1+\alpha\varepsilon}\, dx\bigg)^{\frac{1}{p-\varepsilon}},$$

where

$$\int\limits_\Omega w(x)^{1+\alpha p}\, dx < \infty.$$

The main statement. If operator Hardy

$$(Hf)(x) = \frac{1}{x} \int\limits_0^x f(t)\, dt, \quad x \in R_+$$

bounded from $L^p(R_+, v)$ to $L^p(R_+, w)$ and fanctions $w(x)x^\nu$ and $v(x)x^\nu$ is equivalent to a non-decreasing function on R_+ for some $\nu \geq 0$, then

$$\|Hf\|_{\mathscr{L}_\alpha^{p),\theta}(R_+,w)} \leq C\|f\|_{\mathscr{L}_\alpha^{p),\theta}(R_+,v)}$$

for every number α:

$$\int\limits_0^\infty w(x)^{1+\alpha p}\, dx < \infty, \quad \int\limits_0^\infty v(x)^{1+\alpha p}\, dx < \infty.$$

Complex Analysis

Professor David Kveselava

(To the 100th Birthday Anniversary)

Short biography

The childhood of David Kveselava went off in the village of Letsitskhvaie not far from Martvili (Georgia). He was born to the family of local villager Aleqsandre Kveselava on August 25, 1911. The family was close-knit where both national traditions and education were respected. Along with other virtues, the parents inculcated in their sons, David and Michael[1], the love for native land, for dignity and diligence. Besides, children inherited naturally the thirst to knowledge. Actually in those early years their future successful creative stile of life was initiated.

In 1930 to give children a better education Kveselavas moved to Tbilisi. A year later David graduated from a secondary school and continued his education at the faculty of Physics and Mathematics of Tbilisi State University. His diligence and skills were not left unnoticed. In 1937, as a high achiever student and diplomant, he called the attention of Niko Muskhelishvili who was at the time Chair of the State Examination Council. Due to Muskhelishvili's recommendation, David started his research activity under the supervision of Michael Alekseevich Lavrent'ev, the prominent Soviet mathematician and mechanic.

After graduating from the university David entered the graduate school (aspirantura) at Institute of Mathematics of the Georgian Branch of Academy of Sciences of USSR. (At that time Georgian Academy of Sciences did not exist independently so far.) The years of graduate studies were both fruitful and interesting. His research activity was very intense, by itself it required a lot of energy and patience. In spite of such a big load, he always found time and attention for an active pedagogical activity. From the very first stage of his graduate studies he taught at Tbilisi Institute of Railway Engineering as well as Tbilisi State University and Gogebashvili Telavi Pedagogical Institute where he headed the Mathematical Department. It was the time when David was being formed as a professional teacher and lecturer. Along with his unique manner of confident explanation of sometimes very complicated topics. And his laconic and clear language (both Georgian and Russian). As time passed, his pedagogical skills and style got perfect. David's numerous students and listeners will remember forever his colorful speech and vivid image. In 1940 the All-Union Supreme Certifying Commission conferred David the rank of Associate Professor (Dozent) for his successful pedagogical work. This was the year when the first triennial of David's creative work was summed up: he has completed his PHD (candidate dissertation) and defended it successfully. The title of the dissertation was "On the theory of conformal mappings". The notion of "conformal" in the late Latin means "like" or "similar". For example, the mapping of a sphere onto the plane is conformal, if the angle between two arbitrary directions emanating from any point of the sphere transforms into the same angle (in value) on the plane. Such mappings are somewhat related to creation of geographic maps, so that they didn't lose their meaning in cartography (and not only in cartography). If a plane is being transformed onto the plane, then the mapping is conformal, if an additional condition is satisfied. Namely, a circle should be transformed onto a circle, and the circumference should be transformed onto a circumference. However here by circles and circumferences we mean not just usual ones, but those whose radii are infinitely small. It turned out that such mappings are nothing but analytic functions of a complex argument. A function is analytic in a neighborhood of a point, if it can be represented by the sum of infinitely many powers of the argument multiplied by a corresponding complex coefficients. And therefore, it naturally arose the necessity of comparison of the properties of conformal mappings on the one hand and the analytic functions on the other. David Kveselava faced the problem of estimation of parameters of a mapping when the difference between the image and pre-image domains was insignificant.

[1]Michael Kveselava (1913–1981) – accomplished philologist (German studies), writer and philosopher; he was the only Georgian who attended the Nuremberg Trials where served as a translator. He wrote books: "Faustian Paradigms" (1961), "Adam Mickiewicz" (1965), "A hundred and fifty days" (about World War II and the Nuremberg Trials, 1967-1971), "Poetical Integrals" (1977).

The problem was studied before by well-known experts. Professor Lavrent'ev wanted his student to solve out the problem in much greater generality. David jointly with his adviser succeeded in significant generalization of results of prominent Russian mathematician A.Ostrogradskii. David's contribution was so important and interesting that Michael Alekseevich decided that the paper should have been signed solely by David. After the successful defense of the dissertation David was left at Institute of Mathematics. His first position there was that of Academic Secretary of the institute. However shortly he was appointed to the position of Senior Staff Scientist, and in 1941 he got the corresponding official rank.

When it became clear that David for fair possesses a talent of a scientific organizer, he was appointed to the position of Academic Secretary of Department of Natural Sciences of the Georgian Academy of Sciences. The administrative activity never hampered his research work.

In 1952 David defended his doctoral dissertation at Moscow Steklov Institute of Mathematics, the leading Soviet Mathematics Institute. The title of the dissertation was "Some boundary problems of the function theory and singular integral equations". The decision of unprejudiced Council was unanimous in favor of David. Meanwhile David continued his pedagogical activity. He was an extraordinary lecturer at the university. Many remember his calm, but sometimes hot-tempered manner of teaching. Shortly, 2 years later in 1954 he got the official Professor's rank.

October 8 of 1956 is a special day in David Kveselava's biography: he was appointed the director of newly created Computer Center of Academy of Sciences of Georgia. It was the time when science, engineering and technology started developing at a briskly pace. All the related branches, computer engineering, computer mathematics and numerical analysis were facing new problems to handle the development. To solve out the emerged problems, in all developed nations new research centers and institutions were created. The Soviet Union of those days was not an exception. Under these incredible conditions everybody had considerations of his own how to handle this new and unusual situation. Many things depended on the talent of a scientific organizer and managerial abilities of the principal. David Kveselava was given both from heaven. He gathered a strong team of young skilled scientific-technical personnel that created an effectively acting Computer Center. David Kveselava was the unchallenged leader of the institute till the end of his life. David Kveselava passed away on November 6, 1978. Besides huge organizational contribution, his scientific and research legacy consists of many scientific works and a monograph. The ideas and traditions laid by him are still alive: the Niko Muskhelishvili Institute of Computational Mathematics continues its fruitful work as a part of Georgian Technical University. The scientific community always highly appreciated David Kveselava's scientific and pedagogical merits. The title "Honored Georgian Scientist" as well as other awards were conferred on him. However, the main award is probably the great love and respect of his pupils and collaborators who will remember him forever.

Short review of David Kveselava's scientific legacy

David always was fair and favorable to every person irrespectively of his rank or age, in both personal and business relationship. Everything this along with his natural skills of the teamwork left its mark on his creative work. A large part of his joint works with colleagues of him (with his teachers or students among them) represent excellent examples of scientific collaboration.

His results worked out in collaboration with his associates or solely by him belong to several directions which are closely related to each other. As we have mentioned above, these include theories of conformal mappings and analytic functions of complex variable, as well as theory of singular integral equations and systems of integral equations. The problems arose in these directions were prompted by the practical need, and many of them were of theoretical importance at that time. Nowadays these directions belong to classics, and solid part of them belongs to David Kveselava. The directions below are so interlaced, that their separation is a pretty difficult problem. Nevertheless, in order to make a short review of Professor Kveselava's legacy transparent, we split it into several parts. It is natural to start with his first results.

1. Methods of approximations in conformal mappings.

Here we speak of conformal mappings f and f_1 of mutually close simply connected domains D and D_1 respectively, given on the complex plane $z = x + iy$, onto the circle $|w| < 1$ and the estimation of $|f - f_1|$, the modulus of the difference of the mappings in question. In the joint paper [1] with academician Lavrent'ev the domain D of the complex plane z contains the origin, and its boundary lies completely in the ring $\theta < |z| < 1$, $\theta \geq 1/2$. Note that this sort of conditions does not cause a significant loss of generality. The parameter θ is not specified so far; the smaller the difference $1 - \theta$ is, the closer is the domain D to the unit circle D_1 of the z-plane. Obviously, the identity function $w = f_1(z) = z$ maps the unit circle D_1 of the z-plane onto the unit circle $|w| < 1$ of the w-plane. And if the function $w = f(z)$ also maps the domain D onto $|w| < 1$, then for any z with $|z| \leq \theta$ the following estimation holds

$$|f(z) - f_1(z)| = |f(z) - z| \leq k(1 - \theta) \log \frac{1}{1 - \theta}\,.$$

By means of this inequality the authors also approved analytically that the closer are the domains, the lesser is the difference between the corresponding mappings. Moreover, it turned out that at the neighborhood of the origin the difference is lesser than at a distant points. The coefficient on the left-hand side is an absolute constant. Also, the coefficient is an absolute constant, if the argument is inside the aforementioned circle, i.e.

$$|z| \leq \theta_1\theta$$

where $0 < \theta_1 < 1$. In this case the estimation has the following refined form:

$$|f(z) - z| \leq k(1 - \theta) \log \frac{1}{1 - \theta_1}\,.$$

Here the transforming mapping solely is estimated. Also of interest are results related to inverse mappings when the unit circle is being conformly transformed onto two mutually close simply connected domains. One of the Kveselava's results in this direction sounds as follows: Let D and D_1 be ε-radially close domains such that D_1 is contained in D and let the functions f, $f(0) = 0$ and f_1, $f_1(0) = 0$, transform conformly the circle $|z| < 1$, respectively on the domains D and D_1. Then the following inequality holds

$$|f'(0)| \leq |f_1'(0)| + 4\varepsilon\,.$$

The well-known Lindelof principle deals with the sign of this variation only, and therefore, Kveselava's result represents a significant refinement of the principle.

The last result has an interesting application to the estimation of the modulus of the derivative in the case of an approximate conformal mapping. Let us single out one of them: If the boundary of D belongs to the ring $1 < |w| < 1 + \varepsilon$ and the function f, $f(0) = 0$, transforms conformly the circle $|z| < 1$ onto the domain D, then for $|z| < r_0$, $r_0 \geq 0.1$, we have

$$1 \leq |f(z)| \leq 1 + \varepsilon\,.$$

In the direction of conformal mappings we have also to mention the following Kveselava's theorem: Let D and D_1 be two star domains with respect to the origin $w = 0$ which are radially close. If the functions f, $f(0) = 0$, $f'(0) > 0$, and f_1, $f_1(0) = 0$, $f_1'(0) > 0$, transform the circle $|z| < 1$, conformly onto the domains D and D_1 respectively, then

$$|f(z) - f_1(z)| < M(r)\varepsilon, \ \text{ for } |z| \leq r < 1\,,$$

where $M(r)$ depends only on r. He also has shown that the result cannot be extended to arbitrary Jordan domains.

D. Kveselava also has original and interesting results on the behavior of the derivatives of two conformal mappings on the joint part of their boundaries. We give here one of them. Let D and D_1 be neighboring domains with the right part γ of the joint boundary and let D and D_1 contain points z^0 and z_1^0 respectively. Assume that functions f, $f(z^0)=0$, and f_1, $f_1(z_1^0)=0$, transform conformly the domains D and D_1 onto the circle $|z|<1$. Then for any point t of the arc γ the following inequality holds

$$|f'(t)f_1'(t)| \leq \frac{4}{\rho^2(t;z^0,z_1^0)}\,,$$

where $\rho(t;z^0,z_1^0)$ is the minimum between the distances $\rho(t;z^0)$ and $\rho(t;z_1^0)$ considered respectively in the domains D and D_1.

D. Kveselava has got significant results on conformal mappings by use of methods of the theory of integral equations. For finding needed conformal mappings (for both simply connected and multiply connected canonical domains) he introduced new integral equations. A significant advantage of these equations compared with analogous equations existed before his findings, was the greater possibilities of using numerical methods. D. Kveselava's students have elaborated the methods of approximate solutions to the integral equations in question and estimated the corresponding error. We stress that D. Kveselava has got explicitly the form of the modulus of conformal mappings of doubly-connected domains. In a joint paper with Z. Samsonia he has used this representation to show that for a sufficiently wide class of boundaries of ε-close doubly-connected domains D and D_1 the order of the difference of the comformal moduli equals ε.

2. Boundary problems of the theory of analytic functions.

Linear boundary problems of the theory of analytic functions have been studied by Soviet mathematicians profoundly, and D. Kveselava took an active part in the study. In the joint paper by D. Kveselava and N. Vekua conditions were found under which the Hilbert boundary problem for several unknown functions can be solved out in quadratures. D. Kveselava has found advanced results for the Hilbert boundary problem in the case of a single unknown functions, open contour and discontinuous coefficients. The problem was solved out in several papers jointly with N. Muskhelishvili and included almost completely into his monograph "Singular Integral Equations". One of the most important results of this cycle is the division of the set of solutions by classes and the introduction, in the case of open contour and discontinuous coefficients, of the related fundamental notion, the index of a boundary problem. Their results here are as final and complete as those found before for the case of a closed contour and continuous coefficients. It should be noticed that D. Kveselava has got the solution to the Hilbert boundary problem also for piece-wise meromorphic functions and mutually intersected domains. The last result has many both theoretical and practical applications. However Kveselava's most advanced results are related with shift included boundary problems for analytic functions. In them the boundary conditions represent the linear relations of the boundary values computed at different points of the boundary. Problems of this type go back to Riemann. Namely, he posed the problem of finding an analytic function of a complex variable, if on the boundary of the domain an equation containing the real and imaginary parts of the function is given. Riemann came up with a series of conjectures over solvability of this problem, but he did not give a rigorous proof of the problem. It was Hilbert in the beginning of 20-th century who proved the conjectures in some particular cases. He considered a holomorphic function $\Phi = u + iv$ with the following boundary condition

$$\alpha u + \beta u = \gamma\,. \tag{1}$$

Several works by Hilbert were devoted to this problem. In one of them he showed that this problem can be solved out for the Laplace equation by reduction to two Dirichlet problems. The method was very sophisticated, however the possibilities of its applications are restricted: the problem is solvable completely only for simply connected domains. For problem (1) Hilbert has used the method of integral equations that reduces the problem to a singular integral involving the principle value of the Cauchy integral. However such equations were not studied well at that time. Because of this, while extending the Fredholm's theorems to singular equations, Hilbert got a wrong answer to problem (1).

Nevertheless Hilbert's works were of great importance. For the first time it was shown a close relationship between the boundary problems for analytic functions and singular integral equation with the Cauchy kernel.

Along with problem (1), Hilbert also considered the following problem: find two functions Φ^+ and Φ^- which for a given closed curve L satisfy the condition

$$\Phi^+(t) = G(t)\Phi^-(t) + g(t), \quad t \in L\,, \tag{2}$$

where Φ^+ is holomorphic inside of L, while Φ^- is holomorphic outside of L. Hilbert has solved this problem by means of the methods of integral equations, although he did not get any essential success.

After Hilbert's works the Riemann problem went in two directions: first, the problems posed by Hilbert that found further development with more general domains and coefficients; and there were also attempts of solving more general problems that Hilbert did not consider.

In 1907 Hilbert's student Charles Haseman posed the following problem: find two functions Φ^+ and Φ^- which along a given simple closed curve L satisfy the condition

$$\Phi^+[\alpha(t)] = G(t)\Phi^-(t) + g(t), \quad t \in L\,, \tag{3}$$

where Φ^+ is holomorphic inside of L, while Φ^- is holomorphic outside of L and α is a complex one-to-one function of a variable $t \in L$ which maps bijectively the curve onto itself. If $\alpha(t) = t$, then we obviously get Problem (2).

Problem (1) was generalized by Torsten Carleman, who in 1932 considered the following problem:

$$\Phi^+[\alpha(t)] = G(t)\Phi^+(t) + g(t), \quad t \in L\,, \tag{4}$$

where Φ^+ is holomorphic inside of L, while α, G and g are as in the previous problem.

Along with aforementioned problems, D. Kveselava has considered the following ones:

$$\Phi^+[\alpha(t)] = G(t)\overline{\Phi^-(t)} + g(t), \quad t \in L\,, \tag{5}$$

$$\Phi^+[\alpha(t)] = G(t)\overline{\Phi^+(t)} + g(t), \quad t \in L\,. \tag{6}$$

Let us remark that before Kveselava's works the Hilbert problem (2) was best and completely studied, whereas problems (3)-(6) were investigated pretty superficially. According to academician I. Vekua, D. Kveselava "...succeeded in making an essential step forward in their study. He managed to get results in the same complete form as those that existed in the case of Hilbert problem (2)... I believe, it is necessary to stress that by solving the listed problems D. Kveselava achieved outstanding scientific results. In this direction only Carleman has made a valuable step forward when he constructed simple integral equations for his problem. Whereas D. Kveselava has solved out the most complicated question: he has proved that the Carleman equation, as well as many other integral equations obtained for similar problems are soluble; based on his findings he managed to prove a series of significant theorems for problems (3)-(6)."

These results played an important role in further development of singular integral equation theory related to boundary problems, and became a stimulating factor for other research works in this direction. Many authors have developed methods for generalization of D. Kveselava's theorems. A special attention was paid to the case of unknown systems of functions.

Let us give a result of D. Kveselava, which represents a key for solving problems in the whole area. Let α_+ (α_-) be a differentiable homeomorphism of a simple closed Lyapunov contour L into itself which preserves (changes) the orientation of L. Assume also that $|\alpha_+(t)| > 0$ and

$|\alpha_-(t)| > 0$ for $t \in L$ and α_+ on L satisfies the Holder condition. Then the elementary boundary problems on L

$$\Phi^+[\alpha_+(t)] = \Phi^-(t)\,,$$
$$\Phi^+[\alpha_-(t)] = \Phi^+(t)\,,$$

do not have analytic solutions except constants.

3. Theory of singular integral equations.

The solution of Hilbert boundary problem for an open contour or discontinuous coefficients has given an effective way of construction, in corresponding cases, of the theory of integral equations of the following form (they involve the principal value of the Cauchy integral):

$$\alpha(t)\varphi(t) + \frac{\beta(t)}{\pi i}\int_L \frac{\varphi(\tau)d\tau}{\tau - t} + \frac{1}{\pi i}\int_L K(t,\tau)\varphi(\tau)d\tau = f(t)\,,$$

and, as we have already remarked, the theory was given a completed form, as it was done before for the case of closed contour and continuous coefficients.

In Kveselava's works such a theory has been constructed in the following cases:

(i) The curve L consists of finitely many detached open smooth arcs, and the functions α, β, K belong to the Holder class;

(ii) The curve L consists of finitely many detached closed smooth contours, and the functions α, β, K satisfy the Holder condition everywhere on L except for the finitely many points of simple discontinuity;

(iii) The curve L consists of finitely many closed and open piece-wise smooth contours having finitely many intersections, and the functions α, β, K belong to the Holder class.

Bringing in the affiliated classes of solutions as well as the notion of the corresponding indices in the aforementioned cases admitted to construct a complete theory of such integral equations. Nowadays these results of the singular integral equations are considered as a part of classics.

Vakhtang Kvaratskhelia, Nicholas Vakhania, Jemal Sanikidze,

Duglas Ugulava, Sergei Chobanyan, Vaja Tarieladze

Boundary Value Problems of the Theory of Generalized Analytic Vectors

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Boundary value problems of the theory of generalized analytic vectors in some functional classes are considered. The necessary and sufficient solvability conditions as well as index formulae are established.

On a General Property of Complex Polynomials

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In this talk we present a general principle in the theory of complex polynomials. Qualitatively speaking we show that any cluster of zeros of a given polynomial (even of very few zeros) attracts, in a sense, zeros of its derivatives. The principle is closely connected with Gauss-Lukas theorem, Crace–Heawood theorem, Walsh's two circle theorem.

Univalence Criteria and Loewner Chains

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In this paper we obtain, by the method of Loewner chains, some sufficient conditions for the analyticity and the univalence of the functions defined by an integral operator. In particular cases, we find the well-known conditions for univalency established by Becker for analytic mappings $f : \mathscr{U} \to \mathbb{C}$. Also, we obtain the corresponding new, useful and simpler conditions for this integral operator.

A New Application of Miller and Mocanu Lemma for Certain Multivalent Functions

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Let $\mathscr{A}_p$ denote the class of functions of the form

$$f(z)=z^p+\sum_{n=p+1}^{\infty}a_nz^n \quad (p\in\mathbb{N}=\{1,2,\dots\})$$

which are analytic in the open unit disk $\mathscr{U}=\{z:\ |z|<1\}$. For $f(z)\in\mathscr{A}_1:=\mathscr{A}$, S. S. Miller and P. T. Mocanu (*J. Math. Anal. Appl.* **65** (1978), 289–305) have shown an interesting lemma which was called Miller and Mocanu lemma. The object of the present paper is to consider a new application of Miller and Mocanu lemma for some functions $f(z)\in\mathscr{A}_p$.

Derivability and Representations of Quaternion Functions

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For the quaternion functions of a quaternion variable we introduce the notion of a **Q**-derivative. In particular, it is proved that the elementary functions introduced by Hamilton possess such a derivative. The **Q**-derivation rules are established, and the necessary and sufficient conditions are found for the existence of a **Q**-derivative. The properties of quaternion functions are investigated with respect to two complex variables, and both their integral representation and their representation by power series are given. The properties of right- and left-regular, according to Fueter, quaternion functions are studied with respect to two complex variables. Formulas are obtained for the coefficients of a unilateral power quaternion.

Generating Triple for Irregular Carlemann–Bers–Vekua System

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We consider the impact generating triple for the following irregular Carlemann–Bers– Vekua system (see I. N. Vekua, Generalized analytic functions. *Nauka, Moscow*, 1988, and V. Jikia, On the classes of functions induced by irregular Carleman–Vekua equations. *Georgian Math. J.* **17** (2010)):

$$w_{\overline{z}} + Aw + B\overline{w} = 0, \tag{1}$$

where $A \in L_p^{loc}(\mathbf{C})$, $B \in L_{p,2}(\mathbf{C})$, $p > 2$.

Let $Q(z)$ be the $\partial_{\overline{z}}$-primitive of $A(z)$ (see V. Jikia, On the classes of functions induced by irregular Carleman–Vekua equations. *Georgian Math. J.* **17** (2010)). By definition, it means that $Q_{\overline{z}} = A(z)$ on $\mathbf{C}$. Let (F_1, G_1) be the generating pair (L. Bers, Theory of pseudoanalytic functions. *NYU*, 1953) of the following regular system:

$$w_{1\overline{z}} + B_1\overline{w}_1 = 0,$$

where $B_1(z) = B(z)e^{2i\operatorname{Im} Q(z)}$. Therefore (F_1, G_1) is a generating pair for the class $u_{p,2}(0, B_1)$ such that a) $F_1, G_1 \in C_{\frac{p-2}{2}}(\mathbf{C})$, b) $F_{1\overline{z}}, G_{1,\overline{z}} \in L_{p,2}(\mathbf{C})$ and c) there exists $K_0 > 0$ such that $\operatorname{Im}(\overline{F}_1(z)G_1(z)) \geq K_0 > 0$. It is known that the functions $F(z) = F_1(z)e^{-Q(z)}$ and $G(z) = G_1(z)e^{-Q(z)}$ are solutions of (1). It is clear that $F, G \in C_{\frac{p-2}{p}}(\mathbf{C})$, $F_{\overline{z}}, G_{\overline{z}} \in L_p^{loc}(\mathbf{C})$ and it is easy to check that $\operatorname{Im}(\overline{F(z)e^{Q(z)}}G(z)e^{Q(z)}) > K_0$, implying that (F, G) is a *generating pair*. From this it follows that the functions F and G satisfy the following identities:

$$F_{\overline{z}} + AF + B\overline{F} = 0, \quad G_{\overline{z}} + AG + B\overline{G} = 0. \tag{2}$$

Consider (2) as a linear system of equations with respect to $A(z)$ and $B(z)$. The determinant of this system is equal to $-2\operatorname{Im}(\overline{F}G) \neq 0$. Therefore,

$$A = \frac{\overline{G}F_{\overline{z}} - \overline{F}G_{\overline{z}}}{G\overline{F} - F\overline{G}}\,, \quad B = \frac{FG_{\overline{z}} - GF_{\overline{z}}}{G\overline{F} - F\overline{G}}\,.$$

We call (F, G, Q) a *generating triple* of the irregular system by analogy with the Bers generating pair of the pseudoanalytic functions (L. Bers, Theory of pseudo-analytic functions. *NYU*, 1953). Using this concept we can define irregular pseudoanalytic functions similar to the regular case.

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On the Two Questions of Lohwater and Piranian

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We are dealing with the following problem: find the necessary and sufficient conditions for subset of the circumference at which points the inner function has no radial limits.

Holomorphic Besov Spaces on the Polydisk

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This work is an introduction of weighted Besov spaces of holomorphic functions on the polydisk. Let U^n be the unit polydisk in $\mathbb{C}^n$, and S be the spaces of functions of regular variation. Let $1 \leq p < \infty$, $\omega_j \in S (1 \leq j \leq n)$ and $f \in H(U^n)$. The function f is said to be an element of the holomorphic Besov spaces $B_p(\omega)$ if

$$\|f\|_{B_p(\omega)} = \int\limits_{U^n} |Df(z)|^p \prod_1^n \frac{\omega_j(1-|z_j|)}{(1-|z_j|^2)^{2-p}}\, dm_{2n}(z) < +\infty,$$

where $dm_{2n}(z)$ is the $2n$-dimensional Lebesgue measure on U^n and D stands for a special fractional derivative of f defined in the paper.

We describe the holomorphic Besov spaces in terms of $L_p(\omega)$ spaces. Moreover, projection theorems and theorems of the existence of a right inverse are proved.

The talk is based on the paper [1].

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Expansion of Validity Conditions of Some Relations for Special Functions

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We have investigated convergence of some definite integrals and special functions in the sense of generalized functions and have shown that under some conditions one can expand their domain.

Namely, recall that the Euler integral of the first kind – well known beta function $B\,(\alpha,\,\beta)$ – is defined as

$$B\,(\alpha,\beta)=\int\limits_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\,,$$
$$\operatorname{Re}(\alpha)>0,\quad \operatorname{Re}(\beta)>0.$$

Theorem. *If* $\operatorname{Re}\alpha=\operatorname{Re}\beta=0$, $\operatorname{Im}\alpha=-\operatorname{Im}\beta=x$, *then the formulae*

$$B(i\,x,-i\,x)=\lim_{\varepsilon\to 0+}B(\varepsilon+ix,\varepsilon-ix)=2\pi\,\delta(x)=\int\limits_0^1 dt\,t^{ix-1}(1-t)^{-ix-1}$$

holds.

A number of new relations follows from this result, some of them useful in avoiding the quantum mechanical difficulties.

Moreover, we expand the area of validity of some relations for the Gauss hypergeometric function $F(a,b;c;\xi)=F\left(\begin{array}{c}a\\c\end{array};b;\xi\right)$.

Theorem. *Well known analytic continuation relations for the Gauss hypergeometric function remain valid when the parameters of the function are* $a=c+n$, $b=c+k$, $n,k\in\mathbb{N}$. *In this case one has*

$$F\left(\begin{array}{c}a\\c\end{array};b;\xi\right)=(-1)^{b-c}\frac{\Gamma(c)\Gamma(1+a-c)}{\Gamma(b)\Gamma(1+a-b)}\,(-\xi)^{-a}F\left(\begin{array}{c}a\\1+a-b\end{array};1+a-c;\xi^{-1}\right),$$
$$=(-1)^{b-c}\frac{\Gamma(c)\Gamma(1+a-c)}{\Gamma(b)\Gamma(1+a-b)}(1-\xi)^{-a}F\left(\begin{array}{c}a\\1+a-b\end{array};c-b;(1-\xi)^{-1}\right),$$
$$\Big(|\arg(-\xi)|<\pi,\quad |\arg(1-\xi)|<\pi,\quad c\neq 0,-1,\dots,\quad a-b=m,\quad m=0,1,\dots\Big).$$

Elliptic Systems in the Plane

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We establish sufficiently general result characterizing the behavior of elliptic (regular and singular) systems in the neighborhood of their singularities.

The Lattice of the Parabolic Non-Automorphism in the Hardy and the Dirichlet Space

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In this lecture we will provide a description of the lattice of a composition operator induced by a parabolic non-automorphism in the Hardy space and the Dirichlet space. We will also discuss about the lattice of these operator in other spaces of analytic functions.

Joint work with M. Ponce-Escudero and S. Shkarin.

On Boundary Properties of Analytic and Harmonic Functions in Unit Ball

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We study boundary properties of analytic and harmonic functions in the unit ball with summable gradient. In particular, we show the existence almost everywhere of angular limits for these functions.

On Witsenhausen–Kalai Constants for a Gaussian Measure on the Infinite-Dimensional Complex-Valued Unite Sphere

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We introduce some notations:

$\mathbf{C}$ – the set of all complex numbers;

$W^2 = \{(z_k)_{k\in N}: \ z_k \in \mathbf{C}, \ k \in N, \sum\limits_{k\in N} |z_k|^2 < +\infty\}$ – an infinite-dimensional separable complex-valued Hilbert space equipped with the usual inner scalar product;

$S^\infty = \{(z_k)_{k\in N}: \ (z_k)_{k\in N} \in W_2 \ \& \ \sum\limits_{k\in N} |z_k|^2 = 1\}$ – the unit sphere in W^2;

$SO(1)$ – the group of all proper Euclidean isometries of R^2 that fix the origin.

Lemma ([3, Theorem 3.8, p. 42]). *There is a $(SO(1))^\infty$-invariant Gaussian measure μ_∞ on S^∞.*

Definition. We say that a non-negative real number c is a Witsenhausen–Kalai constant for the measure μ_∞ if for a μ_∞-measurable set $A \subseteq \mathbb{S}^\infty$ the condition $\mu_\infty(A) > c$ implies that A contains two orthogonal points.

Theorem. *The real number $\frac{1}{2}$ is a Witsenhausen–Kalai constant for the measure μ_∞.*

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Sums of Generalized Harmonic Series for Kids from Five to Fifteen

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We reexamine a remarkable connection, first discovered by Beukers, Kolk and Calabi, between $\zeta(2n)$, the value of the Riemann zeta-function at an even positive integer, and the volume of some $2n$-dimensional polytope. It can be shown that this volume equals to the trace of a compact self-adjoint operator. We provide an explicit expression for the kernel of this operator in terms of Euler polynomials. This explicit expression makes it easy to calculate the volume of the polytope and hence $\zeta(2n)$. In the case of odd positive integers, the expression for the kernel leads us to rediscover an integral representation for $\zeta(2n+1)$, obtained originally by a different method by Cvijović and Klinowski. Finally, we indicate that the origin of the miraculous Beukers–Kolk–Calabi change of variables in the multidimensional integral, which is at the heart of this circle of ideas, can be traced to the amoeba associated with the certain Laurent polynomial.

Almost Periodic Factorization of Matrix Functions

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For many functional classes (such as continuous, piece-wise continuous, meromorphic, belonging to Douglas algebras, etc.) existential factorization results are quantitatively the same in the scalar and in the matrix case. It was expected that this pattern persists for almost periodic (AP) functions as well. However, while the scalar AP case was settled completely in [4, 5], it was discovered later (see, e.g., [6]) that for matrices the situation is completely different: not all invertible AP matrix functions (even in the 2×2 triangular case) are factorable. A number of sufficient factorability conditions has been obtained since then, and the state of the matter as of the turn of the century is described in [1]. In this talk, we will discuss further progress in this area, obtained in particular in [2, 3].

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Approximation of Functions and Measures Defined on a Locally Compact Abelian Groups

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Problems of approximation in some space X of functions or measures defined on a locally compact abelian group G are considered. Let $I \subset R^+$ be an ordered unbounded set and consider a generalized sequence of symmetric neighborhoods K_α of the unity in the dual group $\widehat{G}$ such that $K_\alpha \subset K_\beta$ if $\alpha < \beta$ $(\alpha, \beta \in I)$ and $\bigcup\limits_{\alpha \in I} K_\alpha = \widehat{G}$. As X we take the following spaces: $L^p(G) \equiv L^p(G, \mu)$, $1 \le p < \infty$, the space of integrable on G with respect to the Haar measure μ in the p-th order real or complex valued functions; $L^\infty(G) \equiv L^\infty(G, \mu)$, the space of functions, essentially bounded on G with respect to μ; $M(G)$-the space of bounded regular complex valued Borel measures on G. We consider the following sequence $\{\sigma_{K_\alpha}\}$ of positive operators defined on X:

$$\sigma_{K_\alpha}(f)(g) \equiv (f * V_{K_\alpha})(g), \;\; g \in G, \;\; f \in X,$$

where $V_{K_\alpha}(g) = (mes\, K_\alpha)^{-1}(\widehat{(1)}_{K_\alpha}(g))^2$, and $\widehat{(1)}_{K_\alpha}$ is the Fourier transform of the characteristic function of K_α. If $G = R^m$, $I = N$ and K_n, $n \in N$ is the ball of the radius n, then σ_{K_n} coincides with the Feier's well-known integral operator. It is proved, that if $f \in L^p(G)$ and $\lim\limits_{\alpha \to \infty} mes\,(TK_\alpha)/(mes\, K_\alpha) = 1$ for arbitrary fixed symmetric neighborhood T of the unity in $\widehat{G}$, then the sequence $\sigma_{K_\alpha}(f)$ converges to f in the space $L^p(G)$, $1 \le p < \infty$. In the spaces $L^\infty(G)$ and $M(G)$ the analogous convergence is true with respect to the weak* topology of these spaces. The proofs of these statements are based on the Proposition, according to which the sequence of kernels $\{V_{K_\alpha}\}$ represents an approximative unit in the space $L^1(G)$. Moreover it is calculated the order, higher of which the realization of above mentioned convergence is impossible. The obtaining results are illustrated for some concrete locally compact abelian groups.

Mathematical Logic, Applied Logic and Programming

Computer Composition of all Georgian Word-Forms for Given Lexical Unit

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Realization of machine translation from one natural language into Georgian demands composition of Georgian word-form from its invariable part and morphological categories. In addition, for distance learning of Georgian language and teaching Georgian morphology in secondary school by aid of computer needs composition of all correct word-forms from their invariable part. For solving of the problems, we have developed software, which makes possible to resolve the problems. Algorithm is based on Georgian grammar and for verbs we used the system proposed by D. Melikishvili, which simplifies resolution of the problems.

Parts of the software are written in visual csharp language and realized using Visual Studio 2008 Express Edition. It is foreseen presentation of the software's work.

Templates Processing Lists in Haskell

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Programming language Haskell represents a functional language and it has all the characteristics that defines functional paradigm. In the functional paradigm of programming, methods used for building data structure gives ability to create simultaneously templates of typical functions to edit these structures.

Typical tasks that are solved using functional programming methods include the tasks for dynamic structure descriptions and automatic construction of programs and verification for given structures. We will describe these structures using Haskell language and compare with Lisp language capabilities. The classic module of language Haskell defines template of the function for editing lists. Our goal is following: the algorithm that we have used for Lisp functional programs to use for the Haskell typical template as well. This paper describes the method for structural induction that is used for verification of those Haskell programs that can present functions for editing lists.

ლაბორატორიული ექსპერიმენტების მართვა მიკროკონტროლების საშუალებით

ზაზა დავითაძე, დავით დევაძე, გულადი ფარტენაძე

შოთა რუსთაველის სახელმწიფო უნივერსიტეტი

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ნაშრომის მიზანს წარმოადგენს მიკროკონტროლერის გამოყენებით ფიზიკური ლაბორატო რიული სამუშაოს, ექსპერიმენტის ჩატარების შესაძლებლობების გამოკვლევა. ექსპერიმენტები დან გამომდინარე ვასაბუთებთ, რომ ჩვენს მიერ ჩატარებული და გაცილებით რთული ლაბორატორიული სამუშაოებისა და ექსპერიმენტების ორგანიზება ვიზუალურ და ავტომატურ რეჟიმში საკმაოდ მარტივდება მიკროკონტროლერების გამოყენებით, ამასთანავე, რაც მნიშვნელოვანია, მსგავსი მოწყობილობების ფასი გაცილებითი იაფია მზა ექსპერიმენტულ თუ ავტომატურ ლაბორატორიულ მოწყობილობებთან შედარებით. აღნიშნულიდან გამომდინარე უამრავი ლაბორატორიული სამუშაო თუ ექსპერიმენტი მინიმალური დანახარჯებით შეიძლება მომზადდეს და მისაწვდომი გავხადოთ უმაღლესი სასწავლებლებისა და სკოლების ფიზიკის, ქიმიის, ბიოლოგიის და სხვა კვლევითი თუ სამეცნიერო ლაბორატორიების თანამშრომლების ფართო წრისთვის. აღნიშნული მოსაზრების დასასაბუთებლად ჩავატარეთ ექსპერიმენტი ფოტორეზისტორის წინაღობის სინათლის ინტენსივობაზე დამოკიდებულების ამოცანისთვის. ექსპერიმენტში გამოყენებულია მიკროკონტროლერის ანაწყობის მზა დაფა "ARDUINO DUEMILANOVE" მიკროკონტოლერის ბაზაზე ATmega328 დაფაზე ინტეგრირე- ბული USB პორტით [1,3]. ARDUINO-ს პროგრამირების ენაზე [2,4] ჩვენს მიერ დაწე- რილი იქნა შესაბამისი პროგრამა (სკეჩი), რომელიც ციფრული სიგნალებით ავტომატურად მართავს ექსპერიმენტში გამოყენებული სინათლის წყაროს (შუქდიოდის) ინტენსივობას და მეორეს მხრივ ავტომატურად ღებულობს და ზომავს ანალოგურ სიგნალს ფოტოწინაღობიდან. ექსპერიმენტის ხანგრძლივობა რამოდენიმე ათეული წამია, რაც რეალურად ჩატარებულ ექსპერიმენტთან შედარებით გაცილებით სწრაფია. საბოლოო რიცხვითი და გრაფიკული შედეგების გამოტანა ხდება კომპიუტერის მონიტორზე. მნიშვნელოვანია აღინიშნოს, რომ ATmega328 მიკროკონტოლერის ბაზაზე შესაძლებელია ერთდროულად 14 ციფრული და 6 ანალოგური პარამეტრის მართვა, რაც რთული ექსპერიმენტის მართვის საშუალებასაც იძლევა.

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ქართული ენის ძირეული ნაწილი როგორც მათემატიკური თეორია

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საქართველოს ტექნიკური უნივერსიტეტი

ქართული ენის ტექნოლოგიზების სასწავლო–სამეცნიერო ცენტრი

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2010 წელს, საქართველოს ტექნიკურ უნივერსიტეტში, დაფუძნდა ქართული ენის ტექნოლოგიების სასწავლო–სამეცნიერო ცენტრი. ცენტრი ამუშავებს გრძელვადიან პროექტს „ქართული ენის ლოგიკური გრამატიკა და ქართულენოვანი კომპიუტერი", რომელიც მანამდე არსებული სახელმწიფო–მიზნობრივი პროგრამის „კომპიუტერის სრული პროგრამულ–მომსახურეობითი მოქცევა ბუნებრივ ქართულ ენობრივ გარემოში" კვლევითი ამოცანების შემდგომი გაფართოების საფუძველზე ჩამოყალიბდა.

პროექტი მიზნად ისახავს ქართული ენის (აქ, ქართული ენის ქვეშ გაიგება ქართული ბუნებრივი ენობრივი სისტემა, რომლის შემადგენლებიცაა ქართული სამწერლობო, სამეტყველო და სააზროვნო ენები) ბუნების სრულად ამსახველი მათემატიკური თეორიის ანუ, მოკლედ, ქართული ენის ლოგიკური გრამატიკის შემუშავებას და, ასევე, ქართულენოვანი კომპიუტერის ანუ ინტელექტუალური უნარებით აღჭურვილი ქართული კომპიუტერული სისტემის აგებას.

ამასთან, ცხადია, რომ ქართული ინტელექტუალური კომპიუტერული სისტემის აგების ერთადერთი გზა ქართულ ენით მოცემული და, ამდენად, ქართულ ენაში არსებული გამოთვლების აზრობრივად მაორგანიზებელი წესების სრულ მათემატიკურ მოდელირებაზე გადის, რაც ქართული ენის სრული მათემატიკური თეორიის შემუშავებას ნიშნავს. ამგვარად, გასაგები ხდება, რომ ცენტრის საპროექტო მიზანთა შორის ქართული ენის სრული მათემატიკური თეორიის შემუშავება ძირეულია.

მოხსენებისას მათემატიკური თეორიის სახით წარმოვადგენთ ქართული ენის ძირეულ ნაწილს (აქ, ქართული ენის ძირეული ნაწილის ქვეშ გაიგება ქართული ენის თხრობითი კილოს წინადადებათა შრე). ამასთან, მიმოვიხილავთ ქართული ენის ლოგიკური გრამატიკის იმ საფუძველმდებით საკითხებს, რასაც ეყრდნობა ქართული ენის ძირეული ნაწილის მათემატიკურ თეორიად გადააზრების ზემოხაზგასმული ხედვა. კერძოდ, მოხსენებისას მიმოხილული იქნება:

1. ლოგიკური ბრუნების და ლინგვისტური პრედიკატის ცნება და ამ ცნებების დაყრდნობით შემუშავებული ახალი მათემატიკური ხედვა ქართული ენის სახელებსა და ზმნებზე;

2. ქართული ენის I საფეხურის მათემატიკური თეორია, რომელიც მათემატიკური თეორიის სახით წარმოგვიდგენს ქართული ენის ძირეული ნაწილს. ამასთან, წარმოდგენილი იქნება ქართული გამაფართოებელი წესები, რომელთა საფუძველზეც ქართული ენის ძირეული ნაწილის სიტყვები გაიგება შ.ფხაკაძისეულ შემამოკლებელ სიმბოლოებად;

3. ქართული ენის ძირეული ნაწილის წინადადებების ქართული ენის I საფეხურის მათემატიკურ თეორიაზე ავტომატურად დამყვანი ანუ ავტომატურად მთარგმნელი ფორმალური წესები.

ბურბაკის τ-ოპერატორის მოდიფიკაცია ხელოვნური ენებისათვის

ხიმურ რუხაია, ლალი ტიბუა

ილია ვეკუას სახელობის გამოყენებითი მათემატიკის ინსტიტუტი

ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი

სოხუმის სახელმწიფო უნივერსიტეტი

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ცნობილია, რომ იმ თეორიაში, რომლის ენის ძირითად სიმბოლოებში შედის ბურბაკის τ-ოპერატორი [1], არსებობისა და ზოგადობის კვანტორები ისაზღვრებიან განსაზღვრებათა წესების რაციონალური სისტემით [2]. ამავე სისტემით ხდება $M\tau SR$-თეორიის ენის [3] დედუქციური გაფართოება-განვითარება და შესაბამისად, მას გააჩნია კარგი გამომსახველობითი უნარი. $M\tau SR$-ენის შემდგომ სრულყოფას მივაღწიეთ τ-ოპერატორის მოდიფიკაციით იმ აზრით, რომ მის ოპერატიულ ასოდ დავუშვით $M\tau SR$-ენის მეტაცვლადები თერმებისათვის. ე.ი. მოცემულ თეორიაში ჩვენ დავუშვით $\tau_T A(T)$ ტიპის თერმის არსებობა. $\tau_T A(T)$ თერმი იკითხება ასე: "ისეთი პრივილეგირებული T თერმი, რომელსაც აქვს $A(T)$ თვისება". შესაბამისად, τ_T კვანტორით განისაზღვრა $\exists T$-არსებობის და $\forall T$-ზოგადობის კვანტორები და დადგინდა მათი ზოგიერთი თვისება.

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Topology

Topology of the Fibres of Proper Quadratic Mappings

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As is known, in many problems of variational calculus and optimal control theory, an important role is played by the topological structure of the fibers of a given quadratic mapping of Euclidean spaces. Similar problems arise in algebraic geometry and singularity theory. We estimate the number of components and the Euler characteristic for the fibres of a stable quadratic map in low dimensions.

Let X and Y be smooth manifolds, f and f' be two elements from $\mathrm{C}^\infty(\mathrm{X},\mathrm{Y})$. Mappings f and f' are called equivalent, if there exist diffeomorphisms $g:\mathrm{X}\to\mathrm{X}$, and $h:\mathrm{Y}\to\mathrm{Y}$, such that

$$h\circ f=f'\circ g.$$

Definition. A mapping $f\in\mathrm{C}^\infty(\mathrm{X},\mathrm{Y})$ is called stable if there exists a neighborhood W_f of f in $\mathrm{C}^\infty(\mathrm{X},\mathrm{Y})$, such that any map $f'\in W_f$ is equivalent to f.

Let $Q:\mathbb{R}^s\to\mathbb{R}^t$ be a *stable proper* quadratic mapping with generic fiber of positive dimension $k=s-t$.

Theorem. *The Euler characteristics of the fibers of a stable proper quadratic mappings* $Q:\mathbb{R}^4\to\mathbb{R}^3$ *fill the integer segment* $[-3,3]\cap\mathbb{Z}$.

Proposition. *For* $n=3$*, the fibres are finite and the number of points in a fibre fills in the integer segment* $[0,8]\cap\mathbb{Z}$.

More precisely, to each number from the integer segment $[0,8]\cap\mathbb{Z}$ corresponds an obvious realization, therefore Euler characteristic takes all integer values from $[0,8]$.

Proposition. *The component number of any fibre does not exceed* 54.

The Shape and Cohomology Exact Sequences of a Map

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In this paper the shape of continuous map $f:X\to Y$ is defined (cf. [2]). Applying Cech and Vietoris constructions then it will be shown that exist two equivalence functors from the category of maps of topological spaces to the pro-category of category of maps of CW-complexes and to the pro-category of appropriate homotopy category of maps of CW-complexes. Next we will give the definitions of functors from the category of maps to the category of long exact sequences of normal homology pro-groups and to the category of long exact sequences of normal cohomology inj-groups ([1, 3, 5, 6]). Using the result of ([1, 2, 4]) we will prove the following theorems.

Theorem 1. *For each map $f : X \to Y$ of topological spaces and abelian group G there exist the long exact sequences of normal homology pro-groups and normal cohomology inj-groups*

$$\cdots \to pro - H_n(X;G) \to pro - H_n(Y;G) \to pro - H_n(f;G) \to \cdots$$

$$\cdots \to inj - H^n(f;G) \to inj - H^n(Y;G) \to inj - H^n(X;G) \to \cdots ,$$

where

$$pro - H_n(f;G) = \{H_n(f_{\alpha\beta\nu};G)\}_{(\alpha,\beta,\nu)\in \mathrm{cov}_N(f)},$$
$$inj - H^n(f;G) = \{H^n(f_{\alpha\beta\nu};G)\}_{(\alpha,\beta,\nu)\in \mathrm{cov}_N(f)},$$
$$H_n(f_{\alpha\beta\nu};G) = H_n(Cyl(f_{\alpha\beta\nu})), X_\beta;G), \quad H^n(f_{\alpha\beta\nu};G) = H^n(Cyl(f_{\alpha\beta\nu})), X_\beta;G),$$

$f_{\alpha\beta\nu} : X_\beta \to Y_\alpha$, $\nu : \beta > f^{-1}(\alpha)$, $\alpha \in \mathrm{cov}_N(Y)$, $\beta \in \mathrm{cov}_N(X)$ *and* $\mathrm{cov}_N(X)$ *and* $\mathrm{cov}_N(Y)$ *are the sets of normal open coverings of* X *and* Y, *respectively.*

Theorem 2. For each map $f : X \to Y$ of topological spaces there exists a long exact sequences of normal cohomology groups

$$\cdots \to \check{H}^n(f;G) \to \check{H}^n(Y;G) \to \check{H}^n(X;G) \to \cdots ,$$

where $\check{H}^n(f;G) = \lim\limits_{\to} inj - H^n(f;G)$.

Corollary 3 (cf. [3]). For each pair (X, A) of topological spaces there exists a long exact sequence of normal cohomology groups

$$\cdots \to \check{H}^n(i;G) \to \check{H}^n(Y;G) \to \check{H}^n(X;G) \to \cdots ,$$

where i is the inclusion map $i : A \to X$.

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On Intuitionistic Fuzzy Soft Topological Spaces

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Let $IFSP\,(X,E)$ be the family of all intuitionistic fuzzy soft sets over X via parameters in E.

Definition 1. Let $\tau \subset IFSP\,(X,E)$ be the collection of intuitionistic fuzzy soft sets over X, then τ is said to be an intuitionistic fuzzy soft topology on X if

(1) $\widetilde{\Phi}, \widetilde{1}$ belong to τ;

(2) the union of any number of intuitionistic fuzzy soft sets in τ belongs to τ;

(3) the intersection of any two intuitionistic fuzzy soft sets in τ belongs to τ.

The triplet (X,τ,E) is called an intuitionistic fuzzy soft topological space over X.

Proposition 2. *Let (X,τ,E) be an intuitionistic fuzzy soft topological space over X and $\tau = \{(F_\alpha, G_\alpha, E)\}_{\alpha\in\Lambda}$. Then the collection $\tau_1 = \{(F_\alpha, E)\}_{\alpha\in\Lambda}$ and $\tau_2 = \{(G_\alpha, E)\}_{\alpha\in\Lambda}$ defines a fuzzy soft topology on X.*

Proposition 3. *Let (X,τ,E) be an intuitionistic fuzzy soft topological space over X. Then the collection $\tau_\alpha = \{(F(\alpha), G(\alpha)) \mid (F,G,E) \in \tau\}$ for each $\alpha \in E$, defines a fuzzy bitopology on X.*

Definition 4. Let (X,τ,E) and (Y,τ',E) be two intuitionistic fuzzy soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. For each $(F,G,E)\in\tau'$, if $f^{-1}(F,G,E)\in\tau$, then $f:(X,\tau,E)\to(Y,\tau',E)$ is said to be intuitionistic fuzzy soft continuous mapping of intuitionistic fuzzy soft topological spaces.

Theorem 5. *Let (X,τ,E) and (Y,τ',E) be two intuitionistic fuzzy soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. Then the following conditions are equivalent:*

(1) *$f:(X,\tau,E)\to(Y,\tau',E)$ is an intuitionistic fuzzy soft continuous mapping;*

(2) *For each intuitionistic fuzzy soft closed set (F,G,E) over Y, $f^{-1}(F,G,E)$ is a intuitionistic fuzzy soft closed set over X;*

(3) *For each intuitionistic fuzzy soft set (F,G,E) over X, $f(\overline{(F,G,E)})\,\widetilde{\subset}\,\overline{(f(F,G,E))}$;*

(4) *For each intuitionistic fuzzy soft set (F,G,E) over Y, $\overline{(f^{-1}(F,G,E))}\widetilde{\subset} f^{-1}(\overline{(F,G,E)})$;*

(5) *For each intuitionistic fuzzy soft set (F,G,E) over Y, $f^{-1}((F,G,E)^\circ)\widetilde{\subset}(f^{-1}(F,G,E))^\circ$.*

On Soft Compactness

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The most important of all covering properties is compactness. In this study, we introduce the concept of soft compactness and study some of its basic properties.

Definition 1. Let (X,τ,E) be a soft topological space and $\emptyset \neq Y \subset X$, $U = (\{F_\alpha, E\})_\alpha \subset \tau$.

A collection U of soft open sets is called a soft open covering of $\widetilde{Y}$ if $\widetilde{Y} \subset \bigcup\limits_\alpha (F_\alpha, E)$.

If $\widetilde{Y} = X$, then the family U is said to be soft open covering of (X,τ,E).

Definition 2. A soft topological space (X,τ,E) is said to be soft compact space, if every soft open covering of X has a finite soft open subcovering.

Definition 3. Let (X,τ,E) be a soft topological space and $\emptyset \neq Y \subset X$. If (Y,τ_Y,E) is a soft compact space, then soft subset $\widetilde{Y}$ is called a soft compact set on $\widetilde{X}$.

Theorem 4. *Let (X,τ,E) be a soft topological space. If (X,τ,E) is a soft compact space, then (X,τ_α) is a compact space, for each $\alpha \in E$.*

Theorem 5. A*ny soft closed subset of a soft compact space is soft compact space.*

Theorem 6. *Any soft compact subset of a soft T_2-space is a soft closed.*

Theorem 7. *Let (X,τ,E) be a soft compact space, (Y,τ',E) be any soft toplogical space. If $f:(X,\tau,E)\to(Y,\tau',E)$ is a soft continuous mapping, then $\left(f(X),\tau'_{f(X)},E\right)$ is soft compact space.*

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Strong Homology Group of Continuous Map

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Using the shape properties of continuous map normal cohomology functor from the category of maps to the category of long exact sequences of groups is constructed by V.Baladze [1].The main aim of this work is to study the strong homology of maps. It is proved that for each **pro**-chain map $\boldsymbol{f:C\longrightarrow C'}$ there exists the long exact strong homological sequence:

$$\cdots\longrightarrow \boldsymbol{H_m(C)}\longrightarrow\boldsymbol{H_m(C')}\longrightarrow\boldsymbol{H_m(f)}\longrightarrow\boldsymbol{H_{m-1}(C)}\longrightarrow\cdots.$$

Using the obtained results strong homology functor $\boldsymbol{H_*(-)}$ is constructed on the category $\boldsymbol{M_{Top}}$ of continuous maps of topological spaces. It is proved that the functor $\boldsymbol{H_*(-)}$: $M_{T0p}\longrightarrow Ab$ satisfies the Boltianski type axioms. Besides, the isomorphism $\boldsymbol{H_*}(f)\approxeq\boldsymbol{H_*}(C_f)$ of strong homology group of continuous map $f:X\longrightarrow Y$ of topological spaces and strong homology group [2] of mapping cone C_f of the map f is proved. As corollary, it is obtained that for each pair (X,A) of topological spaces there exists the long exact strong homological sequence:

$$\cdots\longrightarrow\boldsymbol{H_m}(A)\longrightarrow\boldsymbol{H_m}(X)\longrightarrow\boldsymbol{H_m}(C_i)\longrightarrow\boldsymbol{H_{m-1}}(A)\longrightarrow\cdots,$$

where C_i is mapping cone of inclusion $i:A\longrightarrow X$. In the case when $i:A\longrightarrow X$ is cofibration and (X,A) is normally embedded pair there is the isomorphism $\boldsymbol{H_*}(C_f)\approxeq\boldsymbol{H_*}(X,A)$ of strong homology group of mapping cone C_i and strong homology group of pair (X,A).

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On Equivariant Strong Shape Theory

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For the category of spaces with action of compact group G strong shape theory is constructed and studied.

On Fixed Point Theorems and Nonsensitivity of Dynamical Systems

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Sensitivity is a prominent aspect of chaotic behavior of a dynamical system. We study the relevance of nonsensitivity to fixed point theory in affine dynamical systems. We prove a fixed point theorem which extends Ryll–Nardzewski's theorem and some of its generalizations. Using the theory of hereditarily nonsensitive dynamical systems we establish left amenability of $Asp(G)$, the algebra of Asplund functions on a topological group G (which contains the algebra $WAP(G)$ of weakly almost periodic functions). We note that, in contrast to $WAP(G)$ where the invariant mean is unique, for some groups (including the integers) there are uncountably many invariant means on $Asp(G)$. Finally we observe that dynamical systems in the larger class of tame G-systems need not admit an invariant probability measure. This is a joint work (will appear in *Israel Journal of Mathematics*) with Eli Glasner (Tel Aviv University).

On Derived Functors

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Let $\mathscr{K}$ be an abelian category with enough a) injective or b) projective objects and for any exact sequence

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0 \qquad (*)$$

from $\mathscr{K}$ there exists a commutative diagram

$$\text{a)} \qquad \begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\ & & \| & & \big\downarrow & & \big\downarrow{\scriptstyle\gamma} & & \\ 0 & \longrightarrow & A & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0, \end{array}$$

where γ is a monomorphism, or

$$\text{b)} \qquad \begin{array}{ccccccccc} 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C & \longrightarrow & 0 \\ & & {\scriptstyle\alpha}\big\downarrow & & \big\downarrow & & \| & & \\ 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0, \end{array}$$

where α is an epimorphism.

Assume that a functor $T : \mathscr{K} \to \mathscr{K}'$ is an additive and a) left exact covariant or right exact contravariant, b) right exact covariant or left exact contravariant, where $\mathscr{K}'$ is an abelian category. By T^i, $i \geq 0$, denote derived functors of T.

Theorem A. *For any sequence* $(*)$ *in the case* a) *there exists for* $i \geq 2$ *a naturally split sequence*

$$0 \longrightarrow T^i(A) \longrightarrow T^i(B) \longrightarrow T^i(C) \longrightarrow 0;$$

or

$$0 \longrightarrow T^i(C) \longrightarrow T^i(B) \longrightarrow T^i(A) \longrightarrow 0.$$

Theorem B. *For any sequence* $(*)$ *in the case* b) *there exists for* $i \geq 2$ *a naturally split sequence*

$$0 \longrightarrow T^i(A) \longrightarrow T^i(B) \longrightarrow T^i(C) \longrightarrow 0;$$

or

$$0T^i(C) \longrightarrow T^i(B) \longrightarrow T^i(A) \longrightarrow 0.$$

Partially Continuous Singular Cohomology and Fibration

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Let X be a topological space and G be a topological abelian group. In the papers [1], [2] partially continuous singular cohomology $\overline{h}^*_s(X,G)$ and continuous singular cohomology $h^*_s(X,G)$, respectively, were defined. If X is a metric space and $G = ANR$, then there is an isomorphism $\overline{h}^*_s(X,G) \approx h^*_s(X,G)$ [2, Corollary 13].

Definition. A function $\varphi : S_q(X) \to G$ is said to be partially inessential, if there exists an open covering $\alpha = \{U_\alpha\}$ of X such that the restriction $\varphi|S_q(\alpha)$ is a continuous inessential map, where $S_q(\alpha) = F(\Delta_q, X)$ is the space of all continuous maps from Δ_q to U_α, given the compact-open topology. Denote by $L^q_0(X,G)$ the subgroup of $\widetilde{L}^q(X,G)$ consisting of all locally zero functions and by $\overline{L}^q(X,G) = \widetilde{L}(X,G)/L^q_0(X,G)$. Cohomology of the cochain complex $\overline{L}^q(X,G)$ is denoted by $\widetilde{h}^*_s(X,G)$.

Theorem. *For any topological space* X *and a fibration* $p : E \to B$*, where* E *is a contractible space and* $F = p^{-1}(b_0)$ *is fiber, there is an exact cohomology sequence*

$$\cdots \longrightarrow \overline{h}^q_s(X,F) \longrightarrow \overline{h}^q_s(X,E) \longrightarrow \widetilde{h}^q_s(X,B) \longrightarrow \overline{h}^{q+1}_s(X,F) \longrightarrow \cdots .$$

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ტოპოლოგიური ჯგუფის ალგებრულ და ტოპოლოგიურ თვისებებს შორის ურთიერთკავშირის შესახებ

ონისე სურმანიძე

შოთა რუსთაველის სახელმწიფო უნივერსიტეტი

მათემატიკის დეპარტამენტი

ბათუმი, საქართველო

ტოპოლოგიურ ჯგუფში განმარტებული სტრუქტურები - ჯგუფისა და ტოპოლოგიური სივრცის სტრუქტურები ერთმანეთთან არის ისე მჭიდროდ დაკავშირებული, რომ ერთ-ერთი მათგანი არესებით ზეგავლენას ახდენს მეორეზე.

მახასიათებელთა თეორიის გამოყენებით ლ. პონტრიაგინმა დაამტკიცა კომპაქტური და კომპაქტური წარმოშობის ლოკალურად კომპაქტური ჯგუფებისათვის სტრუქტურული თეორემა, რომელიც წარმოადგენს დისკრეტული სასრულწარმომქმნელიანი ჯგუფების ცნობილი კლასიკური შედეგის განზოგადებას.

სუსტად წრფივად კომპაქტური ჯგუფებისათვის აგებულმა ლ. პონტრიაგინის თეორიის ანალოგიურმა თეორიამ საშუალება მოგვცა დაგვედგინა:

1. წრფივად კომპაქტური ჯგუფების სტრუქტურული დახასიათება - ისინი წარმოადგენენ ცნობილი ჯგუფების $(C(P^n), C(P^\infty), Z_p)$ და $Q_p)$ სრულ პირდაპირ ჯამს;

2. წრფივად დისკრეტული ჯგუფის ალგებრული დახასიათება კონკრეტული სახის (l-ტოპოლოგია) ტოპოლოგიაში. კერძოდ მტკიცდება, რომ დისკრეტული ჯგუფი წრფივად დისკრეტულია l-ტოპოლოგიაში, როდესაც იგი შემოსაზღვრული p-ჯგუფია.

On Internal Tensor Structures of the Tangent Bundle of Space $Lm(Vn)$ with Triplet Connection

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Consider a vector bundle space $Lm(Vn)$ with a triplet connection Γ_i^α, $\Gamma_{\beta i}^\alpha$, Γ_{jk}^i. Local coordinates of the space $Lm(Vn)$ transform in the following way:

$$\overline{x}^i = \overline{x}^i(x^k);\ \ \overline{y}^\alpha = A_\beta^\alpha(x)y^\beta, \quad \det\left\|\frac{\partial\overline{x}i}{\partial x^k}\right\| \neq 0;$$

$$\det\left\|A_\beta^\alpha\right\| \neq 0;\ \ i,j,k = 1,2,\dots,n;\ \ \alpha,\beta,\gamma = 1,2,\dots,m.$$

If $\{e_i, e_\alpha\}$ is a frame of the tangent space T_{n+m} at the point $z=(x,y) \in Lm(Vn)$ then vectors $E_i = e_i - \Gamma_i^\alpha e_\alpha$ define invariant equipment of the tangent space. Let T_B^A be $GL(n,m,R) \times GL(n,m,R)$-tensor field, then

$$T(\xi) = T_B^A \xi^B e_A, \quad A,B,C = 1,2,\dots,n+m$$

is tanhe element of the space T_{n+m}. A space $Lm(Vn)$,in which we define tensor field T_B^A, satisfying conditions

$$T_C^A T_B^C = \lambda\delta_B^A,$$

we shall call a vector bundle space with a tensor structure. If $\lambda = 0$, then the tensor structure we shall call almost dual tensor structure, if $\lambda = -1$, it will be called almost complex tensor structure, if $\lambda = 1$, then-almost product tensor structure.

Theorem 1. *If in the space $Lm(Vn)$ with the triplet connection $GL(n, R)$-vector field $\xi^\alpha(x, y)$, $GL(n, R)$-covector field $\eta_i(x, y)$ are given, then in the tangent bundle of the space $Lm(Vn)$ there exist two two-parameter families of tensor structures, which include duai tensor structures and almost product structures and no almost complex structures.*

Theorem 2. *If in the space $Lm(Vn)$ with the triplet connection $GL(n, R)$-vector field $\xi^i(x, y)$, $GL(n, R)$-covector field $\eta_\alpha(x, y)$ are given, then in the tangent bundle of the space $Lm(Vn)$ there exist two two-parameter families of tensor structures, which include duai tensor structures and almost product structures and no almost complex structures.*

ალექსანდერ-სპანიერის კოჰომოლოგიური თეორიების შესახებ

რუსლან ცინარიძე

შოთა რუსთაველის სახელმწიფო უნივერსიტეტი

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ტოპოლოგიურ სივრცეთა სხვადასხვა კატეგორიებზე აგებული იქნება სასრულ და ლოკალურად სასრულ მნიშვნელობებიან კოჯაჭვებზე დაფუძნებული ალექსანდერ-სპანიერის კოჰომოლოგიური თეორიები და მათი კომპაქტურმატარებლიანი და უწყვეტი კოჰომოლოგიური ნაირსახეობანი. დადგენილი იქნება მათი კავშირები კლასიკურ კოჰომოლოგიის თეორიებთან, შემოწმებული იქნება სტინროდ-ეილენბერგის ტიპის აქსიომები და დამტკიცებული იქნება უწყვეტი ასახვის ზუსტი კოჰომოლოგიური მიმდევრობების არსებობის თეორემები.

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ზოგიერთი კავშირი ტოპოლოგიურ სივრცეთა ბაუერის ძლიერ შეიპურ თეორიასა და თანაბრულ ძლიერ შეიპურ თეორიებს შორის

ლელა თურმანიძე
შოთა რუსთაველის სახელმწიფო უნივერსიტეტი
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ფ. ბაუერის მიერ განმარტებული და გამოკვლეული იქნა ძლიერი შეიპური თეორია ტოპოლოგიურ სივრცეებისთვის [1]. მოხსენებაში გადმოცემული იქნება თანაბარი სივრცეებისთვის ბაუერის ტიპის ძლიერი შეიპური თეორიის აგება, მისი ძირითადი თვისებები, ინვარიანტები და კავშირი ტოპოლოგიურ სივრცეთა ძლიერ შეიპურ თეორიასთან.

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Algebra and Number Theory

გაერთიანების ნახევარმესერების ელემენტთა წარმოდგენისა და ჯაჭვზე განსაზღვრული $B(X, D)$ ნახევარჯგუფის ელემენტის იდენპოტენტობის შესახებ.

ზებურ ავალიანი
შოთა რუსთაველის სახელმწიფო უნივერსიტეტი
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X-ის ქვესიმრავლეთა ისეთ $D = \{Z_1, Z_2, ..., Z_n, ...\}$ სისტემას, რომელიც ჩაკეტილია გაერთიანების ოპერაციის მიმართ, გაერთიანების ნახევარმესერი ეწოდება. $\Sigma_1(X, 5)$ ტიპის ნახევარმესერების ელემენტების წარმოდგენის ამოცანა გადაწყვეტილი იქნა შრომაში [1]. შემდგომში, მიღებული შედეგები ფორმალური ტოლობებით განზოგადდა ი. დიასამიძის მიერ [3]. მოხსენება ეხება სხვა გზით, შინაარსობრივი ტოლობების გამოყებებით აღნიშნული ამოცანის შესწავლას, რომელიც გაცილებით მარტივია.

როცა D არის ჯაჭვი, რომლის სიგრძეა 2 ან 3, $B(X, D)$ ნახევარჯგუფის ელემენტების იდენპოტენტურობის პირობა პირდაპირი გზით მოძებნილი იქნა ჩვენს მიერ. შემდგომში, $V(D, \alpha)$-ს გამოყენებით ნებისმიერი სასრული სიგრძის ჯაჭვი შესწავლილი იქნა შრომაში [4].

მოხსენებაში გადმოცემული იქნება აღნიშნული პირობების გამოყვანა ბინარულ მიმართებათა გამრავლების თვისებებით, რომელიც გაცილებით მარტივი და თვალსაჩინოა.

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Abelian and Nilpotent Varieties of Power Groups

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The present paper continues the series of the papers [1]–[3] and is dedicated to the construction of basic principles of the theory of power groups varieties and tensor completions of groups in a variety. We study the relationship between free groups of a given variety for various rings of scalars. Varieties of abelian power groups are described.

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Idempotent Elements of Complete Semigroups of Binary Relations Defined by the Finite X-Semilattices of the Rooted Tree Class

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Let D be an arbitrary nonempty set, D be an X-semilattice of unions, i.e. a nonempty set of subsets of the set X that is closed with respect to the set-theoretic operations of union of elements from D, f be an arbitrary mapping from X into D. To each such mapping f there corresponds a binary relation α_f on the set X that satisfies the condition $\alpha_f = \bigcup\limits_{x\in X} (\{x\} \times f(x))$. The set of all such α_f is denoted by $B_X(D)$. It is easy to prove that $B_X(D)$ is a semigroup with respect to the operation of multiplication of binary relations, which is called a complete semigroup of binary relations defined by an X-semilattice of unions D.

We denote by $\varnothing$ the empty binary relation or empty subset of the set X. The condition $(x, y) \in \alpha$ will be written in the form $x\alpha y$. Further let $x, y \in X$, $Y \subseteq X$, $\alpha \in B_X(D)$, $T \in D$ and $\breve{D} = \bigcup D$. Then by symbols we denote the following sets: $y\alpha = \{x \in X | y\alpha x\}$, $Y\alpha = \bigcup\limits_{y\in Y} y\alpha$, $V(D, \alpha) = \{Y\alpha | Y \in D\}$.

Definition 1. The finite X-semilattice of unions D is called rooted tree, if for the every element Z $(Z \neq \breve{D})$ of the semilattice D, there exists a unique element of the semilattice D, which covers the element Z.

By $D = \{Z_1, Z_2, \ldots, Z_{n-1}, \breve{D}\}$ and D_M respectively let us denote finite rooted tree D and all minimal elements of the rooted tree D.

Let $T, Z \in D$, $T \neq Z$ and $T \subset Z$. By $c(T, Z)$ will be denoted those subsets of the rooted tree D, which are maximal chain in the given rooted tree having smallest element T and largest element Z.

Definition 2. Let $N(D) = \{|c(T', \breve{D})||T' \in D_M\}$ and let $h(D)$ be the largest natural number of the set $N(D)$. By the symbol Q_k $(1 \leq k \leq h(D))$ we denote a chain of the form $T_1 \subset T_2 \subset \cdots \subset T_k$.

Definition 3. Let $D_M = \{Z_2, Z_1\}$ and $d = 2^{|c(Z_2 \cup Z_1, \breve{D})|-1}$. By the symbol Q'_s $(1 \leq s \leq d)$ we denote any X-semilattice which satisfies the conditions $Z_1 \cap Z_2 = \varnothing$ and $Q'_s = \{Z_2, Z_1, Z_2 \cap Z_1\} \cup D_1$, where $D_1 \subseteq c(Z_2 \cup Z_1, \breve{D})$.

Theorem 1. *Let D be any rooted tree and $|D_M| = 1$ or $|D_M| \geq 3$. Then a binary relation α of the semigroup $B_X(D)$ which has a quasinormal representation of the form $\alpha = \bigcup\limits_{i=1}^{k} (Y_i^\alpha \times T_i)$ is an idempotent element of the semigroup $B_X(D)$ iff for all k $(1 \leq k \leq h(D))$ the semilattice $V(D, \alpha)$ is a chain $T_1 \subset T_2 \subset \cdots \subset T_k$ and $Y_1^\alpha \cup Y_2^\alpha \cup \cdots \cup Y_p^\alpha \supseteq T_p$, $Y_q^\alpha \cap T_q \neq \varnothing$ for any $p = 1, 2, \ldots, k-1$ and $q = 1, 2, \ldots, k$.*

Theorem 2. *Let D be any rooted tree, $D_M = \{Z_2, Z_1\}$ and $Z_2 \cap Z_1 = \varnothing$. Then a binary relation α of the semigroup $B_X(D)$ is an idempotent element of the semigroup $B_X(D)$ iff it satisfies the following conditions:*

a) *For any $1 \leq k \leq h(D)$ the binary relation $\alpha = \bigcup\limits_{i=1}^{k} (Y_i^\alpha \times T_i)$, where $V(D, \alpha)$ is a chain of the form $T_1 \subset T_2 \subset \cdots \subset T_k$; $Y_1^\alpha, Y_2^\alpha, \ldots, Y_k^\alpha \notin \{\varnothing\}$; $Y_1^\alpha \cup Y_2^\alpha \cup \cdots \cup Y_p^\alpha \supseteq T_p$; $Y_q^\alpha \cap T_q \neq \varnothing$ for any $p = 1, 2, \ldots, k-1$ and $q = 1, 2, \ldots, k$;*

b) *$\alpha = (Y_2^\alpha \times Z_2) \cup (Y_1^\alpha \times Z_1) \cup (Y_0^\alpha \times (Z_2 \cup Z_1)) \cup \beta$ and $\beta = \bigcup\limits_{j=1}^{s} \left(Y_j^\alpha \times T_j\right)$ for any $0 \leq s \leq d$, where $V(D, \beta)$ is a chain having the smallest element $Z_2 \cup Z_1$; $Y_2^\alpha, Y_1^\alpha \notin \{\varnothing\}$; $Y_1^\alpha \supseteq Z_1$, $Y_2^\alpha \supseteq Z_2$, $Y_1^\alpha \cup Y_2^\alpha \cup \cdots \cup Y_p^\alpha \supseteq T_k$, $Y_q^\alpha \cap T_q \neq \varnothing$ for any $k = 4, 5, \ldots, s-1$ and $q = 4, 5, \ldots, s$.*

Theorem 3. *Let D, D_M and ε be finite rooted tree, all minimal elements of the rooted tree D and any idempotent element of the semigroup $B_X(D)$ respectively. Then for the order of the maximal subgroup $G_X(D, \varepsilon)$ of the given semigroup we have: a) if $|D_M| = 1$ or $|D_M| \geq 3$, then $|G_X(D, \varepsilon)| = 1$; b) if $|D_M| = 2$, then $|G_X(D, \varepsilon)| \leq 2$.*

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The Pair of Operations with the Generalized Entropic Property

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For an algebra $A = (A, F)$ we define the complex operations for every $\emptyset \neq A_1, \ldots, A_n \subseteq A$ and every n-ary $f \in F$ on the set $\rho(A)$ of all non-empty subsets of the set A by $f(A_1, \ldots, A_n) = \{f(a_1, \ldots, a_n) : \ a_i \in A_i\}$. The algebra $CmA = (\rho(A), F)$ is called the complex algebra of A. An algebra $A = (A, F)$ is called entropic (or medial) if it satisfies the identity of mediality: $g(f(x_{11}, \ldots, x_{n1}), \ldots, f(x_{1m}, \ldots, x_{nm})) = f(g(x_{11}, \ldots, x_{1m}), \ldots, \ g(x_{n1}, \ldots, x_{nm}))$, for every n-ary $f \in F$ and m-ary $g \in F$. In other words, the algebra A is medial if it satisfies the hyperidentity of mediality ([3, 4]). Note that a groupoid is entropic if and only if it satisfies the identity of mediality [2] $xy.uv \approx xu.yv$. An idempotent entropic algebra is called a mode [5]. We say that a variety V (respectively, the algebra A) satisfies the generalized entropic property if for every n-ary operation f and m-ary operation g of V (of A) there exist m-ary term operations $t_1, \ldots, t_n$ such that the identity: $g(f(x_{11}, \ldots, x_{n1}), \ldots, f(x_{1m}, \ldots, x_{nm})) = f(t_1(x_{11}, \ldots, x_{1m}), \ldots, t_n(x_{n1}, \ldots, x_{nm}))$ holds in V (in A) [1].

Theorem 1. *Every algebra in a variety V has the complex algebra of subalgebras, iff the variety V satisfies the generalized entropic property.*

We define concept of the generalized entropic property for the pair of operations, (f, g), of the algebra, $A = (A, f, g)$, and we investigate the relations between the entropic property and the generalized entropic property.

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Irreducible Generating Sets of Complete Semigroups of Unions $B_X(D)$ Defined by Semilattices of Class $\Sigma_2(X,4)$, when $X = Z_4$

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In complete semigroups of unions $B_X(D)$, generated by semilattices of the class $\Sigma_2(X,4)$, where $X = Z_4$ and $|X| = 3$, subsets of certain type are selected, on which equivalent relations are defined. Using these relations irreducible generating sets of considered semigroups are described.

On Some Estimate Problems for the Number of Representations of Numbers by Quadratic Forms

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In the talk some estimate problems for the arithmetic function $r(f;m)$ – the number of representations of a natural number m by the positive definite n-ary, $n \geq 4$, quadratic forms f are discussed.

We continue investigation of asymptotic behavior of $r(f;m)$ and its corresponding singular series $\rho(f;m)$ with respect to the determinant d of the form f and the representable number m.

The Condition Similar to Full Transitivity for Cotorsion Hull

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In group theory it is important to establish the condition, when one of the elements of the group maps on the other element via some endomorphism. I. Kaplansky showed that the condition of full transitivity represents such condition for separable abelian p-groups. In [1] the author showed that for cotorsion hulls of separable p-groups generally the condition of full transitivity is not fulfilled.

A new function is given in the talk, which gives the chance to fulfil the same condition of full transitivity for some classes of cotorsion hulls of separable p-Groups.

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Some Combinatorial Problems Concerning Infinite Mono-Unary Algebras

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In the papers [6] and [7], for any infinite cardinal number κ, we have constructed a root tree of power κ, which has the trivial automorphisms group. Now we consider some mono-unary algebras and relational structures that are built with the aid of this tree, and some of their applications to combinatorial problems considered in [1–5].

Theorem 1. *Let E be an infinite set of cardinality κ, and let n be a positive integer. Then there are 2^κ isomorphism types of connected mono-unary algebras (E, f) such that, each of this has exactly n automorphisms.*

Theorem 2. *For any infinite group G of power κ there exists an undirected graph H of the same power κ with $Aut(H) \cong G$.*

Corollary. *For any positive integer n and for any infinite set E of cardinality k there are 2^κ isomorphism types of symmetric binary relations on the set E each of which has exactly n automorphisms.*

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Inverse and Directed Systems of Soft Modules

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Let SM be the category of soft modules and I a directed poset (considered as a category). For every $i \in I$, let (F_i, A_i) be a soft module over M_i and for every $i < i'$, let $(p_i^{i'}, q_i^{i'}) : (F_{i'}, A_{i'}) \to (F_i, A_i)$ be soft homomorphism of soft modules.

Definition 1. If the conditions

(1) For $i = i'$ $p_i^{i'} = 1_{M_i}, q_i^{i'} = 1_{A_i}$;

(2) For $i < i' < i'' p_i^{i''} = p_i^{i'} \circ p_{i'}^{i''}$, $q_i^{i''} = q_i^{i'} \circ q_{i'}^{i''}$

are satisfied, then the family $\Big(\{(F_i, A_i)\}_{i\in I}, \{(p_i^{i'}, q_i^{i'})\}_{i<i'}\Big)$ (1) is said to be inverse system of soft modules.

Theorem 2. *Every inverse system in the category SM has a unique limit.*

Theorem 3. *Let $Inv(SM)$ be a category of all inverse systems in SM . Then $\varprojlim$ operation is a functor from the category of $Inv(SM)$ to the category of SM.*

Theorem 4. $\varprojlim \Big[(F_i, A_i) \,\widetilde{\cup}\, (G_i, B_i)\Big] \,\tilde{\to}\, \Big[\varprojlim(F_i, A_i)\Big] \,\widetilde{\cup}\, \Big[\varprojlim(G_i, B_i)\Big]$.

Direct systems of soft modules are defined by duality.

Theorem 5. *Every direct system in the category SM has a unique limit.*

Some Class of Semigroups of Binary Relations

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In this paper the semigroups $B_X(D)$ defined by semilattices of the class $\Sigma_4(X,7)$ are studied.The set $B_X(D)$ of all a binary relations α_f $(f : X \to D)$, $\alpha_f = \cup(\{x\} \times f(x))$ is a semigroup with respect to the operation of multiplication of binary relations [1].

We give a full description of regular elements of these semigroups. We have received formulas that allow to calculate the number of regular elements when X is a finite set.

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On the Dimension of Some Spaces of Generalized Theta-Series

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Let

$$Q(x) = Q(x_1, \cdots, x_f) = \sum_{0 \le r \le s \le f} b_{rs} x_r x_s$$

be an integral positive definite quadratic form in an even number f of variables.

Let $R(\nu, Q)$ denote the space of the spherical polynomials $P(x)$ of even order ν with respect to $Q(x)$ and let $T(\nu, Q) = \{\vartheta(\tau, P, Q) : \ P \in R(\nu, Q)\}$ is the space of generalized theta-series, where

$$\vartheta(\tau, P, Q) = \sum_{x \in \mathbb{Z}^f} P(x) z^{Q(x)}, \quad z = e^{2\pi i \tau}, \ \operatorname{Im} \tau > 0, \ \tau \in \mathbb{C}.$$

In [1–3] is obtained the upper bound for the dimension of the space $T(\nu, Q)$ for some quaternary quadratic forms. Here is calculated the dimension of the space $T(4, Q)$ and $T(8, Q)$.

We calculate the dimension of the space $T(6, Q)$.

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Modular Functions and Representations of Positive Integers by Quadratic Forms

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The modular properties of generalized theta-functions with characteristics and spherical polynomials are used to build a cusp form of weight 9/2. It gives the opportunity of obtaining exact formulas for the number of representations of positive integers by some quadratic forms in nine variables.

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Differential Equations and Applications

Cauchy Problem for a System of Hyperbolic Equations with Damping Terms

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Let's consider the system of semilinear hyperbolic equations

$$u_{1_{tt}} + u_{1_t} + (-1)^{l_1} \Delta^{l_1} u_1 = \lambda_1 |u_1|^{p-1} |u_2|^{q+1} u_1, \quad t > 0, \quad x \in R^n, \tag{1}$$

$$u_{2_{tt}} + u_{2_t} + (-1)^{l_2} \Delta^{l_2} u_2 = \lambda_2 |u_1|^{p+1} |u_2|^{q-1} u_2, \quad t > 0, \quad x \in R^n, \tag{2}$$

$$u_k(0,x) = \varphi_k(x), \quad u_{k_t}(0,x) = \psi_k(x), \quad k = 1,2, \quad x \in R^n. \tag{3}$$

Let

$$\lambda_1 > 0, \quad \lambda_2 > 0, \tag{4}$$

$$p \geq -1, \quad q \geq -1, \quad p+q > 0; \quad \frac{p+1}{l_1} + \frac{q+1}{l_2} > \frac{2}{n} + r(p,q), \tag{5}$$

where $r(p,q) = \frac{1}{l_1}$, if $p > 1$, $q \geq -1$; $r(p,q) = \frac{p}{2l_1} + \frac{2-p}{2l_2}$, if $-1 \leq p \leq 1$, $q > -1$.

Theorem. *Let condition* (4)–(5) *be satisfied, then there exists* $\delta > 0$ *such that, for any*

$$(\varphi_k, \psi_k) \in U_\delta =$$

$$= \Big\{ (u,v) : \|u\|_{W_2^{l_k}(R^n)} + \|u\|_{L_1(R^n)} + \|v\|_{L_2(R^n)} + \|v\|_{L_1(R^n)} < \delta \Big\}, \quad k = 1,2$$

problem (1)–(3) *has a unique solution* (u_1, u_2)*:*

$$u_k \in C([0,\infty); W_2^{l_k}(R^n)) \cap C^1([0,\infty); L_2(R^n)), \quad k = 1,2,$$

which satisfies the following estimates:

$$\sum_{|\alpha|=r} \|D^\alpha u_k(t,\cdot)\|_{L_2(R^n)} \leq C(1+t)^{-\frac{n+2r}{4l_k}}, \quad t > 0, \quad r = 0,1,\dots,l_k,$$

$$\|u_{k_t}(t,\cdot)\|_{L_2(R^n)} \leq C(1+t)^{-\min\left(1+\frac{n}{4l_k}, \gamma_k\right)},$$

where

$$\gamma_k = \frac{n}{2} \sum_{i=1}^{2} \frac{\rho_{ki}}{l_i} - r_k, \quad k = 1,2.$$

Next we discuss the counterpart of the condition (5).

On the Domains of Propagation of Characteristic Curves of Non-Strictly Hyperbolic Equations

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We consider the quasi-linear hyperbolic equation with possible parabolic degeneration:

$$u_y(u_y-1)u_{xx}+(u_y-u_x-2u_xu_y+1)u_{xy}+u_x(u_x+1)u_{yy}=0.$$

The Cauchy problem is studied for this equation. The general integral of the given equation plays the major role in the process of studying of the Cauchy problem, namely the characteristic curves. The general integral is obtained in explicit form.

$$f(u+x)+g(u-y)=y.$$

The structure of the domain of definition of the solution for the given equation has been studied. The cases of formation of sub-areas of non-existence of solutions inside the area of propagation of solution are considered in the work. Sufficient conditions for existence of such sub-areas, where the characteristic curves do not propagate, have been obtained. It's shown that in some cases the strong parabolic degeneracy of the equation may stimulate the formation of sub-areas of non-existence of solutions.

Analysis of Four-Port Rectangular Waveguide Junctions with Two Resonance Regions

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Waveguide junctions are widely used to compose directional couplers, power dividers, phase shifters, filters, multiplexers and other microwave devices. However, a rigorous analysis of them has been conducted only for relatively simple constructions, such as waveguide tees, cruciform waveguide junctions, etc.

In this work, using the Mode Matching Technique (MMT), a rigorous solution has been obtained for a boundary-value problem on four-port H-plane rectangular waveguide junction with 2 resonance regions.

First, electromagnetic fields in various waveguide regions has been written as Fourier series and integrals with yet unknown coefficients of discrete and continues Fourier spectra of waveguide harmonics. Next, applying the boundary conditions, a system of functional (integral and summation) equations in terms of unknown coefficients of Fourier spectra has been obtained.

Further, using the Fourier Transform Technique (FTT), the filtering properties of Dirac function in infinite domain, and orthogonality properties of transverse eigen-functions in a waveguide cross-section, a functional system of equations has been reduced to a dual linear algebraic system of equations. Finally, this system has been reduced and numerically solved in computer with controlling a reasonable accuracy of solution.

Using the created computer program, various electrodynamic characteristics of four-port waveguide junctions has been simulated and analysed. The validity of the obtained results will be illustrated, and different characteristics of waveguide junctions will be demonstrated. In particular, a near-field structure of the total electric field in a waveguide junction will be shown, the reflected and transmitted powers in different waveguide regions will be analysed, and a power balance in a waveguide junction will be demonstrated.

Mixed Boundary-Value Problems for Polymetaharmonic Equations

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The Riquier-type mixed boundary-value problems are considered for the polymetaharmonic equation. We investigate these problems by means of the potential method and the theory of pseudodifferential equations, prove the existence and uniqueness of solutions and establish their regularity properties in Sobolev–Slobodetski spaces. We analyse the asymptotic behaviour of solutions near the curve, where the different boundary conditions collide, and establish smoothness properties in Hölder spaces.

Localized Boundary Domain Integral Equations Approach to the Boundary-Value Problems for Inhomogeneous Elastic Solids

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We consider the Dirichlet boundary-value problem of elastostatics for anisotropic inhomogeneous solids and develop the generalized potential method based on the application of a localized parametrix. By means of the localized layer and volume potentials we reduce boundary-value problem to the localized boundary-domain integral equations (LBDIE) system. First we establish the equivalence between the original boundary-value problem and the corresponding LBDIE system. Afterwards, we establish that the localized boundary-domain matrix integral operator obtained belongs to the Boutet de Monvel algebra of pseudodifferential operators and with the help of the Vishik-Eskin theory, based on the factorization method (Wiener–Hopf method), we investigate Fredholm properties and prove invertibility of the localized operator in appropriate function spaces.

Localization of Boundary Value Problems

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Localization is a powerful tool in the investigation of the Fredholm properties of a boundary value problem for a partial differential equation in a domain with the smooth or piecewise-smooth boundary. It provides a better insight into the role of the Shapiro-Lopatinsky condition and, in combination with the uniqueness result and the index theorem, allows to prove the unique solvability of the boundary value problem.

We investigate a boundary value problem

$$\begin{cases} \mathbf{A}(x,D)u(x) = f(x), & x \in \Omega, \\ (\gamma_{\mathscr{S}}\mathbf{B}_j u)(t) = G_j(t), & j = 0,\dots,\ell-1, \ \ t \in \mathscr{S} := \partial\Omega, \end{cases} \tag{1}$$

with matrix $N \times N$ partial differential operators

$$\mathbf{A}(x,D) := \sum_{|\alpha|\leqslant m} a_\alpha(x)\partial^\alpha, \quad \mathbf{B}_j(x,D) = \sum_{|\alpha|\leqslant m_j} b_{j\alpha}(x)\partial^\alpha, \quad a_{\alpha,j,k}, b_{j\alpha,m,k} \in C^\infty(U_{\mathscr{S}}),$$

where $U_{\mathscr{S}} \subset \overline{\Omega}$ is a small neighborhood of the boundary $\mathscr{S}$. The BVP (1) we consider in generalized settings, including the spaces of distributions

$$\begin{gathered} f \in \mathbb{H}^m(\Omega), \ \ u \in \mathbb{H}^{m+\ell}(\Omega), \ \ G_j \in \mathbb{H}^{m+\ell-m_j-1/2}(\mathscr{S}), \\ m_j := \operatorname{ord} \mathbf{B}_j(x,D), \ \ j = 0,1,\dots,\ell-1, \end{gathered} \tag{2}$$

where m is an arbitrary integer, negative or positive. Under the single constraint on f that the Newtons's potential from it $N_\omega f$ has traces on the boundary, we prove that all traces in BVP (1) exist and investigate the solvability of the BVP (1)–(2) for negative $m = -1, -2, \dots$. The localization of BVP was investigated in [1, 2], while BVPs in generalized setting was partly discussed in [3].

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Fredholmity Criteria for a Singular Integral Operator on an Open Arc in Spaces with Weight

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In this paper we study a singular integral operator (SIO) with the Cauchy kernel

$$\mathbf{A}\varphi(t)=a(t)\varphi(t)+\frac{b(t)}{\pi i}\int_{\Gamma}\frac{\varphi(\tau)d\tau}{\tau-t},\quad A:\mathbb{L}_p(\Gamma,\rho)\to\mathbb{L}_p(\Gamma,\rho) \tag{1}$$

and continuous coefficients $a,b\in C(\Gamma)$ in the Lebesgue spaces $\mathbb{L}_p(\Gamma,\rho)$ with an exponential "Khvedelidze" weigh $\rho:=(t-c_1)^{\alpha}(t-c_2)^{\beta}$, $1<p<\infty$, $1/p-1<\alpha$, $\beta<1/p$. The underlying contour $\Gamma=\smile{c_1c_2}$, is an open arc with the endpoints c_1 and c_2.

It is well known, that the condition $\inf\limits_{t\in\Gamma}|a(t)\pm b(t)|\neq 0$ is necessary for the operator in (1) to be Fredholm, but is not sufficient. A necessary and sufficient condition is the so called "Arc Condition", which means that a chords of a circle, depending on the exponents of the space α,β and p and connecting the disjoint endpoints of the graph $a(c_1)\pm b(c_1)$ and $a(c_2)\pm b(c_2)$, does not cross zero 0. The "Arc Condition" was found by I. Gohberg and N. Krupnik in 1965 for the Lebesgue spaces $\mathbb{L}_p(\Gamma,\rho)$ (also see the earlier paper by H. Widom for $p=2$). The result was carried over in 1970 to the space of Hölder continuous functions $\mathbf{H}^0_{\mu}(\Gamma,\rho)$ with an exponential "Khvedelidze" weight $\rho:=(t-c_1)^{\alpha}(t-c_2)^{\beta}$, $0<\mu<1$, $\mu<\alpha,\beta<\mu+1$ by R. Duduchava in his doctor thesis.

Based on the Poincare–Beltrami formula for a composition of singular integral operators and the celebrated N. Muskhelishvili formula describing singularities of Cauchy integral, the formula for a composition of weighted singular integral operators $(-1<\gamma,\ \delta<1)$

$$S_{\gamma}S_{\delta}\varphi=\varphi(t)\ +\ i\,\cot\pi(\gamma-\delta)\,[S_{\gamma}-S_{\delta}]\,(\varphi),\quad S_{\delta}\varphi(t):=\frac{t^{\delta}}{\pi i}\int_0^{\infty}\frac{\varphi(\tau)}{\tau^{\delta}(\tau-t)}\,d\tau \tag{2}$$

is proved. Using the obtained composition formula (2), the localization (which means "freezing the coefficients") we derive the criterion of fredholmity of the SIO (1) (the "Arc Condition") by looking for the regularizer of the operator $\mathbf{A}$ in the form $\mathbf{R}=a^*I+b^*S_{\gamma}$, $a^*=a(a^2-b^2)^{-1}$, $b^*=-b(a^2-b^2)^{-1}$ and choosing appropriate γ. To the composition $\mathbf{RA}$ is applied the formula (2) and coefficients of non-compact operators are equated to 0 to get $\mathbf{RA}=I+T$, where T is compact. The "Arc Condition" follows. Further the index formula and the necessity of the "Arc condition" are proved by using a homotopy and the stability of the index of a Fredholm operators. Absolutely similar results with a similar approach are obtained for SIO (2) with Hölder continuous coefficients in the space of Hölder continuous functions $\mathbf{H}^0_{\mu}(\Gamma,\rho)$ with an exponential "Khvedelidze" weight.

Behaviour of Solutions to Degenerate Parabolic Equations

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For linear elliptic and parabolic equations the questions on behavior of solutions near the boundary were studied on the papers of O.A.Oleinik and his followers [1]. For quasilinear equations, similar result were obtained in the T. S. Gadjiev [2]. S. Bonafade [3] and others studied qualitative properties of solutions for degenerate equations.

We obtained some estimates that are analogies of Saint-Venaut's principle known in theory of elasticity. By means of these estimations we obtained estimations on behavior of solutions and their derivative on bounded domains up to boundary.

In the cilindric domain $Q = \Omega \times (0, T)$, $T > 0$, where $\Omega \subset R^n$, $n \geq 2$ bounded domain, a generalized solution from the Sobolev space $\dot{W}^{m,1}_{p,\omega}(\Omega)$ of the mixed problem for the equation

$$\frac{\partial u}{\partial t} - \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^{\alpha} A_{\alpha}(x, u, \nabla u, \dots, \nabla^m u) = \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^{\alpha} F_{\alpha}(x), \tag{1}$$

$$u/_{t=0} = 0\,, \tag{2}$$

where $D^{\alpha} = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \cdots \partial x_n^{\alpha_n}}$, $|\alpha| = \alpha_1 + \alpha_2 + \cdots + \alpha_n$, $m \geq 1$ is considered. Also we suppose Dirichlet conditions on boundary satisfying.

Our main goal is to obtain estimations of behavior of the integral of energy

$$I_{\rho} = \int_{\Omega_{\rho}} \omega(x) \, |\nabla^m u|^p \; dx \, dt,$$

for small ρ, dependent on Ω_{ρ} geometry of Ω in the vicinity of the point 0.

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Limit Cycle Problems in Neural Dynamical Systems

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We consider two planar cubic dynamical systems which are used for neural modeling. First, we study the classical FitzHugh–Nagumo planar cubic dynamical system which models the spike dynamics in biological neurons. Such a cubic model was studied earlier but its qualitative analysis was incomplete, since the global bifurcations of multiple limit cycles could not be studied properly by means of the methods and techniques which were used in the qualitative theory of dynamical systems. Applying the Wintner–Perko termination principle for multiple limit cycles and new geometric methods of the global bifurcation theory, we prove that the FitzHugh–Nagumo model can have at most two limit cycles. Then, we carry out the global bifurcation analysis of a higher-dimensional polynomial dynamical system as a learning model of neural networks (the Oja model). Learning models are algorithms, implementable as neural networks, that aim to mimic an adaptive procedure. A neural network is a device consisting on interconnected processing units, designated neurons. An input presented to the network is translated as a numerical assignment to each neuron. This will create a sequence of internal adjustments leading to a learning process. For two input neurons, e.g., the model can be written as a planar cubic centrally symmetric dynamical system. Applying to this system the Wintner–Perko termination principle and our bifurcationally geometric methods, we prove that the planar Oja neural network model has a unique limit cycle.

The Method of Operator Power Series

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The method of operator power series is based on a symbolic method of co-representation of solutions of differential equations in partial derivatives. This method goes on-justification in the theory of pseudodifferential operators [1]. The evolution of representations of the symbolic method and the theory of operators in relation to the method of operator power series method and mechanics of deformable solids – the method of initial functions, has its own literature [2]. The solution of homogeneous linear partial differential equation in the derivatives of n-th order with constant or variable coefficients,

$$(D_0\partial_1^n + D_1\partial_1^{n-1} + \cdots + D_{n-1}\partial_1 + D_n)\, F(x_1, x_2, \dots, x_m) = 0\,,$$

where $\partial_1 = \partial/\partial x_1$ is the partial derivative with respect to x_1; D_i, $i = 1, 2, \dots, n$, are operators which consist of derivatives $\partial_k = \partial/\partial x_k$, $k = 2, 3, \dots, m$, and their various combinations with some constant or variable coefficients; $F(x_1, x_2, \dots, x_m)$ is an unknown function of m variables representable as $F = \sum\limits_{k=0}^{n-1} L_k\, f_k$, where L_k are the operators-functions:

$L_k = \sum\limits_{m=0}^{\infty} l_{km}\, x_1^m/m!$, where l_{km} are differential operators in which the composition includes derivatives ∂_k, $k = 2, 3, \dots, m$, with some constant coefficients, and $l_{km} = 1$ for $k = m$ and $l_{km} = 0$ for $m < k$; $F = f_0(x_2, \dots, x_m)$, $\partial_1 F = f_1(x_2, \dots, x_m)$, $\partial_1^{n-1} F = f_{n-1}(x_2, \dots, x_m)$. Thus, the problem of finding the function F is reduced to finding of arbitrary functions f_k, defined on a surface $x_1 = 0$. Sometimes in order to facilitate the satisfaction of the boundary conditions of the definition of f_k is derived by establishing more links with some of these functions, the values that have clear physical (mechanical) sense. In this paper some properties of operator-functions, the correctness of the Cauchy problem, algebra of operators and operator-functions are discussed.

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On Asymptotic Behaviour of Solutions of Third Order Linear Systems of Differential Equation with Deviating Arguments

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The problem of oscillation of solutions is well studied for differential equations of high order. In particular, many interesting results of optimal types have been obtained (see, for example [1]). The purpose of the present report is to establish some optimal sufficient conditions for the oscillation of solutions of three-dimensional linear systems. More precisely, we present necessary and sufficient conditions for the oscillation of proper solutions of the system

$$\begin{aligned} x_1'(t) &= p_1(t)x_2(\tau_1(t)),\\ x_2'(t) &= p_2(t)x_3(\tau_2(t)),\\ x_3'(t) &= -p_3(t)x_1(\tau_3(t)), \end{aligned}$$

where $p_i \in L_{loc}(R_+, R_+)$, $\tau_i \in C_{loc}(R_+, R_+)$, $\lim\limits_{t\to+\infty} \tau_i(t) = +\infty$ $(i = 1, 2, 3)$.

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Boundary Value Problem for Klein–Gordon Equation in Space $\mathbb{R}^3$

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Let Ω be a smooth bounded domain of the class C^2 and $f \in C(\partial\Omega)$ be a boundary function. Define potential

$$U^{\psi}(x) = \int_{\partial\Omega} \Gamma(x,y)\psi(y)\,dS_y,$$

where

$$\Gamma(x,y) = \frac{e^{-k|x-y|}}{4\pi|x-y|}\,, \quad k = constant > 0.$$

Find a solution v of the Klein-Gordon equation in the domain Ω which satisfies the condition

$$q_1(x)\,\frac{\partial^2 v(x)}{\partial \nu_x^2} + q_2(x)\,\frac{\partial v(x)}{\partial \nu_x} + q_3(x)x(x) = f(x), \quad x \in \partial\Omega,$$

where $q_i(x) > 0$, $x \in \partial\Omega$, $q_i(x) \in C(\partial\Omega)$, $i = 1,2,3$.

We prove unique solvability of this problem.

Similar assertion holds true for elliptic partial differential equation

$$\sum_{i,k=1}^{3} \frac{\partial}{\partial x_i}\left(a_{ik}\,\frac{\partial u}{\partial x_k}\right) + a(x)u(x) = 0,$$

where $a_{ik} \in C^{(3,\alpha)}(R^3)$, $a \in C^{(2,\alpha)}(R^3)$, $a(x) < -\lambda^2$, $x \in R^3$, $\lambda = constant > 0$.

$$m\sum_{i=1}^{3}\xi_i^2 \le \sum_{i,k=1}^{3} a_{ik}(x)\xi_i\xi_k \le M\sum_{i=1}^{3}\xi_i^2, \quad m = constant > 0, \quad M = constant > 0.$$

On the Solvability of Cauchy Spatial Characteristic Problem for One Class of Second Order Semilinear Wave Equations

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Consider the semilinear wave equation of the type

$$\frac{\partial^2 u}{\partial t^2} - \sum_{i=1}^{3}\frac{\partial^2 u}{\partial x_i^2} + f(u) = F, \tag{1}$$

where f and F are given real functions, f is a nonlinear function, and u is unknown real function.

For the equation (1) we consider the Cauchy characteristic problem: find in the frustrum of the light cone of the future $D_T:\ |x|<t<T$, $x=(x_1,x_2,x_3)$, $T=const>0$, a solution $u(x,t)$ according to the boundary condition

$$u\big|_{S_T}=0, \tag{2}$$

where $S_T:\ t=|x|$, $t\le T$, is the characteristic conic surface.

Let $\overset{\circ}{W}{}_2^k(D_T,S_T):=\{u\in W_2^k(D_T):\ u|_{S_T}=0\}$, where $W_2^k(D_T)$ is the well-known Sobolev space, consisting of elements from $L_2(D_T)$ which have generalized derivatives in $L_2(D_T)$ up to order k, inclusively, and the equality $u|_{S_T}=0$ is understood in the sense of the trace theory.

We consider certain conditions imposed on the function f, which for every solution $u\in \overset{\circ}{W}{}_2^2(D_T)$ of the problem (1), (2) provide the validity of the following a priori estimate

$$\|u\|_{\overset{\circ}{W}{}_2^2(D_T)}\le c\Big[1+\|F\|_{L_2(D_T)}+\lambda\|F\|^3_{L_2(D_T)}+\|F\|_{W_2^1(D_T)}\exp\big(c\|F\|^2_{L_2(D_T)}\big)\Big] \tag{3}$$

when $F\in \overset{\circ}{W}{}_2^k(D_T,S_T)$, with a positive constant c not depending on u and F.

Using the estimate (3) we prove that the problem (1), (2) has a unique solution $u\in \overset{\circ}{W}{}_2^2(D_T)$. Whence, in turn, it follows a global solvability of the problem (1), (2) in the light cone of the future $D_\infty:\ t>|x|$ in the following sense: for any $F\in \overset{\circ}{W}{}^1_{2,loc}(D_\infty,S_\infty)$ there exists a unique solution $u\in \overset{\circ}{W}{}^2_{2,loc}(D_\infty,S_\infty)$ of the problem (1), (2), where

$$\overset{\circ}{W}{}^k_{2,loc}(D_\infty,S_\infty):=\Big\{v\in L_{2,loc}(D_\infty):\ v\big|_{D_T}\in \overset{\circ}{W}{}_2^k(D_T,S_T)\ \forall\, T>0\Big\}.$$

Optimal Systems of One-Dimensional Subalgebras of the Symmetry Algebra of Hyperbolic Equations of Perfect Plasticity

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The present paper is devoted to application of the Lie group theory to a three-dimensional non-linear system of partial differential equations known from the mathematical theory of perfect plasticity. The Coulomb–Tresca yielding criterion and associated flow rule are employed to formulate the system of differential equations. If an actual stress state corresponds to an edge of the Coulomb–Tresca prism then the stress tensor are determined by the maximal (or minimal) principal stress and the unit vector field directed along the principal stress axis related with that principal stress, thus allowing the static equilibrium equations can be formally considered independently of equations sequent to associated flow rule. The system first obtained by D. D. Ivlev in 1959 in an attempt to find new approaches to correct mathematical study of three-dimensional perfectly plastic problems is of crucial importance for continuum mechanics and its numerous applications. It is of hyperbolic type thus predicting slip-lines mechanism of perfectly plastic flow in accordance with contemporary point of view and providing significant mathematical advantages for the present study.

The original essentially non-linear system of partial differential equations is transformed to a special coordinate system defined in the space by the stress principal lines (isostatic coordinate net). Group analysis of the obtained in such a way system of partial differential equations of three-dimensional perfect plasticity is carried out. The symmetry group of this system is obtained. A natural 12-dimensional symmetry algebra and a first order optimal system of one-dimensional subalgebras of the symmetry group of partial differential equations of the three-dimensional mathematical theory of plasticity are studied. The optimal system of one-dimensional subalgebras constructing algorithm for the 12-dimensional Lie symmetry algebra is proposed. The optimal system (total 187 elements) is shown consist of a 3-parametrical element, twelve 2-parametrical elements, sixty six 1-parametrical elements and one hundred and eight individual elements.

By the Lie technique new exact solutions in the analytically closed forms for the case of axial symmetry are obtained. Some of them are represented by the canonical Legendre elliptic integrals.

Fredholmity Criterion for Teplitz and Winer–Hopf Operators

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We study Teplitz operators $T_a = [a_{j-k}]_{j,k=0}^{\infty} : \ell_p \to \ell_p$ in the space of sequences ℓ_p for $1 \le p \le \infty$, The symbol $a(\zeta)$ is a continuous ℓ_p-Multiplier on the unit circle a $|\zeta| = 1$ and $\{a_j\}_{j=-\infty}^{\infty}$ are its Fourier coefficients. Also we study Wiener–Hopf operators

$$W_a\varphi(x) = c\varphi(x) + \int\limits_0^\infty k(x-y)\varphi(y)dy, \quad W_a : \mathbb{L}_p(\mathbb{R}^+) \to \mathbb{L}_p(\mathbb{R}^+)$$

in the Lebesgue space $\mathbb{L}_p(\mathbb{R}^+)$ of p-integrable functions for $1 \le p \le \infty$ on the half axis $\mathbb{R}^+ = (0,\infty)$. The symbol $a(\xi) = c + (\mathscr{F}k)(\xi)$, $\xi \in \mathbb{R} = (-\infty,\infty)$, is a continuous $\mathbb{L}_p$-Multiplier and $\mathscr{F}$ is the Fourier transform.

It is well-known, that the ellipticity of the symbol $\inf\limits_{|\zeta|=1} |a(\zeta)| \ne 0$ is a necessary and sufficient condition for operators W_a and T_a to be Fredholm (have a closed range, the finite dimensional kernel and the finite dimensional cokernel) in $\mathbb{L}_p(\mathbb{R}^+)$ and ℓ_p spaces, respectively (see, for example, article of M. Krein published in 1957, I. Gohbergs and I. Feldmans monograph).

We will give a new proof of the Fredholmity criteria for the operators above, which applies a homotopy and stability of the index of Fredholm operators. A similar approach was used in an article published in 1970 by R.Duduchava, where criterion of Fredholmity of a singular integral equations in Hö lder spaces with weight was proved.

Theorem. *Let a be a continuous ℓ_r-multiplier for $p-\varepsilon < r < p-\varepsilon$ for some $\varepsilon > 0$ and $1 < p < \infty$. For an operator T_a to be Fredholm in the space $\ell_p(N)$ it is necessary and sufficient that $\inf\limits_{|\zeta=1} |a(\zeta)| \ne 0$. If these conditions are hold, then index of the operator equals*

$$\operatorname{Ind} T_a = -\operatorname{ind} a.$$

Moreover, the invertibility of the operator T_a is agreed with the index ind a*. In other words that means that the operator T_a is invertible from the left (is invertible from the right) if only $n = \operatorname{ind} a \geq 0$ (Or, respectively, $n = \operatorname{ind} a \leq 0$). On the dense set in the space $\ell_p(N)$ the corresponding inverse operator from the left (from the right) is written as follows $(T_a)^{-1}_{\text{left}} = (T_a)^{-1}_{\text{right}} = T^{-1}_{a_+} T_{r_{-n}} T_{a_-^{-1}}$, where $a = a_- r_n a_+$ is a generalized p-factorization of the symbol a (see I. Simonenko's and I. Gohber, N. Krupnik's papers).*

In particular, if $\operatorname{ind} a = 0$*, then the operator T_a is invertible and the bilateral inverse operator read $T_a^{-1} = T^{-1}_{a_+} T^{-1}_{a_-}$.*

The result is quite similar to that of Wiener–Hopf operator W_a.

Localized Boundary-Domain Integral Equations for Acoustic Scattering by Inhomogeneous Anisotropic Obstacle

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We consider the acoustic wave scattering problems when the material parameters and speed of sound are functions of position within the inhomogeneous anisotropic bounded obstacle. The problem is formulated as a transmission problem (TP) for a second order elliptic partial differential equation with variable coefficients in the inhomogeneous region and for the "anisotropic" Helmoltz type equation with constant coefficients in the unbounded homogeneous region. The transmission problem treated in the paper can be investigated by the variational method and also by the classical potential method when the corresponding fundamental solution is available in explicit form. Our goal here is to show that the above mentioned TP with the help of localized potentials corresponding to the Laplace operator can be reformulated as a coupled localized boundary-domain integral equations (LBDIE) system and prove that the corresponding localized boundary-domain integral operator (LBDIO) is invertible. Beside a pure mathematical interest these results seem to be important from the point of view of numerical analysis, since LBDIE can be applied in constructing convenient numerical schemes in applications. In our case, we apply the localized parametrix which is represented as the product of the fundamental solution function of the Laplace operator and an appropriately chosen cut-off function supported on some neighbourhood of the origin. Evidently, the kernels of the corresponding localized potentials are supported in some neighbourhood of the reference point and they do not solve the original differential equation. By means of the usual and localized layer and volume potentials we reduce the TP to the localized boundary-domain integral equations system. First we establish the equivalence between the original boundary-transmission problems and the corresponding LBDIE systems which plays a crucial role in our analysis. Afterwards, we establish that the localized boundary domain integral operators obtained belong to the Boutet de Monvel algebra of pseudo-differential operators and on the basis of the Vishik–Eskin theory based on the factorization method we investigate corresponding Fredholm properties and prove invertibility of the LBDIO in appropriate function spaces.

This is a joint work with Otar Chkadua and Sergey Mikhailov.

Reduction Principle in the Theory of Stability of Impulsive Differential Systems

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We consider the system of impulsive differential equations in Banach space that satisfies the conditions of integral separation. We prove the theorem of asymptotic phase. Using this result and the centre manifold theorem we reduce the investigation of stability of the trivial solution of initial impulsive differential system to investigation of stability of simpler impulsive differential system.

The research was supported by the grant 09.1220 of the Latvian Council of Science and by the grant 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund.

About Development of Elliptic Theory

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I would like to remind the colleagues about researches of I. G. Petrovskii, N. I. Muskhelishvili, A. V. Bitsadze, I. N. Vekua, F. D. Gahov, N. E. Tovmasyan, Ja. B. Lopatinskii, S. Agmon, A. Douglis, L. Nirenberg, M. I. Vishik, M. S. Agranovich, L. R. Volevich, A. Dynin, V. A. Solonnikov, V. V. Grushin, B. R. Vainberg and other.

Their results make the base of the elliptic theory. A one-dimensional singular integral operator of normal type [1] we consider as elliptic operator of order $(0,0)$.

We denote by $P\cdot EL(X)$, $DN\cdot EL(X)$, $VG\cdot EL(X)$ the classes of pseudo differential operators on a closed manifold X, defined by Petrovskii [2], Douglis and Nirenberg [3], Vainberg and Grushin [4]. In my paper [5] a class $GEL(X)$ of generalized elliptic operators was introduced and I've proved the following inclusions:

$$P\cdot EL(X)\subset DN\cdot EL(X)\subset GEL(X),\quad VG\cdot EL(X)\subset GEL(X).$$

The operators from $GEL(X)$ which don't belong to $DN\cdot EL(X)$ are called weakly elliptic and the set of such operators we denote by $WEL(X)$. Operators $rot+\lambda I$ and $\nabla div+\lambda I$ are weakly elliptic if $\lambda\neq 0$. A analogous class of (p,q) elliptic one-dimensional singular integro-differential operators was studied in my book "*Boundary value problems...*", 1975.

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The Weighted Cauchy Problem for Linear Functional Differential Equations with Strong Singularities

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The sufficient conditions of well-posedness of the Cauchy weighted problem for linear functional equations of higher order with deviating arguments whose coefficients have nonintegrable singularities at the initial point, are found.

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Asymptotic Behaviour of Solutions of Mixed Problem for Hyperbolic Equations with Periodic Coefficients, when the Corresponding Hill'S Operator is Non-Positive

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The talk will discuss asymptotic behaviour of solutions of the problem

$$u_{tt} - u_{xx} + q(x)u = 0, \tag{1}$$

$$u(x,0) = 0, \;\; u_t(x,0) = f(t), \;\; t \geq 0, \quad x \in [0,b], \quad u(0,t) = 0, \;\; t > 0 \tag{2}$$

as $t \to \infty$. Here $q(x)$ continuous periodic function with period one, $f(x) \in C_0^\infty(R^1)$ such that $\sup f \subset [0,1]$, and b a positive number.

The main result is the following

Theorem. *Assume that the left end point of the corresponding Hill's operator is negative and coincides with $-\alpha^2$ where $\alpha > 0$. Then the solution u of the problem (1), (2) admits the gollowing representation,*

$$u(x,t) = t^{-3/2}e^{\alpha t}\big(h(x) + v(x,t)\big).$$

Here $h(t)$ is a known function, and $v(x,t)$ is a function which admits the estimate

$$|v(x,t)| \leq \frac{C(b)}{t}\,\|f\|_{L^2}.$$

Probability & Statistics and Financial Mathematics

Integral Limit Theorems for the First Achievement Time of High Level by Branching Processes

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Recently appeared some papers on the investigation of boundary value problems for random walks described by the Markov chains and also branching processes (see, for example [1]).

In the present paper, the integral limit theorems for the moment of the first achievement of certain level by Galton–Watson critical and supercritical branching processes are proved.

Let Z_n, $n = 0, 1, 2, \dots$ be the Galton–Watson branching process c $0 < EZ_1^2 < \infty$.

Consider the first achievement moment $\tau_c = \inf\{n \geq 0 : \ Z_n > c\}$ of the level $c > 0$ by the process Z_n.

The following limit theorems are valid.

Theorem 1. *Let $Z_n \uparrow$ as $n \to \infty$ almost sure, and $EZ_1 = \mu > 0$, then, $P\{\tau_c < \infty\} = 1$, $\forall\, c \geq 0$; $\tau_c \to \infty$ as $c \to \infty$; $\frac{\tau_c}{c} \to \frac{1}{\mu}$ as $c \to \infty$.*

Theorem 2. *Let $Z_n \uparrow$ almost sure, $\mu = EZ_1 = 1$ and $0 < DZ_1 = 2\sigma < \infty$. Then for $x \geq 0$*

$$P\left\{\frac{\tau_c}{c} \leq x \mid \ Z_n > 0\right\} \to e^{\frac{1}{\sigma x}} \ \text{ as } \ c \to \infty.$$

Theorem 3. *Let $Z_n \uparrow$ almost sure, $\mu = EZ_1 > 1$, $0 < DZ_1 < \infty$. Then there exists a random variable T with the continuous distribution function $F(x) = P\{T \leq x\}$ for $x \neq 0$, such that*

$$P\left\{\frac{\mu^{\tau_c}}{c} \leq x \mid \ T > 0\right\} \to \frac{1 - F(1/x)}{1 - F(0)} \ \text{ as } \ c \to \infty.$$

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On the Absolute Continuity of the Distribution of a Schödinger Type Equation with Random Perturbation

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Let $H_+ \subset H \subset H_-$ be an equipped Hilbert space with quasi-kernel embeddings. We consider in H_- the problem

$$L\xi + (a, \xi)g = \eta, \quad \xi_\Gamma^+ = \zeta. \tag{1}$$

The terms and notation are the same as in [1–2]. It is assumed that H_- is a local Hilbert space, and $L : D(S) \to H_-$ is a linear continuous local operator. We denote by $L^* : H_- \to D'(S)$ the Banach conjugate to L. In addition to (1) we also consider the system

$$L\hbar = \eta, \quad \hbar_\Gamma^+ = \zeta. \tag{2}$$

Let A be the operator generated by equality (1) with a homogeneous initial condition.

Theorem. *Let the following conditions be fulfilled for* (1) *and* (2) :

1) *L is a linear continuous local operator with the domain of definition $D(L)$ densely embedded in H_-;*
2) *$g \in H_+$ and $|(a,u)| \cdot \|g'A^{-1}\| < 1$;*
3) *η is a random element in H_- with distribution μ_η and having a logarithmic derivative $\lambda(x)$ along H_+.*

Then $\mu_\xi \sim \mu_\hbar$ and

$$\frac{d\mu_\xi}{d\mu_\hbar}(u) = \det{}_H\big(I + (a,u)A^{-1}g\big)\exp\left\{\left(\int_0^1 \lambda(Au + t(a,u)g)dt, (a,u)g\right)_H\right\},$$

if η is a Gaussian random element with a unit correlation operator in H, then

$$\frac{d\mu_\xi}{d\mu_\hbar}(u) = \det{}_H\big(I + (a,u)_H A^{-1}g\big)\exp\left\{-(a,u)_H(Au,g)_H - \frac{1}{2}(a,u)_H^2\|g\|_H^2\right\}.$$

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Convergence Almost Surely of Summands of a Random Sum

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We announce the following

Theorem. *Let (Y_n) and (Z_n) be two sequences of random variables such that $Y_n + Z_n \to 0$ a.s. Then $Y_n \to 0$ a.s. (and $Z_n \to 0$ a.s.) provided that the following condition is satisfied*

$$P(|X_n + Y_n| \geq |Y_n| \quad | \quad \mathscr{F}_n) \geq c,$$

where $\mathscr{F}_n$ is the σ-algebra generated by $|Y_1|,\dots,|Y_n|$, and c is a positive constant independent of n.

The theorem holds true in the general case of normed-space-valued random variables (just the absolute value should be replaced by the norm). The theorem implies the well-known theorems belonging to Loeve [1] and Martkainen [2].

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On One Problem of Disorder

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On a probability space $(\Omega,\mathscr{F},\mathscr{P})$ consider the mutually independent random variables θ and η with values in $[0,+\infty)$ and the standard Wiener process $w_t, t\geq 0$. Suppose that

$$P(\theta=0)=\pi,\quad P(\theta\geq t|\,\theta>0)=e^{-\lambda t},\ \ 0<\lambda<\infty,\ \ 0\leq\pi\leq 1,$$

$$P(\eta\leq x)=1-e^{-\nu x},\ \ 0<\nu<\infty.$$

We also assume that the observable random process ξ_t, $t\geq 0$, has the stochastic differential

$$d\xi_t=r\chi(t-\theta)dt+\sigma dw_t,\ \ r\neq 0,\ \ \sigma^2>0, \tag{1}$$

where $\chi(t)=0$ if $t<0$ and $\chi(t)=1$ if $t\geq 0$. We consider the problem of the earliest detection of θ, i.e. the problem of disorder (disruption) for a Wiener process (1) in the Bayes formulation (see Shiryayev 1978). Let

$$\rho(\tau)=cP(\tau<\theta)+\nu\int_0^\infty P(\theta\leq\tau\leq\theta+\eta)e^{-\nu\eta}\,d\eta,\ \ c>0. \tag{2}$$

We say that the τ^* is Bayes stopping time, if $\varrho=\varrho(\tau^*)=inf\varrho(\tau)$, where inf is taken over the class of some stopping times.

Theorem. *The Bayes stopping time*

$$\tau^*=inf\{t\geq 0:\ \psi_t\geq A^*\}, \tag{3}$$

where ψ_t, $t\geq 0$, is random process with the stochastic differential

$$d\psi_t=[1+(\lambda-\nu)\psi_t]dt+\frac{r}{\sigma^2}\,\psi_t d\xi_t \tag{4}$$

and the threshold A^* is the unique solution of the equation

$$\big[-A^2\lambda(1-\lambda)+A(1-c)+c\big]\Phi(A)-c-\lambda A=0, \tag{5}$$

where

$$\Phi(x)=x^{\lambda}\cdot e^{\frac{1}{x}}\cdot\int\limits_0^x u^{-(\lambda+2)}e^{-\frac{1}{u}}\,du.$$

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Partially Independence of Random Variables

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1. We say that real random variables X and Y on the probability space $(\Omega,\mathscr{F},\mathscr{P})$ are $\mathscr{A}$-independent ($\mathscr{A}$ is the subset of R^2) iff $F_{XY}(x,y)=F_X(x)F_Y(y)$ for all $(x,y)\in\mathscr{A}$. Here $F_{XY}(x,y)=P(X\le x,Y\le y)$ is joint distribution function of X and Y; $F_X(x)$ and $F_Y(y)$ are the probability distribution functions of X and Y, respectively.

It is clear, that usually independence of random variables X and Y coincides with their $\mathscr{A}=R^2$-independence.

2. Let $f(x)$ be standard normal distribution density and $f(x,y)$ is joint normal distribution density with correlation matrix $\begin{pmatrix}1&\rho\\ \rho&1\end{pmatrix}$, $|\rho|<1$ and $f(x\mid y)=\frac{f(x,y)}{f(y)}$, $f(y\mid x)=\frac{f(x,y)}{f(y)}$ are the conditional distribution densities.

Let $\mathscr{A}_{++}=\{(x,y)\in R^2:\ x\ge0,\ y\ge0\}$, $\mathscr{A}_{--}=\{(x,y)\in R^2:\ x<0,\ y<0\}$, $\mathscr{A}_{+-}=\{(x,y)\in R^2:\ x\ge0,\ y<0\}$, $\mathscr{A}_{-+}=\{(x,y)\in R^2:\ x<0,\ y\ge0\}$ and

$$\begin{aligned}g(x,y)=C_+^2I_{\mathscr{A}_{++}}(x,y)f(x,y)+C_-^2I_{\mathscr{A}_{--}}(x,y)f(x,y)+\\ +I_{\mathscr{A}_{+-}}(x,y)u_+(x)f(x)u_-(y)f(y)+I_{\mathscr{A}_{-+}}(x,y)u_-(x)f(x)u_+(y)f(y),\end{aligned} \tag{1}$$

where $I_{\mathscr{A}}(x,y)$ is the indicator of $\mathscr{A}$ and

$$u_+(x)=\begin{cases}\alpha^{-\frac12}C_+\int\limits_0^\infty f(y\mid x)\,dy, & x\ge0\\ 0, & x<0.\end{cases};\quad u_-(x)=\begin{cases}\alpha^{-\frac12}C_-\int\limits_{-\infty}^0 f(y\mid x)\,dy, & x<0\\ 0, & x\ge0.\end{cases};$$

$$u(x)=\begin{cases}u_+(x), & x\ge0\\ u_-(x), & x<0\end{cases};\quad \alpha=\int\limits_0^\infty\int\limits_0^\infty f(x,y)\,dx\,dy.$$

We choose the constants C_+ and C_- such a way, that both are positive and $C_+ + C_- = \alpha^{-\frac{1}{2}}$. For example $C_+ = C_- = \frac{1}{2\alpha^{\frac{1}{2}}}$ or $C_+ = \frac{1}{3\alpha^{\frac{1}{2}}}$, $C_- = \frac{2}{3\alpha^{\frac{1}{2}}}$.

Theorem. *The real function $g(x,y)$, defined on R^2 by* (1) *is the probability distribution density and marginal distribution densities are $g(x) = u(x)f(x)$, $g(y) = u(y)f(y)$. If $g(x,y)$ is the joint distribution density of random variables X and Y, then they are $(\mathscr{A}_{+-} \cup \mathscr{A}_{-+})$-independent.*

დისპერსიული ანალიზის მეთოდის შესახებ

თემურ კოკობინაძე

შოთა რუსთაველის სახელმწიფო უნივერსიტეტი

მათემატიკის დეპარტამენტი

ბათუმი, საქართველო

გამოკვლეული იქნა დისპერსიული ანალიზის მეთოდით სარგებლობის საკითხი.

ერთფაქტორიანი დიპერსიული ანალიზის მეთოდით შეიძლება შესწავლილი იქნას სხვადასხვა სახის პროცესის დამახასიათებელ შედეგობრივ მახასიათებელზე რაოდენობრივი ან ხარისხობრივი შემთხვევითი და არაშერმთხვევითი ფაქტორების გავლენა. საწყის ეტაპზე ფიშერის კრიტერიუმის გამოყენებით ორი ან მეტი რაოდენობის ამოკრეფისათვის მოწმდება ჰიპოთეზა მათი საშუალოების ტოლობის შესახებ. აღნიშნული ჰიპოთეზის უკუგდების შემთხვევაში სხვადასხვა კრიტერიუმების გამოყენებით ხდება დაკვირვებათა იმ ორი ჯგუფის ამორჩევა რომელთა საშუალოებიც მნიშვნელოვნად განსხვავდებიან ერთმანეთისგან ან ფაქტორის იმ დონის განსაზღვრა, რომლის გავლენა შედეგობრივ მახასიათებელზე ნიშანდობლივია. სტატისტიკური დასკვნები კეთდება იმისდა მიხედვით, თუ რამდენად განსხვავდებიან ერთმანეთისგან ჯგუფური საშუალოების ზოგადი საშუალოსგან გადახრის კვადრატებისა და დაკვირვებათა შედეგების ჯგუფური საშუალოებიდან გადახრის კვადრატების ჯამები.

ორფაქტორიანი დისპერსიული ანალიზის მეთოდით სარგებლობისას ჯერ მოწმდება ფაქტორების ურთიერთგავლენა. შემდეგ ცალკეული ფაქტორების გავლენა შედეგობრივ მახასიათებელზე და ბოლოს განისაზღვრება ფაქტორების ის მნიშვნელობები რომელთა გავლენა შედეგობრივ მახასიათებელზე განსაკუთრებულია.

შედეგობრივ მახასიათებელზე ორზე მეტი ფაქტორის გავლენის წინასწარი გამოკვლევა უმჯობესია ჩატარდეს მრავლობითი რეგრესიული ანალიზისა და კორელიაციური ანალიზის მეთოდების გამოყენებით.

დისპერსიული ანალიზის მეთოდი შეიძლება გამოყენებული იქნას მედიცინაში, ბიოლოგიაში, სოფლის მეურნეობაში, ეკონომიკური, სოციოლოგიური, ეკოლოგიური, ბუნებრივი და სხვა სახის პროცესების შესწავლისას.

Banach Space Valued Ito Processes, the Ito Formula

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The stochastic integral (generalized stochastic integral) from Banach space valued non-anticipating random process with respect to the one dimensional Wiener process and from operator valued non-anticipating random process with respect to a Banach space valued Wiener process are considered. For the corresponding Ito processes the Ito formula is proved.

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Modelling of Comparison Sequential Test for Reliability

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The paper deals with development of a planning method for a sequential probability ratio test (SPRT) for reliability. In the course of this test, a hypothesis is checked whereby the mean time between failures (MTBF) of the tested item exceeds that of another item, chosen as base – by not less than a specified factor. It is assumed that the TBF's of both items are exponentially distributed. The particular feature of this work that the MTBF of the basic item is known only as a sample-derived estimate (see for example [1-3]).

The planning assignment consists in determining, the boundaries at which a decision is reached on acceptance or rejection of the checked hypothesis. The initial planning data are two points on the test's operating characteristic – associated with the error probabilities of the I-st and II-nd kind. As this problem does not lend itself to exact analytical treatment, an alternative approach, based on modeling, is proposed.

The presented solution comprises a search algorithm and the calculation results. Also presented are approximative formulae which make for a simpler search process, are sufficiently accurate for industry purposes, and obviate the need for extra searching.

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Limit Distribution of a Quadratic Deviation for Nonparametric Estimate of the Bernoulli Regression

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Let Y be a random variable with Bernoulli distribution and $p = p(x) = P\{Y = 1/x\}$. Suppose that x_i, $i = \overline{1,n}$ are points of partition of $[0,1]$ which are chosen from relation $\int\limits_0^{x_i} h(x)\,dx = \frac{i-1}{n-1}$, where $h(x)$ is the known positive density of the distribution on the interval $[0,1]$. Furthermore Y_{ij}, $i = \overline{1,n}$, $j = \overline{1,N}$, are independent Bernoulli random variables with $P\{Y_{ij} = 1/x_i\} = p(x)$, $P\{Y_{ij} = 0/x_i\} = 1 - p(x)$, $i = \overline{1,n}$, $j = \overline{1,N}$. Then $Y_i = \sum\limits_{j=1}^{N} Y_{ij}$ has the binomial distribution law $B(N, p_i = p(x_i))$. We observe values $Y_1, \dots, Y_n$. Our aim is to construct estimate for $p(x)$ analogously of the Nadaraya–Watson regression function estimation: $\widehat{p}_{nN}(x) = \frac{\varphi_{nN}(x)}{f_n(x)}$, $\varphi_{nN}(x) = \frac{1}{nb_n} \sum\limits_{i=1}^{n} K\big(\frac{x-x_i}{b_n}\big) \frac{1}{h(x_i)} Y_i$, $f_n(x) = \frac{1}{nb_n} \sum\limits_{i=1}^{n} K\big(\frac{x-x_i}{b_n}\big) \frac{1}{h(x_i)}$, where $b_n > 0$, $b_n \to 0$ as $n \to \infty$. Suppose that following properties are fulfilled:

i) $K(x)$ is the distribution density, $\sup\limits_x K(x) < \infty$, $K(-x) = K(x)$, $\operatorname{supp}(K) \subset [-\tau, \tau]$ and has bounded derivative; ii) $p \in C^2[0,1]$; iii) $h(x) \geq \mu > 0$ and $h \in C^1[0,1]$.

We study the properties of such estimations. Let's give one property

Denote $U_{nN} = nNb_n \int\limits_{\Omega_n} (\varphi_{nN}(x) - E\varphi_{nN}(x))^2\,dx$, $\Omega_n = [\tau b_n, 1 - \tau b_n]$, $\Delta_n = EU_{nN}$, $\sigma_n^2 = 4(nb_n)^{-2} \sum\limits_{k=2}^{n} p(x_k)(1 - p(x_k)) \sum\limits_{i=1}^{k-1} p(x_i)(1 - p(x_i)) Q_{ik}^2$, $Q_{ij} = \frac{1}{h(x_i)h(x_j)} \int\limits_{\Omega_n} K\big(\frac{x-x_i}{b_n}\big) K\big(\frac{x-x_j}{b_n}\big)\,dx$.

Theorem. *If $nb_n^2 \to \infty$ as $n \to \infty$, then $\Delta_n = \Delta(p) + O(b_n)$ where*

$$\Delta(p) = \int\limits_0^1 p(x)(1 - p(x)) h^{-1}(x)\,dx \int\limits_{-\tau}^{\tau} K^2(u)\,du,$$

$b_n^{-1}\sigma_n^2 = \sigma^2(p) + O(b_n) + O(n^{-1}b_n^{-2})$, $\sigma^2(p) = 2\int\limits_0^1 p^2(x)(1 - p(x))^2 h^{-2}(x)\,dx \int\limits_{-2\tau}^{2\tau} K_0^2(u)\,du$, $K_0 = K * K$ and $\sigma^{-1} b_n^{-1/2}(U_{nN} - \Delta) \Rightarrow N(0,1)$, where $\Rightarrow$ denotes weakly convergence.

Differentiation Rule for Poisson Functionals

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It is well-known that the Ito formula is the main tool of the stochastic calculus. In the anticipative case the Ito type formula was obtained by Ustunel (1986) for random fields $F(x, \omega)$. This fields are fast decreasing with respect to x and variable x is replaced by the so-called Ito's anticipative process (with respect to Wiener process). The general case was considered by Nualart and Pardoux (1988). In case when $F(t, x)$ (for any x) is adapted diffusion process and x is replaced by Ito's anticipative process the anticipative Ito–Ventsel type formula was established by Martias (1988). The case where both $F(t, x, \omega)$ (for any x) and u_t are Ito's anticipative processes the Ito–Ventsel type formula and an integral variant of the Ito–Ventsel formula was obtained by Purtukhia (1991, 1998). In the Poisson case the similar questions was studied by Peccati and Tudor (2005) and anticipative Ito type formula was established in terms of nonanticipative Ito integrals. Our aim is to derive anticipative Ito type formula for the so-called an anticipative Poisson semimartingales in terms of anticipative Skorokhod integrals.

Let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\in[0,T]})$ be a filtered probability space satisfying the usual conditions. Suppose that N_t is the standard Poisson process ($P(N_t = k) = \frac{t^k e^{-t}}{k!}$, $k = 0, 1, 2, \dots$) and $\mathscr{F}_t$ is generated by $N(\mathscr{F}_t = \mathscr{F}_t^N), \mathscr{F} = \mathscr{F}_T$. Let M_t be the compensated Poisson process ($M_t = N_t - t$). Denote by $D_\cdot^M G$ the stochastic derivative of functional G.

Definition. The stochastic process $\xi_t(\omega)$ is called an anticipative Poisson semimartingale, if it has the representation $\xi_t(\omega) = \xi_0(\omega) + \int_0^t a_s(\omega)\,ds + \int_0^t b_s(\omega)\delta M_s(\omega)$, where the last integral is the Skorokhod anticipative integral. In this case we use the notation $d\xi_t = a_t dt + b_t \delta M_t$.

Theorem. *If ξ_t is an anticipative Poisson semimartingales with $d\xi_t = b_t\delta M_t$ and $F \in C_b^2$, then the process $F(\xi_t)$ admits the following integral representation*

$$F(\xi_t) = F(\xi_0) + \int_0^t F'(\xi_{s-})b_s\delta M_s + \int_0^t D_s^M[F'(\xi_{s-})]b_s\delta M_s + \frac{1}{2}\int_0^t F''(\xi_{s-})b_s^2\,ds +$$
$$+ \int_0^t D_s^M[F'(\xi_{s-})]b_s\,ds + \sum_{0<s\le t}\Big\{F(\xi_s) - F(\xi_{s-}) - F'(\xi_{s-})\Delta\xi_s\Big\}.$$

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A Rearrangement Theorem for Cesaro Summability

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It will be discussed the question of validity of following statement for the Cesaro summability instead of usual convergence.

A Rearrangement theorem (E. Steinitz). *Let* (x_n) *be an infinite null-sequence of elements of a finite-dimensional Banach space such that some subsequence of the sequence* $\big(\sum\limits_{k=1}^{n} x_k\big)$ *converges. Then there exists a permutation* $\sigma : N \to N$ *for which the series* $\sum\limits_{n} x_{\sigma(n)}$ *converges.*

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არაექპონენციალური მასობრივი მომსახურების ჩაკეტილი სისტემა აღდგენებითა და ჩანაცვლებით

ნინო სვანიძე

შოთა რუსთაველის სახელმწიფო უნივერსიტეტი

მათემატიკის დეპარტამენტი

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რიგების ჩაკეტილი სისტემის სახით განხილულია დარეზერვებული სისტემა ორი ტიპის მომსახურებით - აღდგენებითა და ჩანაცვლებებით. სისტება შედგება ერთი ძირითადი და ორი სარეზერვო ელემენტისგან, ძირითადი ელემენტის მტყუნება ხდება α, სარეზერვოსი - β, $0 < \beta \leq \alpha$ ინტენსივობით. ინტენსივობით. მტყუნებული ძირითადი ელემენტი ჩაინაცვლება ქმედუნარიანი სარეზერვო ელემენტით, მტყუნებული ელემენტები კი გადაეცემა აღსადგენად. იმ შემთხვევაში, თუ სისტემაში არ არის ქმედუნარიანი სარეზერვო ელემენტი, ჩანაცვლება იწყება პირველივე აღდგენის დასრულებისთანავე. სისტემაში ფუნქციონირებს ერთი ორგანო ჩანაცვლებისათვის და ერთი ორგანო მტყუნებული ელემენტის აღდგენისათვის. ჩანაცვლების დროის განაწილების ფუნქცია $H(t)$ ნებისმიერი დიფერენცირებადი ფუნქციაა, აღდგენის დრო კი ექსპონენციალური კანონით განაწილებული შემთხვევითი სიდიდეა μ პარამეტრით.

მიღებულია სისტემის ანალიზური მოდელი, რომელიც წარმოადგენს მასობრივი მომსახურების ჩაკეტილ სისტემას ორი ტიპის შემავალი ნაკადით. ანალიზური მოდელი მიღებულია როგორც გარდამავალ, ასევე სტაციონალურ რეჟიმში და იგი წარმოადგენს მათემატიკური ფიზიკის სასაზღვრო ამოცანას არაკლასიკური სასაზღვრო პირობებით. ამოცანის ანალიზური ამონახსნები მიღებულია სტაციონალურ რეჟიმში.

A Characterization of Subgaussian Random Elements in a Separable Hilbert Space

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Let X be a real normed space with the dual space X^*. We say that a random element ξ with values in X is

- *weakly subgaussian* if $\langle x^*, \xi\rangle$ is a subgaussian random variable for every $x^* \in X^*$;
- *subgaussian* if there exists centered Gaussian random element η in X such that $E\,e^{\langle x^*,\xi\rangle} \leq E\,e^{\langle x^*,\eta\rangle}$ for every $x^* \in X^*$.

If ξ is subgaussian then it is weakly subgaussian too. The converse statement is true in finite-dimensional case but it is not true in general [1, 2].

Our main result is the following theorem.

Theorem. *Let ξ be weakly subgaussian random element with values in a separable Hilbert space H. Then ξ is subgaussian if and only if*

$$\sum_{k=1}^{\infty} \tau^2(\langle \xi, e_k\rangle) < \infty$$

for every orthonormal basis (e_k) of H.

Here $\langle \cdot,\cdot\rangle$ is the scalar product and for a fixed $h \in H$ the quantity $\tau(\langle \xi, h\rangle)$ stands for the subgaussian standard of the subgaussian random variable $\langle \xi, h\rangle$.

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Mathematical Modelling

On Some Nonlocal in Time Problems for One Modification of Navier–Stokes Equations

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The present paper is devoted to the investigation of nonclassical initial-boundary value problems for one modification of Navier–Stokes equations, where the nonlinear term having polynomial structure with respect to the gradient of the unknown vector-function is added. We consider the variational formulation of the nonclassical problem in suitable spaces of vector-valued distributions, which is equivalent to the original differential formulation of the problem in the spaces of smooth enough functions. The nonclassical problem with nonlocal initial condition defining the relationship between the initial values of the unknown vector-function and its values at later times is studied. We determine the class of nonlocal operators and formulate the corresponding conditions for which the nonlocal problem has a solution in suitable spaces. An algorithm of approximation of the nonclassical problem by a sequence of classical ones is constructed. The result on the uniqueness of solution of the nonlocal in time problem is obtained and if suitable conditions are fulfilled the convergence of the sequence of solutions of the constructed classical problems to the solution of the nonclassical problem is proved. Some applications of the obtained general results to initial-boundary value problems for one nonlinear modification of Navier–Stokes equations with nonlinear discrete-integral nonlocal initial conditions are considered.

General Continuous Linear Mathematical Model of Information Warfare

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In the presented work provides a general, linear, continuous mathematical model of Information warfare between two antagonistic states (or two states of the Union, Ann the two powerful economic institutions – a consortium), taking into account the fact that there is a third, the peacekeeping side. Model includes as an equal, as well as significantly different associations with the power of controversy. We believe that the information warfare against each other, providing the first and second side, and the third party to consider the international organizations. At the moment of time $t \in [0, \infty)$ quantity of the information spread by each of the sides we will accordingly designate by $N_1(t)$, $N_2(t)$, $N_3(t)$. Built General continuous linear mathematical model of information warfare have form:

$$\begin{aligned}
\frac{dN_1(t)}{dt} &= \alpha_1 N_1(t) + \alpha_2 N_2(t) - \alpha_3 N_3(t),\\
\frac{dN_2(t)}{dt} &= \beta_1 N_1(t) + \beta_2 N_2(t) - \beta_3 N_3(t),\\
\frac{dN_3(t)}{dt} &= \gamma_1 N_1(t) + \gamma_2 N_2(t) - \gamma_3 N_3(t).
\end{aligned}$$

with initial conditions $N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $N_3(0) = N_{30}$ where, α_1, α_3, β_2, $\beta_3 \geq 0$, $\gamma_i \geq 0$, $i = 1, 2, 3$; α_2, β_1 are constant factors.

Solving model for concrete value of constants and initial a condition numerical methods in package Matlab, conditions are received, at which: 1. The antagonistic sides, despite increasing appeals of the third side, intensify information attacks. 2. One of the antagonistic sides, under the influence of the third side eventually stops information warfare (an exit of the corresponding solution on zero) while another strengthens it. 3. Both antagonistic sides, after achieving maximum activity, reduce it under the influence of the third side, and through finite time, stop information attack at all (an exit of solutions to zero).

Cubature of the Solution of the Dirichlet Problem for Euler–Poisson–Darboux Equation in the Half-plane by Approximate Quasi-Interpolation

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Our aim is to construct the approximate quasi-interpolant (see, e.g., [1] and [2]) of the solution of the Dirichlet problem for Euler–Poisson–Darboux Equation in the half-plane. The solution of Euler–Poisson–Darboux equation

$$y(u_{xx} + u_{yy}) + bu_y + au_x = 0, \quad b = \text{const},$$

in the half-plane $y > 0$, under the following boundary condition $u(x, 0) = g(x)$, $g(x) \in C(R)$, can be written as follows (see, [3])

$$u(x, y) = \frac{y^{1-b}}{\Lambda(a, b)} \int\limits_{-\infty}^{+\infty} g(\xi) e^{a\theta} \rho^{b-2} d\xi, \quad b < 1,$$

where

$$\Lambda(a, b) := \int\limits_0^{\pi} e^{a\eta} \sin^{-b} \eta d\eta, \quad \theta := \operatorname{arccot} \frac{x - \xi}{y}, \quad \rho := \Big[(x - \xi)^2 + y^2\Big]^{1/2}.$$

The approximate quasi-interpolant on a uniform grid $\{hm\}$ is constructed, the error estimate is determined. The corresponding numerical realization is carried out.

This is a joint work with G. Jaiani and F. Lazara.

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Numerical Modeling of Spreading of Oil Pollution in the Georgian Black Sea Coastal Zone

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In the last decades the study of regularities of space-temporal distribution of anthropogenic admixtures in the Black Sea becomes extremely important and urgent because of sharp deterioration of an ecological situation of this unique sea basin. Among different pollutants an oil and oil products present the most widespread and dangerous kind of pollution for the separate regions of the World ocean including the Black Sea. They are able to cause significant negative changes in hydrobiosphere and to infringe natural exchange processes of energies and substances between the sea and atmosphere. Potentially the most dangerous regions from the point of view of oil pollution are coastal zones of the sea, which are exposed to the significant anthropogenic loadings.

Modeling of distribution of oil spill enables to estimate pollution zones and probable scales of influence of the pollution on the water environment with the purpose to reduce to a minimum negative consequences of oil pollution in case of emergencies.

In this study, spreading of the oil pollution in the Georgian Black Sea coastal zone on the basis of a 2-D numerical model of distribution of oil pollution is simulated. The model is based on a transfer-diffusion equation with taken into account reduction of oil concentrations because of physical - chemical processes. The splitting method is used for solution of the transfer-diffusion equation. The surface current field is determined from the regional baroclinic model of the Black Sea dynamics developed at M Nodia Institute if Geophysics of Iv. Javakhishvili State University. The regional domain, which was limited by Caucasus and Turkish shorelines and the western liquid boundary coinciding with a meridian 39.360E was covered with a grid having 193 x 347 points and grid step equal to 1 km spacing. Numerical experiments are carried out for different hypothetical sources of pollution in case of different sea circulation regimes.

Numerical experiments showed that character of regional sea circulation predetermines main features of spatial distribution of the oil pollution in the Georgian Black Sea coastal zone.

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Numerical Investigation of Spilled Oil Spreading Into Soil for Underground Water Pollution Risk Assessment

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A number of alternative options were assessed during preliminary work relating to the export of oil and gas from the Shah Deniz field. There was considering the best transportation method for oil and gas (followed by a detailed assessment of the best pipeline corridor, the best possible markets) culminating in the final route selection. Gas and oil transportation by pipeline and railway routs from the Shah Deniz field via Azerbaijan and Georgia was defined as the most acceptable commercial and environmental solution. The probability of crashes for pipeline transport rises with the age of the oil pipelines in service, and with the extent of their network. But there are able to take place non ordinary situations too. As foreign practice of pipeline exploitation shows, that the main reason of crashes and spillages (and fires as a consequence) is destruction of pipes as a result of corrosion, defects of welding, natural phenomena and so on (including terrorist attacks and sabotage). In West Europe it has been found, that 10 - 15 leakages occur every year in a pipeline network of around 16,000 km length resulting in a loss of 0.001 per sent of transferred products. The proposed transport corridor via Georgia is characterized by very diverse ecological conditions and by abundant biodiversity. The route crosses a multitude of minor watercourses with broad seasonal variations of surface water flow. Six major river crossing occur along the route on the territory of Georgia. Ground water along the route is also abundant and generally of high quality. So that it is necessary: to design a new high-quality soil pollution models by oil, to develop new algorithms and means of the control and detection of emergency places of underground water pollution. We have created a new numerical model and scheme describing oil infiltration into soil. The constructed scheme is investigated and error of the approximated solution is estimated. Using this scheme, we have carried out numerical experiments for four types of soils dominated within the transport corridor of Georgia. The results of calculations are presented.

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Mathematical Modelling of Hydrates Origin in the Gas Pipelines

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At present pipelines have become the most popular means of transporting natural gas. As our practice shows while transportation of natural gas by pipelines over the territory of Georgia , pressure and temperature changes cause formation of a liquid phase owing to partial condensation of the gaseous medium. There are many scientific articles denoted to the problem of prediction of possible points of hydrates origin in the main pipelines. There are several methods for avoiding gas hydrate problems, but generally modern methods for prevention of hydrate formation are based on the following techniques: injection of thermodynamic inhibitors, use of kinetic hydrate inhibitors to sufficiently delay hydrate nucleation/intensification, and maintain pipeline operating conditions outside the hydrate stability zone by insulation, heating and controlling pressure. However, the above techniques may not be economical and practical From existing methods the mathematical modelling with hydrodynamic method is more acceptable as it is very cheap and reliable and has high sensitive and operative features. In the present paper the problem of prediction of possible points of hydrates origin in the main pipelines taking into consideration gas non-stationary flow and heat exchange with medium is studied. We have created a new method prediction of possible points of hydrates origin in the main pipelines taking into consideration gas non-stationary flow. For solving the problem of possible generation point of condensate in the pipeline under the conditions of non-stationary flow in main gas pipe-line the system of partial differential equations is investigated. For learning the affectivity of the method quite general test was created. Numerical calculations have shown efficiency of the suggested method.

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Iteratively Regularized Gradient Method for Determination of Source Terms in a Linear Parabolic Problem from the Measured Final Data

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This paper investigates a numerical computation for determination of source terms in a linear parabolic problem. The source term $w := \{F(x,t), p(t)\}$ is defined in the linear parabolic equation $u_t = (k(x)u_x)_x + F(x,t)$ and Robin boundary condition $-k(l)u_x(l,t) = \nu[u(l,t) - p(t)]$ from the measured final data $\varphi(x) = u(x,T)$. We demonstrate how to compute Fréchet derivative of Tikhonov functional $J_\alpha(w) = \|\Phi[w] - \varphi^\delta\|^2_{L^2(0,l)} + \alpha\|w\|^2_W$ based on the solution of the adjoint problem. Lipschitz continuity of the gradient is proved. Iteratively regularized gradient method is applied for numerical solution of the problem. We conclude with several numerical test by using the theoretical results.

Numerical Model of Local Circulation of Atmosphere in Case of Difficult Temperature Inhomogeneity of a Underlying Surface

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It is traditional any big town has the satellite-town. In this case we actually have two thermal "islands".

In this work the two-dimensional, non-stationary problem about local circulation over these "islands" and sreading in it of an aerosol from different sources (point, surface, instant, constantly acting etc.) is statement.

Now the problem only describing thermohydrodynamics and humidity processes is programmed and numerically realised on the computer. Qualitatively real space-time fields of speed of air, temperatures, pressure, water-vapor and liquid-water mixing ratios are received.

On the basis of this model optimum control of two virtual objects-towns (distances variation between them, forecasting of meteorological fields, creation of recreational zones, influence of prevailing wind, playing of different ekometeorological interactions scenario etc.) is possible.

On the Numerical Experiment About One Special Formation of a Fog and a Cloud in Mesoboundary Layer of Atmosphere

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Rather actual it is considered research temperature inversion processes in connection with the environmental problems which are taking place in a mesoboundary layer of atmosphere (MBLA). there is an accumulation of polluting substances in these inversion layers. Such anomalies arise at formation of clouds and fogs when allocation of the latent warmth of condensation of water steam takes place.

There was local circulation over thermal "island" is numerically simulated at its periodic heating with the help of the developed by us numerical model and under certain physical conditions is received against the MBLA thermohydrodynamics simultaneously four humidity processes: it is a fog and a cloud direct over "island" and two clouds on each side. Because of them aforementioned inversion layers arise.

This interesting numerical experiment is quite logical physically. On the future we plan to get corresponding materials of meteorological supervision for the purpose of comparison of theoretical results with experimental data.

Electrodynamic Analysis of Five-Port Waveguide Junctions

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The urgency of the purposes of accomplished work is caused by the contemporary trends of development of radio-electronic and communication systems facing increasing requirements of improvement in the effectiveness of functioning as systems as a whole, and, also, as their component elements. These requirements concern also multi-port waveguide junctions which are widely used in the technology of superhigh frequencies as power dividers, antenna switchers, filters, phase inverters, etc.

The stated problem could be solved by the method of decomposition, which would considerably simplify mathematical solution of the problem, significantly decreasing resources spent during computer implementation. However, the negative side of this method is impossibility of conducting a full-scale analysis of the proceeding process and detection of a whole series of practically important electrodynamic properties of the system.

In the process of mathematical solution it was used the method of partial domains. Within the framework of this method the fields in each chosen domain were written as the solutions of wave equation taking into account the special features of these domains. After application of the continuity conditions of the field on all interfaces the infinite system of linear algebraic equations, consisting of three linearly independent systems with respect to three sequences of unknown coefficients, was obtained. These coefficients represent the wave amplitudes of those waves reflected in each of the side arms.

The analysis of matrix elements and free terms of the obtained system showed that this system is quasi-regular, which made it possible to realize it on the computer with application of a method of reduction.

As a result of numerical calculation the graph of dependence of the coefficients (amplitudes) of waves propagating in each port on the frequency parameter and the figures of field distribution are built (2D and 3D graphs). The program for visualization of the wave propagation is compiled.

On the Spectrum of the Helmholtz Equation for the Hexagonal Type Stripe

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The problem connected with the quantum properties of the hexagonal type crystal structure such as Carbon allotrops is considered. The movement of the particles at such structure is described by the Schrodinger Equation. In the stationary case this equation is reduced to the Helmholtz Equation with the appropriate boundary conditions [1–3].

In this work the spectrum of the Helmholtz Equation in the hexagonal type stripe is estimated by means of the conformal mapping and Fourier series.

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Design of High Rise Buildings on Seismic Effects of Spectral and Nonlinear Dynamic Methods

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The 45-storey dual system high-rise reinforced frame-wall concrete building is investigated. A dual system is a favourable system to use due to the ability to dissipate large amounts of energy.

Two dimensional mathematical model of the case study structure is considered using SEISMOSTRUCT, ANSYS, LIRA, NASTRAN software. Response spectrum and response-history analysis are used. The first four modes value of natural vibrations are: 6.63, 1.60, 0.68, 037sec. The real displacement of the building – 1.68m – was received.

According to Eurocode-8, instead of the full nonlinear analysis of structures, the linear analysis of the modified spectrum of elastic reaction with the introduction of the reduction factor q is allowed. As a seismic action elastic displacements spectrum with an enhanced control period to start a permanent segment of the spectrum is used, becouse the building is long period one.

Second order $(P-\Delta)$ effects that include all of the building dead load and permanent live load are considered in linear as well as nonlinear analysis. If $0,1 < \Theta \leq 0.2$ (the interstorey drift sensitivity coefficient), the second-order effects may approximately be taken into account by multiplying the relevant seismic action effects by a factor equal to $1/(1-\Theta)$.The value of the coefficient Θ shall not exceed 0,3.

For all floors of the building a total drift value (max. $\Theta = 0.26\%$) is received. In such a case the secondary $P-\Delta$ effects must be evaluated by nonlinear analysis

Nonlinear behaviour of materials - concrete and reinforced – are taken into account. For nonlinear analysis Imperial Velley-El-Centro accelerogram - $M = 7.1$, record length – 30sec. – is used.

In frame-wall structures the wall element provides an increase of stiffness which is beneficial in terms of drift control. The wall dominates the structural behaviour in the lower levels of the structure and the frame controls the behaviour in the upper levels of the structure. In frame-wall structures the overturning resistance that frames offer at height tends to restrain the tops of the walls such that a point of contra-flexure develops in the walls at a height.

Linear Central Algorithms for the First Kind Integral Equations

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It is considered ill-posed problem $Ku = f$, where K is a selfadjoint, positive, compact, one-to-one operator, having everywhere dense range in a Hilbert space H.

In [1] for the operator K was introduced the Frechet space $D(K^{-\infty}) = \bigcap\limits_{n=1}^{\infty} D(K^{-n})$, whose topology is given with the sequence of norm

$$\|x\|_n^2 = \|x\|^2 + \|K^{-1}x\|^2 + \cdots + \|K^{-n+1}x\|^2, \;\; n \in N,$$

which are generated by the inner product

$$(x, y)_n = (x, y) + (K^{-1}x, K^{-1}y) + \cdots + (K^{-n+1}x, K^{-n+1}y), \;\; x, y \in D(K^{-\infty}).$$

As well, the operator $K^{-\infty}(x) = \{K^{-1}x, K^{-2}x, \cdots, K^{-n}x, \cdots\}$, which is defined on the whole Frechet space $D(K^{-\infty})$, is considered. $K^{-\infty}$ is continuous, selfadjoint and positive definite operator in the Frechet space $D(K^{-\infty})$ [2]. For the operator $K^{-\infty}$ there exists the inverse operator $(K^{-\infty})^{-1}$, which is also continuous. The operator $(K^{-\infty})^{-1}$ is selfadjoint on the Frechet space $D(K^{-\infty})$. Therefore $K^{-\infty}$ is an isomorpism of the Frechet space $D(K^{-\infty})$ onto itself. Let us denote the operator $(K^{-\infty})^{-1}$ by K_∞. The equation $K_\infty u = f$ has unique and stable solution in the Frechet space $D(K^{-\infty})$. This operator K_∞ topologically coincides with the restriction of the operator K^N from the Frechet space H^N on the Frechet space $D(K^{-\infty})$. On the other hand the solution of the equation $K_\infty u = f$ on the Frechet space $D(K^{-\infty})$ is the same as the solution of the equation $Ku = f$ on H.

Central algorithm for an approximate solution of the equation $K_\infty u = f$ in the Fréchet space $D(K^{-\infty})$ is constructed. Application of the received results for some integral equation of the first kind are given.

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Computer Modeling of White Noise and its Applications

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In the report it is generally considered the issues of correlation between "white noise", a random process, well-known in practice, and two-dimensional and spatial boundary problems which are considerably applied in ICT.

As it is well-known, "white noise" itself is widely used in informatics. The example is information security. In particular, "white noise" is used in hardware and software encryption where random numbers generation is needed.

To illustrate these relations and applications the report presents a computer simulation algorithm of the diffusion process based on "white noise". A specific problem is brought in the case of the three-dimensional area, an ellipsoid in particular. Corresponding numerical experiments have been conducted.

Numerical Analysis

Finite Difference Solution for MEW Wave Equation

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It is considered an initial value problem for modified equal width (MEW) wave equation

$$\frac{\partial u}{\partial t}+\gamma u^2\frac{\partial u}{\partial x}-\mu\,\frac{\partial^3 u}{\partial x^2\partial t}=0, \tag{1}$$

where γ and μ are positive constants. Physical boundary conditions require that $u\to 0$ for $x\to\pm\infty$.

For numerical solution of (1) via initial condition $u(x,0)=u_0(x)$ artificial boundaries can be selected at some points $x=a,\ x=b,\ a<b$ and in the domain $Q_T=[a,b]\times[0,T]$ the initial boundary-value problem with the conditions

$$u(a,t)=u(b,t)=0,\ \ t\in[0,T),\quad u(x,0)=u_0(x),\ \ x\in[a,b] \tag{2}$$

can be considered.

For the problem (1), (2) a three level conservative finite difference scheme is studied. The obtained algebraic equations are linear with respect to the values of a desired function for each new level. It is proved that difference scheme is absolutely stable with respect to the initial data and converges with the rate $O(h^2+\tau^2)$ when the exact solution belongs to the Sobolev space $W_2^3(Q_T)$.

The accuracy and conservation properties of the proposed scheme are examined by numerical experiments.

On the Parabolic Regularization of One Nonlinear Diffusion System

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In the rectangle $[0,1]\times[0,T]$ the following initial-boundary value problem is considered:

$$\begin{gathered}\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(v\frac{\partial u}{\partial x}\right),\\ \frac{\partial v}{\partial t}=g\left(v\frac{\partial u}{\partial x}\right)+\varepsilon\frac{\partial^2 v}{\partial x^2},\\ u(0,t)=u(1,t)=0,\\ \left.\frac{\partial v(x,t)}{\partial x}\right|_{x=0}=\left.\frac{\partial v(x,t)}{\partial x}\right|_{x=1}=0,\\ u(x,0)=u_0(x),\quad v(x,0)=v_0(x),\end{gathered}\tag{2}$$

where g, u_0, v_0 are given sufficiently smooth functions and following conditions are satisfied: $0<\gamma_0\le g(\xi)\le G_0$, $\gamma_0=Const$, $G_0=Const$, $T=Const$, $\varepsilon=Const>0$.

System (1) is the parabolic regularization of the one-dimensional analogue of the nonlinear partial differential equations which describes the vein-formation in meristematic tissues of young leaves (G. J. Mitchison, A model for vein formation in higher plants. *Proc. R. Lond. B.* **207** (1980), No. 116, 79–109).

The asymptotic behavior as $\varepsilon\to 0$ of the solution of the problem (1) is studied. The decomposition and finite difference schemes are constructed and investigated.

Numerical Integration of Hyperbolic Partial Differential Equations along Characteristics on an Adaptive Mesh

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We propose an adaptive mesh scheme for integration of hyperbolic systems of partial differential equations by modifying the method of integration along characteristics and, consequently, incorporating the mesh adaptation rules into the original system. Within the approach we suggest straightforward mesh stabilization mechanism in the most critical regions of the integration domain, such as discontinuities and shock front. Using the method, we simulate density waves propagating in thin gaseous media.

Asymptotics of Solution of Initial-Boundary Value Problem with Mixed Boundary Conditions for a Nonlinear Integro-Differential System

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It is well-known that the process of the magnetic field penetration into a substance is modeled by Maxwell's system of partial differential equations. If the coefficient of thermal heat capacity and electroconductivity of the substance depend on temperature, then Maxwell's system can be rewritten in an integro-differential form. Assuming the temperature of the considered substance depending on time, but independent of the space coordinates, the same process is also modeled by the integro-differential system, one-dimensional analogue of that, in case of two-component magnetic field, has the form:

$$\begin{aligned} \frac{\partial U}{\partial t} &= a\left(\int_0^t \int_0^1 \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] dx\, d\tau\right)\frac{\partial^2 U}{\partial x^2}, \\ \frac{\partial V}{\partial t} &= a\left(\int_0^t \int_0^1 \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] dx\, d\tau\right)\frac{\partial^2 V}{\partial x^2}, \end{aligned} \tag{1}$$

where $a = a(S)$ is a known function of its argument.

In the domain $(0,1)\times(0,\infty)$ we consider the following initial-boundary value problem for the system (1):

$$\begin{gathered} U(0,t) = V(0,t) = 0, \quad \frac{\partial U(x,t)}{\partial x}\Big|_{x=1} = \frac{\partial V(x,t)}{\partial x}\Big|_{x=1} = 0, \ \ t \geq 0, \\ U(x,0) = U_0(x), \quad V(x,0) = V_0(x), \ \ x \in [0,1], \end{gathered} \tag{2}$$

where $U_0 = U_0(x)$ and $V_0 = V_0(x)$ are the given functions.

We study the large time behavior of solutions and numerical resolution of the initial-boundary value problem (1), (2). In particular, we prove the following statement

Theorem. *If* $a(S) = (1+S)^p$, $p > 0$; $U_0, V_0 \in H^3(0,1)$, $U_0(0) = V_0(0) = 0$, $\frac{\partial U_0(x)}{\partial x}\Big|_{x=1} = \frac{\partial V_0(x)}{\partial x}\Big|_{x=1} = 0$, *then for the solution of problem* (1), (2) *the following estimate holds*

$$\left|\frac{\partial U(x,t)}{\partial x}\right| + \left|\frac{\partial U(x,t)}{\partial t}\right| + \left|\frac{\partial V(x,t)}{\partial x}\right| + \left|\frac{\partial V(x,t)}{\partial t}\right| \leq C \exp\left(-\frac{t}{2}\right).$$

On One Interactive Method of Solving Multicriterion Discrete Optimization Problem

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Among of applied optimization problems discrete ones take up the important place. Among them enough actual are schedule theory problems which generally represent following: on the basis of certain set of resources or means of service the given system of tasks must be implemented. The effective algorithm of task implementation sequence must be constructed under conditions of tasks system and resources given properties and also their constraints must be taking into account. This algorithm satisfies certain criteria of effectiveness simultaneously (multicriterion condition). As a measure of optimization we can consider length of schedule in terms of time, or maximum cost of system. From the application point of view such problems include all spheres of human activities. Therefore for this or that specific problem it is urgent to construct comparatively exact mathematical model and such algorithms, which maximally will use specific of these problems and in polynomial time will give optimal decision possibility. The work is devoted to the research of one schedule theory specific problem. In particular, tasks implementation is possible by means of one-step multiprocessor system, where processors are completely interchangeable. In addition, set of partial ordering is not empty and set of additional resources is restricted; to implement entered tasks into system a number of necessary processors is infinite. For the given problem it is taken into account to construct a new optimization algorithm and state its effectiveness. For this purpose there were used methods of combinatorial analysis and knot theory. To solve multicriterion problem specially worked out interactive method is applied.

On Approximate Approximations

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This talk is devoted to a new concept of approximation procedures called approximate approximations, proposed by Vladimir Maz'ya in the late of 80's. The quasi-interpolants are linear combinations of scaled translates of a sufficiently smooth and rapidly decaying basis function η and depend on two positive parameters, the meshsize h and the scale parameter D. If $\mathscr{F}\eta - 1$ has a zero of order N at the origin ($\mathscr{F}\eta$ denotes the Fourier transform of η) then the quasi-interpolant on uniformly distributed nodes gives an approximation of order $\mathscr{O}(h^N)$ up to a *saturation error* which can be made arbitrarily small if D is chosen large enough. The lack of convergence in approximate approximations is compensated by the flexibility in the choice of the basis functions and by the simplicity of the multidimensional generalization. Another advantage is the possibility of obtaining explicit formulas for values of various integrals and pseudodifferential operators of mathematical physics applied to basis functions (see V. Maz'ya, G. Schmidt, Approximate approximations. *Math. Surveys and Monographs*, v. 141, AMS 2007).

Here we present results concerning Hermite quasi-interpolation on uniform grids with applications to the approximation of solutions to elliptic PDE and quasi-interpolation on nonuniform grids based on approximate approximations. These results were obtained together with V. Maz'ya (University of Liverpool, UK; Department of Mathematics, Linköping University, Sweden) and G. Schmidt (Weierstress Institute for Applied Analysis and Stochastics, Berlin, Germany).

Finite Difference Schemes for Systems of ODE on Graphs

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Mathematical modeling of various processes in the nets of gas pipeline, system of submission and distribution of water, drainpipe, also long current lines and different types of engineering constructions quite naturally leads to the consideration of differential equations on graphs. In this paper we consider the mathematical model of electro power system, which is the boundary value problem for ordinary differential equations, defined on graphs. Correctness of the problem is investigated. Constructed and investigated the corresponding finite-difference scheme. Double sweep type formulas for finding solutions of finite difference scheme are offered. This algorithm is essentially a parallel algorithm and efficiently implemented on computers with parallel processors.

Convergence of an Iteration Method for a Kirchhoff Problem

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The following initial boundary value problem is considered

$$w_{tt}-\bigg(\lambda+\frac{8}{\pi^3}\int\limits_{\Omega}|\nabla w|^2dx\bigg)\Delta w=0,\ \ 0<t\leq T,\ \ x\in\Omega, \tag{1}$$

$$\begin{gathered} w(x,0)=w^0(x),\quad w_t(x,0)=w^1(0),\ \ x\in\Omega,\\ w(x,t)=0,\ \ x\in\partial\Omega,\ \ 0\leq t\leq T, \end{gathered} \tag{2}$$

where $x=(x_1,x_2,x_3)$, $\Omega=\{(x_1,x_2,x_3)|\ 0<x_i<\pi,\ \ i=1,2,3\}$, $\partial\Omega$ is the boundary of the domain Ω, $w^0(x)$ and $w^1(x)$ are given functions, $\lambda>0$ and T are the known constants.

Equation (1) is a three-dimensional analogue of the Kirchhoff equation [1],

$$w_{tt} - \left(\lambda + \frac{2}{\pi}\int\limits_0^\pi w_x^2\,dx\right)w_{xx} = 0$$

describing the oscillation of a string. The problem of solvability of this equation was for the first time studied by S. Bernstein. Later, many researchers showed an interest in equations of Kirchhoff type (see e.g. [2, 3]).

Here we present a numerical algorithm of problem (1), (2). Step-by-step discretization with respect to a spatial and a time variable is carried out. To solve the resulting cubic system we use the Jacobi iteration method. The error of this method is estimated.

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A Numerical Algorithm of Solving a Nonlinear System for a Plate

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Let us consider the nonlinear system of differential equations

$$\begin{gathered}
u'' + \frac{1}{2}\left(w'^2\right)' + p(x) = 0,\\
k_0^2\,\frac{Eh}{2(1+\nu)}\left(w'' + \psi'\right) + \frac{Eh}{1-\nu^2}\left[\left(u' + \frac{1}{2}\,w'^2\right)w'\right]' + q(x) = 0, \qquad (1)\\
\frac{h^2}{6(1-\nu)}\,\psi'' - k_0^2\left(w' + \psi\right) = 0, \ \ 0 < x < 1,
\end{gathered}$$

with the boundary conditions $u(0) = u(1) = 0$, $w(0) = w(1) = 0$, $\psi'(0) = \psi'(1) = 0$. Here $u = u(x)$, $w = w(x)$ and $\psi = \psi(x)$ are the functions to be determined and $q(x)$ is the given function, ν, E, h and k_0 are the given positive constants, $0 < \nu < \frac{1}{2}$. System (1) describes the static behaviour of the plate under the action of axially symmetric load. It is obtained from the two-dimensional system of Timoshenko equations for a plate [1, p. 24] or for a shell [2, p. 42]. By Green functions two sought functions $u(x)$ and $\psi(x)$ are expressed through the $w(x)$ for which the nonlinear integro-differential equation

$$\frac{Eh}{1-\nu^2}\left[\left(\frac{1-\nu}{2}\,k_0^2+\frac{1}{2}\int\limits_0^1 w'^2\,dx+\int\limits_0^1(1-x)p(x)\,dx-\int\limits_0^x p(\xi)\,d\xi\right)w''-p(x)w'\right]-$$
$$-\frac{3Ek_0^4}{h\operatorname{sh}\sigma}\,\frac{1-\nu}{1+\nu}\left(\operatorname{sh}\sigma(x-1)\int\limits_0^x\operatorname{ch}\sigma\xi\,w'(\xi)\,d\xi+\operatorname{sh}\sigma x\int\limits_x^1\operatorname{ch}\sigma(\xi-1)w'(\xi)\,d\xi\right)+q(x)=0 \quad (2)$$

with condition $w(0)=w(1)=0$ and $\sigma=k_0\sqrt{6(1-\nu)}/h$ is obtained. To approximate the solution of equation (2) a variational-iteration method is used. The problem of algorithm accuracy is discussed.

Acknowledgements

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Special Centrosymmetric Matrices in Numerical Solutions of Elliptic Equations

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Many problems of technique and science are reduced to the numerical solution of partial differential equations with different conditions. To receive the numerical solutions of such problems, diversity of difference schemes of different order of accuracy are used. One of such approximating methods is known as a method without saturation. One approach to this method was received by means of the approximating method of the academician Shalva Mikeladze for the ordinary differential equations, which gave possibility to raise the order of accuracy by increasing a number of approximating points. Using discrete and semi-discrete approximating methods for solving the boundary value problem for the second order partial differential equations of elliptic type with Dirichlet condition gave certain type matrices known as centrosymmetric or doublesymmetric matrices. Properties and peculiarities of such matrices are presented.

On Higher Accuracy Discrete Singularity Methods

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Some quadrature formulas of higher accuracy are constructed for singular integrals with Cauchy kernels. Their applications are indicated.

Numerical Solutions of the Helmholtz Equation by Multivariate Padé Approximation

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In this paper, numerical solution of the Helmholtz partial differential equation is considered by Padé Approximation. We applied this method to two examples. First, the Helmholtz partial differential equation has been converted to power series by two-dimensional differential transformation, then the numerical solution of equation was put into Padé series form. Thus we obtained numerical solution of Helmholtz-type partial differential equation.

Continuum Mechanics

On a Numerical Solution of a Problem of Non-Linear Deformation of Elastic Plates Based on the Refined Theory

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We consider a numerical solution of a problem of non-linear deformation of elastic plates based on the refined theory. This theory takes into account the deformations which may not be homogeneous along the shifts. The obtained numerical results are compared with the results based the another theory.

The Neumann BVP of Thermoelasticity for a Transversally Isotropic Half-Plane with Curvilinear Cuts

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In the present paper the second boundary value problem of the theory of thermoelasticity is investigated for a transversally isotropic half-plane with curvilinear cuts. For solution we used the potential method and constructed the special fundamental matrices, which reduced the problem to a Fredholm integral equations of the second kind.The solvability of a system of singular integral equations is proved by using the potential method and the theory of singular integral equations. For the equation of statics of thermoelasticity we construct one particular solution and we reduce the solution of the second BVP problem of the theory of thermoelasticity to the solution of the second BVP problem for the equation of transversally-isotropic body.

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Existence Theorems of Solutions of the Third and Fourth BVPS of the Plane Theory of Thermoelasticity with Microtemperatures

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In the present paper the third and fourth boundary value problems of the plane theory of thermoelasticity with microtemperatures are investigated. To this connection see also [1–4]. We use the potential method and construct the special fundamental matrices which reduces the problems to a system of singular integral equations of the second kind. The solvability of the system of singular integral equations is proved by using the potential method and the theory of singular integral equations.

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Finite Element Method for 2D Shell Equations: Existence and Convergence of Approximated Solutions

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We consider linear asymptotic model of 2D shell based on the calculus of tangent Günter's derivatives. For the boundary value problem stated for this model equivalent boundary pseudodifferential equation is written in terms of Günter's derivatives on the middle surface and Korn's inequality is proved. This problem possesses a unique solution in appropriate Bessel potential space.

We describe the discrete counterpart of the problem based on Finite Element Method, employing Korn's inequalities we prove the existence and uniqueness of approximated solutions in suitable finite dimensional spaces and their convergence to the solution of the boundary pseudodifferential equation.

General Theory of Micropolar Anisotropic (Orthotropic) Elastic Multi-Layered Thin Shells

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Elastic shell of constant thickness consisting of micropular orthotropic layers is considered. It is assumed that the layers of the shell are tightly linked and work together without slide and separation. Here we shall reduce three-dimensional equations to two-dimensional equations. Constructing the applied theory for micropular layered shells following hypotheses are accepted:

a. The hypothesis of straight line for all packet of the shell is accepted as initial hypothesis [1] $(\alpha_i, \alpha_3, (i = 1, 2)$ are curvilinear orthogonal coordinates)

$$\nu_1^i = u_1(\alpha_1, \alpha_2) + \alpha_3 \psi_1(\alpha_1, \alpha_2), \quad \nu_2^i = u_2(\alpha_1, \alpha_2) + \alpha_3 \psi_2(\alpha_1, \alpha_2), \quad \nu_3^i = w(\alpha_1, \alpha_2).$$

We shall also assume $\omega_1^i = \Omega_1(\alpha_1, \alpha_2)$, $\omega_2^i = \Omega_2(\alpha_1, \alpha_2)$, $\omega_3^i = \Omega_3(\alpha_1, \alpha_2) + \alpha_3 \iota(\alpha_1, \alpha_2)$.

b. In the generalized Hook's low force stress σ_{33}^i can be neglected in relation to the force stresses σ_{11}^i, σ_{22}^i.

c. Quantities $\frac{\alpha_3}{R_i}$ can be neglected relative to one (R_i are the main radii of curvature of coordinate surface).

d. During determination of deformations, bending-torsions, force and moment stresses in all layers, first for the force stresses σ_{31}^i, σ_{32}^i and moment stress μ_{33}^i we shall take:

$$\sigma_{31}^i = \overset{0}{\sigma}{}_{31}^{i}(\alpha_1, \alpha_2), \quad \sigma_{32}^i = \overset{0}{\sigma}{}_{32}^{i}(\alpha_1, \alpha_2), \quad \mu_{33}^i = \overset{0}{\mu}{}_{33}^{i}(\alpha_1, \alpha_2). \tag{1}$$

After determination of mentioned quantities, values of σ_{31}^i, σ_{32}^i and μ_{33}^i will be finally defined by the addition to values (1) summed up, obtained by integration of correspondent equilibrium equations, for which the condition will be required that quantities averaged along the shell's thickness are equal to zero.

On the basis of accepted hypotheses general mathematical model of micropolar elastic anisotropic (orthotropic) layered shells with free fields of displacements and rotations is constructed.

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Signorini Problem with Natural Nonpenetration Condition in Elasticity

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In the recent work the contact problem of an elastic anisotropic inhomogeneous body with a rigid body (frame) is considered. Usually such contact is described by *Signorini boundary conditions* including normal displacement and normal stress (also the tangential components of the friction arise between bodies). These conditions are written after some linearization and other simplifications of the *Natural Nonpenetration Condition*:

$$x_3 + u_3 \le \psi(x' + u'), \tag{1}$$

where $u = (u_1, u_2, u_3)$ is a displacement vector, $x = (x_1, x_2, x_3)$ belongs to the contact part of boundary of the elastic body, $u' = (u_1, u_2)$, $x' = (x_1, x_2)$ and ψ describes the contact surface of the frame.

Our aim is to avoid the simplification procedure which moves away the mathematical model from the physical one, and describe the mentioned contact by the initial Nonpenetration Condition (1). The only assumption to this end is that ψ should be concave and continuous.

Suppose that the elastic body is subjected to volume and external forces, then the Condition (1) leads to the variational inequality on the close convex set. When the body is fixed along a part of its boundary, i.e., we have the Dirichlet condition, then the variational inequality has a unique solution. Without the condition the necessary result of the existence of solution is obtained. When $\psi \in C^2(R^2)$ then we write the boundary conditions corresponding to the variational inequality. The stability result is also obtained when the problem is uniquely solvable and $\psi \in C^0(R^2)$.

A Boundary Contact Problem of Stationary Oscillations of the Elastic Mixture Theory for a Domain Bounded by a Spherical Surface

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A fundamental boundary contact problem of stationary oscillations of the elastic mixture theory is considered for a domain bounded by a spherical surface when on the contact surface a difference of displacement vectors and a difference of stress vectors are given. A representation formula is obtained for a general solution of a system of homogeneous differential equations of stationary oscillations of two-component elastic mixture theory. The formula is expressed in terms of six metaharmonic functions. A new version of the proof of the uniqueness theorem of the considered contact problem is given. The problem solution is obtained in the form of absolutely and uniformly convergent series.

Fundamental Solution of the System of Differential Equations of Stationary Oscillations of Two-temperature Elastic Mixtures Theory

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The paper deals with construction of fundamental matrix of solutions for the system of differential equations of stationary oscillations of the theory of two-temperature elastic mixtures. The entries of the matrix are represented in explicit form as linear combinations of metaharmonic functions. With the help of the fundamental matrix the corresponding single layer, double layer and Newtonian potentials are constructed and their properties are studied. The results are then applied to prove the existence theorems to boundary value problems by the potential method and the theory of integral equations.

On Construction of Approximate Solutions of Equations of the Non-Shallow Spherical Shells

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The purpose of this paper is to consider the non-shallow spherical shells. By means of I. Vekua method the system of equilibrium equations in two variables is obtained [1, 2]. Using complex variable functions and the method of the small parameter approximate solutions are constructed for $N = 1$ in the hierarchy by I. Vekua. Concrete problem is solved, when the components of the external force are constants.

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The Interior Neumann type Boundary Value Problem of the Thermoelasticity Theory of Hemitropic Solids

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We investigate the interior Neumann type boundary value problem (BVP) for the equations of statics of thermoelasticity theory of hemitropic solids. By the potential method we reduce the BVP to the boundary integral equations. Afterwards we study the Fredholm properties of the corresponding boundary integral operator. The null space of the boundary integral operator is not trivial and therefore the corresponding nonhomogeneous integral equation is not solvable for arbitrary right hand side data. We constructed all linearly independent solutions of the corresponding homogeneous adjoint integral equation and established the explicit form of necessary and sufficient conditions for the integral equation corresponding to the Neumann type boundary value problem to be solvable.

Investigation of Boundary Value Problems of the Theory of Thermoelasticity

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We investigate the interior and exterior Dirichlet and Neumann type boundary value problems (BVP) for the equations of statics of thermoelasticity theory. By the potential method we reduce the BVPs under consideration to the boundary integral equations. Afterwards we study the Fredholm properties of the corresponding boundary integral operators and prove the existence theorems for the BVPs in various function spaces. In the case of the interior Neumann type BVP we establish the necessary and sufficient conditions of solvability in the explicit form.

On Some Problems of Thermoelasticity for the Rectangular Parallelepiped with Non-Classical Conditions on the Surface

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The elastic equilibrium of the rectangular parallelepiped is considered, when the symmetry or asymmetry conditions are given on the lateral faces of the parallelepiped and stresses on the upper and lower faces are equal to zero.

The problem consists in choosing a temperature distribution on the upper and lower faces so that normal or tangential displacements on these faces take the prescribed values.

The solution of the problem is found in analytic form by means of the method of separation of variables.

Nonlinear Parametric Vibrations of Viscoelastic Medium Contacting Cylindrical Shell Stiffened with Longitudinal Ribs with Regard to Lateral Shift

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In the paper, a problem on parametric vibration of a longitudinally stiffened cylindrical shell contacting with external viscoelastic medium and situated under the action of external pressure is solved in a geometric nonlinear statement by means of the variation principle. Lateral shift of the shell is taken into account. Influences of environment have been taken into account by means of the Pasternak model. The curve separating the stability and instability domains of parametric vibrations have been constructed on the plane "load-frequency".

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The Neumann Boundary Value Problems of Statics of Thermo-Electro-Magneto Elasticity Theory

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We investigate the interior and exterior Neumann boundary value problems for the system of statics of the thermo-electro-magneto elasticity theory. Using the potential method and the theory of integral equations we prove the existence results. We show that the solutions can be represented by the single layer potentials. In the case of an exterior unbounded domain, it is shown that the unknown density of the potential is defined uniquely by the corresponding system of integral equations, while in the case of an interior domain of finite diameter the corresponding system of integral equations are not solvable for arbitrary right hand side data. We establish the necessary and sufficient conditions for the system of integral equations (and thus for the interior Neumann boundary value problem) to be solvable. The basis of the null-space of the corresponding adjoint operator is constructed explicitly which gives us possibility to write the necessary and sufficient conditions in efficient form.

სუსტადელექტროგამტარი სითხის პულსაციური დინება სითბოგადაცემით

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ბლანტი არაკუმშვადი სითხის პულსაციური დინების შესწავლას დიდი პრაქტიკული და თეორიული მნიშვნელობა აქვს. ასეთი ამოცანები სითბოგადაცემის გათვალისწინებით თითქმის შეუსწავლელია.

სტატიაში შესწავლილია სუსტადელექტროგამტარი სითხის პულსაციური მოძრაობა ბრტყელ მილში სითბოგადაცემის გათვალისწინებით, როდესაც მოქმედებს გარეგანი ერთგვა-როვანი მაგნიტური ველი. წნევის დაცემა მოიცემა ფორმულით $\left(-\frac{1}{\rho}\frac{\partial p}{\partial z} = Ae^{-i\omega t}\right)$, ხოლო სითბოგადაცემის ცვლილება ხდება პულსაციურად. სითბოგადაცემის განტოლებაში გათვალისწინებულია როგორც ხახუნის შედეგად გამოწვეული დისიპაცია $\eta\left(\frac{\partial V}{\partial x}\right)^2$, ასევე ჯოულის სითბო σV^2. მიღებულია ნავიე-სტოქსის და სითბოგადაცემის განტოლების ზუსტი ამონახსნები სუსტადელექტროგამტარი ბლანტი არაკუმშვადი სითხის არასტაციონარული

მოძრაობის შემთხვევაში. შემოყვანილი მსგავსების კრიტერიუმები ახასიათებენ ხახუნის ძალით გამოწვეულ სითხის რხევით მოძრაობას გარეგანი ერთგვაროვანი მაგნიტური ველის გავლენით.

გამოკვლევები გვიჩვენებს, რომ გარეგანი ერთგვაროვანი მაგნიტური ველის მოქმედება ამუხრუჭებს სითხის პულსაციურ დინებას. მაგნიტური ველის გაზრდისას სითხის დინების სიჩქარე ბრტყელი მილის ღერძზე კლებულობს, ხოლო კედლებთან ახლოს იზრდება, მაშინ როდესაც საშუალო სიჩქარის სიდიდე ბრტყელი მილის კვეთაში არ იცვლება.

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Dynamical Contact Problem for Elastic Halfspace with Absolutely Rigid and Elastic Inclusion

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It is considered the dynamical contact problem for a half space: $(-\infty < x, z < \infty,\ y > 0)$ strengthened with rigid or elastic inclusion in the form strip: $0 \le y \le b$, $-\infty < z < \infty$, $x = 0$.

The border of the half space is free from load, while an evently distributed shifting harmonic load $\tau_0 e^{-ikt}\delta(y)$ is applied on the outer border.

It is requested to find the field of stress and displacement in the condition of antiplane deformations.

The formulated problem is equivalent to the boundary problem for displacement $\omega = \omega(x, y, t)$:

$$\Delta\omega = \rho\frac{\partial^2\omega}{\partial t^2}, \quad |x| < \infty, \quad y > 0, \quad \frac{\partial\omega(x,0,t)}{\partial y} = 0,$$

on the inclusion the tangential stress has discontinuity:

$$\langle \tau_{xz}(0,y,t)\rangle = \left\langle G\frac{\partial\omega(0,y,t)}{\partial x}\right\rangle = \mu(y,t), \quad 0 < y < b,$$

$$\mu(y,t) \equiv 0, \quad y \ge b.$$

Displacement is continuous and constant for absolutely rigid inclusion: $\omega^{(1)}(0,y,t)=\delta_0 e^{-ikt}$, $0\le y\le b$, $\delta=\mathrm{const}$, while for elastic inclusion is satisfies the condition:

$$\frac{\partial^2\omega^{(1)}(0,y,t)}{\partial y^2}-\frac{\rho_0}{E_0}\frac{\partial^2\omega^{(1)}(0,y,t)}{\partial t^2}=-\frac{1}{E_0h}\mu(y,t)-\frac{1}{E_0h}\tau_0 e^{-ikt}\delta(y),$$

where $\omega^{(1)}(0,y,t)$ is displacement of border points of inclusion, $\mu(y,t)$ is unknown contact stresses, ρ_0 is a density of the material, E_0 is the module of elasticity, h is its thickness.

Using the theory of integral transformations the problem can by reduced to solution of singular integral equations of first or second kind.

Using the method of orthogonal polynomials these equations are reduced to the system of infinite linear algebraic equations. It is proved a quasi-full regularity of mentioned system. The numerical results for the low of change of amplitude of tangential contact stresses are obtained.

Investigation of the Dirichlet and Neumann Boundary Value Problems for a Half-Space filled with a Viscous Incompressible Fluid

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In the paper, for a homogeneous system of linear Stokes differential equations the Dirichlet and Neumann boundary value problems are solved for a half-space by means of Papkovich–Neuber representations and integral Fourier transforms. The solutions are obtained in quadratures.

Solution of a Mixed Problem of the Linear Theory of Elastic Mixtures for a Polygonal Domain with an Equi-Strong Boundary Arc

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In the present work we consider a mixed boundary value problem of statics of the linear theory of elastic mixtures for an isotropic polygon one side of which has a cut of unknown shape. On the entire boundary $\sigma_s=0$, and the vector U_n is a constant on the linear part of the boundary; moreover $\sigma_n=P$, on an unknown contour, where $P=(P_1,P_2)^\top$ is a known constant vector,

$$U_n=\begin{bmatrix} u_1n_1+u_2n_2\\ u_3n_1+u_4n_2\end{bmatrix},\quad \sigma_n=\begin{bmatrix}(Tu)_1n_1+(Tu)_2n_2\\ (Tu)_3n_1+(Tu)_4n_2\end{bmatrix},\quad \sigma_s=\begin{bmatrix}(Tu)_2n_1-(Tu)_1n_2\\ (Tu)_4n_1-(Tu)_3n_2\end{bmatrix},$$

u_k and $(Tu)_k$, $k = \overline{1,4}$, are partial displacement and stress vectors components respectively, and $n = (n_1, n_2)^\top$ is the unit outward normal vector. Applying the method of Kolosov–Musxelishvili the problem is reduced to the Riemann–Hilbert problem for a half-plane.

The stressed state of the body is defined and the equation of the unknown contour is obtained, for the condition that so-called the tangential normal stress vector on those boundary takes one and the same constant value.

Using the obtained results, the problem with an unknown boundary for a polygon weakened by holes in the presence of symmetry is solved.

Generation of Clean Renewable Energy and Desalination of Sea Water by "Super Power Energy Towers"

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A clean and renewable energy source that substitutes oil burn and generates fresh water is of major importance to many global factors such as air pollution, economy and climate. A "Super Power Energy Tower" is a gigantic vertical tower at the top of which a large amount of water (that can be sea water) is poured as small drops. The evaporation of the falling drops produces a strong downward flow. At the bottom of the tower, the kinetic energy of the air can be converted to electrical power by the use of turbines. In order to calculate correctly and accurately the flow of the air and the drops in an Energy Tower of large dimensions, an axisymmetric numerical model was developed based on the solution of the Navier-Stokes differential equations for turbulent flow and integro-differential kinetic transfer equations for calculating the drops evaporation, collection and sedimentation processes. . A detailed description of the set of equations, turbulence parameterization, microphysical processes and numerical methods used in the model can be found in previous publications: [S. Tzivion, T. G. Reisin, Z. Levin, A numerical solution of the kinetic collection equation using high spectral grid resolution: a proposed reference. *J. Computation Phys.* **148** (1999), 527–544; Numerical simulation of axisymmetric flow in "Super Power Energy Towers". *Computational Fluid Dynam.* **9** (2001), No. 1, 560–575. Patent in Israel: "Super Power Energy Tower" No. 129129, 26.10.2005]. Based on numerous numerical experiments optimal geometric, physical and atmospheric parameters for such a tower are obtained. The results show that a tower of 800 m height may produce up to 4500 MW of electricity. The out flowing air at the towerś bottom is $12.5°C$ colder than the environmental air and its relative humidity is near 97%. By extracting the small salty drops that remain in the air and by further cooling this air by about $5°C$, it is possible to obtain relatively cheap, essentially fresh water at a rate of $15\,m^3/sec$. These calculations indicate that the idea to use energy from the evaporation of falling drops in relative warm and dry air heated by sun could be realistic. The results of this study can be further used for the development a small-scale experimental prototype of the Energy Tower.

It is important to notice that the above mentioned results are not based on realistic three dimensional conditions of an energy tower with turbines and environmental factors incorporated. In order to eliminate some of these deficiencies in future it is proposed to develop a three dimensional theoretical model of Energy Tower that incorporates influence of turbines and external environmental conditions on the airflow in the Tower and on the efficiency of energy production.

If the idea of producing environmentally clean energy by energy towers is realistic than usage of such towers for energy production will essentially reduce many global negative factors such as air pollution and global climate warming by greenhouse Gases. Currently, production of reasonably environmentally clean energy is one of the most stressing problems. There are many locations in the world having warm and dry climate. The proposed theoretical study is certainly a necessary step in the possible future realization of Energy Towers. It is of high importance and could have important practical engineering outcomes. By using a three dimensional model, we can find the realistic optimal geometric and physical parameters of the Energy Tower.

Mathematical Physics

Shear Flow Non-Normality Induced Mode Coupling in Rotating oUnstably Stratified Flow Systems

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The non-normal nature of shear flow and its consequences became well understood by the hydrodynamic community in the 1990s. Shortcomings of traditional modal analysis (spectral expansion of perturbations in time and, later, eigenfunction analysis) for shear flows have been revealed. Operators in the mathematical formalism of shear flow modal analysis are non-normal and the corresponding eigenmodes are non-orthogonal. The non-orthogonality leads to strong interference among the eigenmodes. Consequently, a proper approach should fully analyze eigenmode interference. While possible in principle, this is in practice a formidable task. The mathematical approach was therefore changed: the emphasis shifted from the analysis of long-time asymptotic flow stability to the study of transient behavior by, so-called, non-modal approach. This approach grasps linear coupling of vortices and waves [1] and different wave modes in shear flows [2]. The wortex-wave coupling is described by second order inhomogeneous differential equation where, the inhomogeneous (vortex) term is the source of wave mode. The talk is based on this route of research and presents our investigation of shear flow non-normality induced linear dynamics of convective and vortex modes (their transient growth and coupling) in differentially rotating flow, when the fluid is Boussinesq with vertical (to the rotation plane) stratification of thermodynamic quantities and the flow has constant shear.

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Regularized Coulomb T-Matrix

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Since the Coulomb potential slowly decreases at the infinity there appear the well known singularities of quantum mechanical functions which describe two charged particles scattering problem. In our investigation of radial part of two charged particles outgoing half-shell T-matrix we have obtained the next results:

The radial part of Coulomb T-matrix having outgoing asymptotics can be expressed as $\langle q|\, T_l^+(E)\, |k\rangle = \int\limits_0^\infty dr\, r\, j_l(qr) R_l^+(kr)$, where k is a kinematic parameter, q is a absolute value of a vector in momentum space, $j_l(qr)$ denotes the spherical Bessel function, and $R_l^+(k\,r)$ is the Coulomb outgoing wave function. Integrating this representation gives en explicit analytic expression of the half-shell Coulomb T-matrix in the next form:

$$\langle q|\, T_l^+(E)\, |k\rangle = \frac{(-1)^{i\gamma}\exp(3\pi\gamma/2)}{2kq}\,\frac{\Gamma(l-i\gamma+1)}{\Gamma(l+i\gamma+1)}\, Q_l^{i\gamma}\left(\frac{q^2+k^2}{2qk}\right), \quad q \neq k.$$

Here $\Gamma\,(z)$ denotes the Euler gamma function. $Q_\nu^\mu(z)$ is the adjusted Legendre function of the second kind. The function $Q_\nu^\mu(z)$ has the cut on the segment $z \in [-1,+1]$ of the complex plane which tends to the ambiguity of obtained expression at the points $q = k$, corresponding to the elastic scattering mode. The relation obtained satisfies two-particle unitary condition and has Born asymptotics. Besides, taking into account the S-matrix perturbation theory results our formula allows us to find out the regular Coulomb T-matrix which satisfies the two particle unitary condition everywhere in the momentum space:

$$\langle q|\, T_l^+(E)\, |k\rangle_R = \begin{cases} \dfrac{(-1)^{i\gamma}\exp(3\pi\gamma/2)}{2kq}\,\dfrac{\Gamma(l-i\gamma+1)}{\Gamma(l+i\gamma+1)}\, Q_l^{i\gamma}\left(\dfrac{q^2+k^2}{2qk}\right), & q \neq k, \\ \dfrac{\exp(2i\delta_l)-1}{2ik}, & q = k. \end{cases}$$

In contrast with the well known Ford rule [1], our results are obtained without modeling consideration and actually gives us a generalization of the Ford rule for arbitrary values of angular momentum. Hence, we get a solution of ambiguity problem in the Coulomb T-matrix theory.

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Singular Value Decomposition for Data Unfolding in High Energy Physics

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An overview of the data unfolding problem in high energy particle physics will be given, followed by a presentation of the algorithm based on the Singular Value Decomposition of the detector response matrix. The ways of regularising the inversion procedure will be considered, and various examples will be shown, together with recommendations and advice for practical uses of the method.

Gauge Invariant Effective Field Theory for Dressed Nucleons

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A gauge invariant formalism is developed which allows one to construct electromagnetic currents in the effective field theory for low energy nucleons. Dressing of nucleons by mesons is important starting from the next to the leading order approximation. The formalism enables one to take into account nucleon dressing and carry out renormalisation in spite of that all problems are reduced to three-dimensional, Lippmann–Schwinger like equations. The talk is partially based on the papers [1, 2].

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New Two-Dimensional Quantum Models with Shape Invariance

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Two-dimensional quantum models which obey the property of shape invariance are built in the framework of polynomial two-dimensional SUSY Quantum Mechanics. They are obtained using the expressions for known one-dimensional shape invariant potentials. The constructed Hamiltonians are integrable with symmetry operators of fourth order in mo- menta, and they are not amenable to the conventional separation of variables.

Mathematical Education and History

მათემატიკის სწავლების თავისებურებანი საერთაშორისო ბაკალავრიატის (IBO) სადიპლომო პროგრამის მიხედვით

თემურ ახობაძე, პეტრე ბაბილუა

ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი

ევროპული სკოლა

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მათემატიკის ხანგრძლივი ისტორიის მანძილზე ყოველთვის იდგა მისი ეფექტურად სწავლების პრობლემა. სხვადასხვა დროს შეიქმნა მრავალი თეორია მათემატიკის სწავლებასთან დაკავშირებით. დღეისათვის მსოფლიოში ერთ-ერთი ფართოდ გავრცელებული და წარმატებულია საერთაშორისო ბაკალავრიატის საგანმანათლებლო პროგრამა. მოხსენების მიზანია ამ პროგრამაში მათემატიკის სწავლების პრობლემატიკის განხილვა. ამ პროგრამაში მათემატიკა ისწავლება სამ დონეზე: Mathematical Studies SL, Mathematics Standard Level, Mathematics Higher Level. სამივე დონეზე, მათემატიკის სწავლების მიზანია მოსწავლეთა შემდეგი უნარების განვითარება: 1) კვლევითი, 2) სხვადასხვა პრაქტიკული ამოცანების მოდელირების, 3) კვლევებში ტექნოლოგიების გამოყენების, 4) კვლევითი პროექტების დამუშავების.

ეს ის ძირითადი უნარებია, რაზეც აქცენტია გაკეთებული ამ პროგრამით მათემატიკის სწავლებაში. ამ პროგრამის ნაწილია გარე და შიგა შეფასებები (Internal Assessment and External Assessment). შიგა შეფასება მოიცავს მთელი შეფასების 20% და ის გულისხმობს მოსწავლის მიერ კვლევითი სახის დამოუკიდებელი ნაშრომების შესრულებას, ხოლო გარე შეფასება გულისხმობს მთელი შეფასების 80% (რაც გამოიხატება ორნაწილიანი გამოცდის ჩაბარებით: Paper I, Paper II).

ასეთი ტიპის სწავლება უკვე ერთი წელია, რაც საქართველოში ინერგება. ერთ-ერთი სკოლა, სადაც ხორციელდება ეს პროგრამა არის შპს ''ევროპული სკოლა''.

ნაშრომის მიზანია დეტალურად განვიხილოთ ის საკითხები, რაც ამ პროგრამით ისწავლება მათემატიკაში.კერძოდ, მათი შედარება ეროვნულ სასწავლო გეგმასთან; მისი დადებითი და პრობლემური საკითხები.

ლიტერატურა

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Mathematics Before and After Gödel

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The beginning of the 20th century may be regarded as crisis of mathematics, because a lot of new paradoxes and questions appeared. The fact that Georg Cantor has founded set theory and then Ernst Zermelo and Abraham Fraenkel proposed an axiomatization of this theory, stated new open problems for mathematicians of the time. Russell has analyzed the Frege's results and during the process of the work did meet with the well-known "Russell Paradox". He co-authored, with A. N. Whitehead, "Principia Mathematica", as an attempt to ground mathematics on logic. In this work it was main idea about formalizing the whole Mathematics. In this period the process of formalizing mathematics was a very actual and interesting question for mathematicians. It was the fundamental problem: is it possible to show that mathematics is consistent? In a 1900 speech to the participants of International Congress of Mathematics, David Hilbert set out a list of 23 unsolved problems in mathematics. The second problem was: is arithmetic or the classical theory of natural numbers consistent? In 1920 he proposed an explicit research project that became known as Hilbert's program. He wanted mathematics to be formulated on a solid and complete logical foundation. He believed that , in principle, this could be done. But in 1931 Gödel published his incompleteness theorems in "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme" (called in English "On Formally Undecidable Propositions of Principia Mathematica and Related Systems"). In the article he proved that it is impossible to show the consistency of arithmetic by means of only finite methods. At present, the general form of Gödel's theorem looks as follows (see[1–3]):

Gödel's Incompleteness Theorem. *For any formal, effectively definable theory T including basic arithmetical truths and also certain truth about formal provability, T includes a statement of its own consistency if and only if T is inconsistence.*

This result turned out to be a cornerstone for further development of mathematical logic and whole mathematics. A lot of new branches of mathematics founded after this theorem (theory of models, universal algebra, theory of algorithms and so on).

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საშუალო სკოლაში მათემატიკის გაღრმავებული სწავლების შესახებ

გურამ გოგიშვილი

საქართველოს საპატრიარქოს წმიდა ანდრია პირველწოდებულის სახელობის ქართული უნივერსიტეტი, სასწავლო ცენტრი

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მოხსენებაში წარმოდგენილია საშუალო სკოლაში გაღრმავებული სწავლების ზოგადი პრინციპებისა და მეთოდიკის ზოგიერთი მნიშვნელოვანი საკითხის ანალიზი; განხილულია სასწავლო თემატიკის შერჩევისა და მისი აქტუალურობის, პრობლემათა შორის კავშირების ძიების, კვლევის მეთოდების დაუფლების, თეორიული მასალის შესაბამისი სამოტივაციო და საილუსტრაციო ამოცანების შერჩევის, კონკრეტული ამოცანების კვლევის სხვადასხვა მეთოდის შედარებითი ანალიზისა და ამოხსნის ოპტიმალური გზების ძიების საკითხები.

მოხსენებაში განხილულია აგრეთვე კონკრეტული პრობლემებიდან აღმოცენებული იდეების გააანალიზების, გაღრმავებისა და განზოგადების საკითხები; შეფასებულია ის წვლილი, რაც მათემატიკის გაღრმავებულ სწავლებას შეაქვს მოსწავლის საგანმანათლებლო სისტემაში, კერძოდ, განათლების შემდგომი საფეხურების გასავლელად მოსწავლეთა წარმატებით მომზადებაში.

Algebraic Curves and their Properties in Higher Mathematics Course

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As usual, elements of the theory of plane curves are included in various lecture courses of Higher Mathematics (Analytic Geometry, Differential Geometry, Calculus). The beginning courses of Analytic Geometry are primarily devoted to algebraic curves of first and second degree (i.e., conical sections) and their properties. In particular, it is demonstrated that any such a curve admits a rational parameterization. In more advanced courses, various parameterizations of much more complicated curves are discussed in Calculus lectures and courses of Differential Geometry. These topics are interesting and important from the purely theoretical point of view and from the view-point of applications (in rigid and arch type constructions). The extensive study of these topics is justified from the didactic and methodological stand-point. In our opinion, it is desirable in various mathematics lectures to pay more attention to questions related to general algebraic curves and their properties. This is very reasonable, because the properties of algebraic curves are tightly connected with Diophantine equations in classical Number Theory and the algebra of polynomials.

Among topics which seem to be of interest to students, we may propose the following ones:

(1) Newton's Theorem on barycenters of sections of an algebraic curve by parallel straight lines (this is one of the first theorems concerning a general property of algebraic curves);

(2) the problem of a rational parameterization of an algebraic curve (which is almost trivial for conical sections but is decided negatively for cubic curves);

(3) the fact that the envelope of an algebraic family of algebraic curves is also algebraic;

(4) the fact that the evolute of an algebraic curve is also algebraic.

As a summary, we may state that the discussion of algebraic curves in Higher Mathematics courses should be more thorough and extensive, which will provide under-graduate students with additional valuable information from algebra and geometry.

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აბუსერისძე ტბელის მათემატიკური ფორმულები მარხვის პერიოდის გამოსათვლელად

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ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი

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აბუსერისძე ტბელის პასქალურ-კალენდარული შრომა დიდი ხანია ცნობილია სამეცნიერო ლიტერატურაში. პასქალურ გამოთვლებში დიდი მნიშვნელობა ენიჭებოდა აღდგომიდან უშუალოდ დაკავშირებულ მოძრავი მარხვების თარიღის დადგენას. ვინაიდან ამ თარიღების დაცილება აღდგომის დღიდან ყოველთვის მუდმივ სიდიდეს შეადგენს, მათი გამოსათვლელი წესებიც ამოსავალ მონაცემად აღდგომის თარიღს იყენებს. აბუსერისძის მიერ შემოთავაზებული წესებიც, რომლებიც ჩვენ თანამედროვე ფორმულების სახით მოგვყავს, ზუსტად აღნიშნულ სქემას ეფუძნება.

ცნობილია, რომ ერთერთი მოსამზადებელი კვირიაკეებიდან - ხორცის აკრების კვირიაკე - და "პირმარხვის" თარიღები შესაბამისად 56 და 48 დღით არიან დაცილებული აღდგომის დღესასწაულიდან. 56 დღიანი ინტერვალი იანვრის, თებერვლის და მარტის ან თებერვლის, მარტის და აპრილის გარკვეულ დღეებზე მოდის. კერძოდ, პირველ შემთხვევაში ის მოიცავს ხორცის აკრეფის თარიღიდან (x) თვის ბოლომდე ათვლილი დღეების ანუ $(31-x)$ დღის, თებერვლის სრული 28 დღის და 1 მარტიდან აღდგომის თარიღის (p) ჩათვლით აღებული დღეების ანუ p დღის ჯამს. მეორე შემთხვევაში შესაკრებებს შეადგენენ თებერვალში $(28-p)$ დღე, მარტში სრული 31 დღე და აპრილში p დღე, რაც ტოლობის სახით შეიძლება ასე ჩავწეროთ:

$(31-x)+28+p=56$ (1) და $(28-x)+31+p=56$ (2). ანალოგიური ტოლობა უნდა იქნეს შედგენილი "პირმარხვის" დღესთან (y) დაკავშირებით, მხოლოდ ამ შემთხვევაში ჯამი 56-ის ნაცვლად 48 დღით უნდა იყოს შემდეგნაირად წარმოდგენილი. $(31-y)+28+p=48$ და $(28-y)+31+p=48$. ნაკიანი წლის შემთხვევაში რიცხვი 28 უნდა შეიცვალოს 29-ით.

აბუსერისძის მიერ სიტყვიერი ფორმით შემოთავაზებული ხორცის აკრების კვირიაკეს და "პირმარხვის" გამოსათვლელი წესები სწორედ ამ ტოლობებში მოყვანილი დღეთა გადანაწილების საფუძველზე არის ჩამოყალიბებული და ისინი თანამედროვე ფორმით შეიძლება წარმოვადგინოთ. სადაც x იანვრის ან თებერვლის თვის თარიღია ხორცის აკრების თარიღისთვის ჩვეულებრივ და ნაკიან წლებში შესაბამისად: $x=p+3$ და $x=p+4$. "პირმარხვის" თარიღისთვის აბუსერისძის შემოთავაზებული წესით $y=p+11$ და $y=p+12$ (ნაკიანი წლისთვის). იმის მიხედვით, თუ როდის არის აღდგომის დღესასწაული, როცა 1) $p+11\leq 28$ და $p+12\leq 29$ ან 2) $p+11>28$ და $p+12>29$, მაშინ ვიღებთ ნაშთს $p+11$ და $p+12$-ის შესაბამისად 31-ზე, 28-ზე და 29-ზე გაყოფის შედეგად.

საინტერესო წესი მოჰყავს აბუსერისძეს მოციქულთა მარხვასთან დაკავშირებით. ამ მარხვის საწყისი აღდგომის დღეზეა დამოკიდებული. პირველი აპრილი არის აღებული საყრდენ წერტილად. ამის შემდეგ ადვილად ითვლება აღდგომის სხვა თარიღების შესაბამისი მარხვების ხანგრძლივობა.

სიტყვიერად გადმოცემული ეს წესი თანამედროვე სიმბოლოებით შემდეგი ფორმულების სახით შეიძლება იქნეს წარმოდგენილი (აქ მარხვის საძიებელი ხანგრძლივობა ასო F-ით არის აღნიშნული):

$F=32+[31-(P-1)]$ (როდესაც აღდგომა მარტის თვეზე მოდის) და $F=32-(P-1)$ (როცა აღდგომას აპრილის თვეში აქვს ადგილი).

მათემატიკის სწავლების საკითხები ბათუმის პირველ ქართულ სასწავლებლებში

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მოხსენების მიზანს შეადგენს მსმენელი კიდევ ერთხელ დააბრუნოს იმ ისტორიულ გარემოში, როდესაც ოსმალთა იმპერიიდან დედა-საქართველოს შემოერთებული საქართველოს უმნიშვნელოვანესი ნაწილის, აჭარის მოსახლეობის წინაშე დადგა მშობლიურ ენაზე განათლების მიღების საკითხი.

სამი საუკუნის განმავლობაში ოსმალთა ბატონობის ქვეშ ყოფნამ ბუნებრივია, დაღი დაასვა ამ კუთხის მცხოვრებთა ყოფა-ცხოვრებისა და განათლების საკითხს. ერის მოჭირნახულენი თავიანთ მიზნად და გადაუდებელ ამოცანად ამ კუთხეში ქართული წიგნის, ქართული საგანმანათლებლო დაწესებულებების გახსნას და ადგილობრივი მოსახლეობის განათლებას ისახავდნენ. ახალგახსნილ ქართულ სასწავლებლებში ერთ-ერთ ძირითად საგნად მათემატიკა ისწავლებოდა. მოხსენებაში მოყვანილია მათემატიკის სწავლებასთან დაკავშირებული საკითხები. თუნდაც ის ფაქტი, რომ მათემატიკის სწავლება ქართულ ენაზე მიმდინარეობდა, იმ დროისათვის მნიშვნელოვანი მონაპოვარი იყო.

რესურსების ოპტიმალური განაწილების ამოცანა

(სწავლების მეთოდიკა და ანალიზი)

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განვიხილავ დინამიური პროგრამირების მეთოდით გამოყენებითი ხასიათის ამოცანების ამოხსნის თავისებურებებს. ამოცანათა ნიმუშებს, რომელთა ამოხსნისათვისაც გამოიყენება დინამიური ოპტიმიზაციის მეთოდები. კერძოდ, რესურსების ოპტიმალური განაწილების ზოგად ამოცანას, როდესაც რაღაც k რაოდენობის რესურსები ისეთნაირად უნდა გადანაწილდეს n სხვადასხვა საწარმოს შორის, რომ ეფექტურობის ჯამური მაჩვენებელი იყოს მაქსიმალური. გამოყვანილია რეკურენტული თანაფარდობა. სტუდენტთა დაინტერესების თვალსაზრისით განიხილება რეალობიდან აღებული შესაბამისი ამოცანების ამოხსნა. როგორიცაა, საწარმოთა შორის კაპიტალდაბანდებათა განაწილების ამოცანა. მოყვანილია აღნიშნული საკითხების სწავლების ანალიზი.

On Equidecomposability Paradoxes

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In 1905, G. Vitali [1] proved the existence of a non-measurable point set (in the Lebesgue sense). With this classical result absolutely new type of researches started in Mathematics, which also reflected in receiving various Equidecomposability Paradoxes in Euclidean space. Namely, the analysis of the proof of Vitali's Theorem has revealed that the existence of a non-measurable set is closely related to uncountable forms of the Axiom of Choice and certain group-theoretical features of the Lebesgue measure (see [2, 3]). As it has become clear later on, this factor has been recognized in a different form in remarkable results obtained by F. Hausdorff, S. Banach, A. Tarski, J. Mycielski and others. Their results may be considered as a continuation of the research initiated by Vitali whose theorem was generalized in various directions along with obtaining many further equidecomposability paradoxes. A special attention deserves a discovery by J. von Neumann that the existence of a free subgroup of the motion group of three-dimensional Euclidean space, with two independent generators, plays a very important role for the aforementioned theorems and similar statements (see again [2, 3]).

The purpose of this report is to review the above-mentioned researches and demonstrate an advisability of their inclusion in an appropriate (more or less adapted) form in the modern courses of Higher Mathematics. This is justified by a lot of interesting connections of the topic with other mathematical disciplines.

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