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On interaction of electromagnetic waves with infinite bianisotropic layered slab

Tengiz Buchukuri[*] [1], **Roland Duduchava**[**] [1], and **Levan Sigua**[***] [2]

[1] A. Razmadze Mathematical Institute, Academy of Sciences of Georgia, 1, M.Alexidze str., Tbilisi 93, Georgia
[2] Tbilisi State University, Department of Mechanics and Mathematuics,University str. 1, Tbilisi 0143, Georgia

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A stratified general bianisotropic medium is considered, consisting of several infinite slabs either between two half-spaces filled with isotropic material, or grounded. The slab is illuminated by an incident plane wave from one of the half-spaces. The excited electromagnetic field inside the bianisotropic slab and in the isotropic domains is described.

Introduction

In three dimensional Euclidean space $\mathbb{R}^3$ with coordinates x, y, z the domain located between the planes $z = 0 = d_0$ and $z = -d_n = -d$ is occupied by n–layered structure, composed of a stratified general bianisotropic medium. The upper half space $\Omega' = \mathbb{R}^3_+ = \{(x,y,z) : z > 0\}$ is filled by an isotropic medium (e.g. with an air) with the scalar dielectric constants ε', μ', whereas the lower half space, the domain $\Omega'' = \{(x,y,z) : z < -d_n\}$, is filled either with an isotropic material with dielectric constants ε'', μ'', or is grounded (see Fig. 1 and Fig. 2).

Ω'

$-d_1$

$-d_2$

$-d_3$

Ω''

Multi-layer slab

Fig. 1

Ω'

$-d_1$

Grounded slab

Fig. 2

The domains $\Omega_j = \{(x,y,z) : -d_j < z < -d_{j-1}\},\ j = 1, \dots, n$ are filled up by a most general bianisotropic material characterized by four constitutive tensors (see [3]):

[*] eMail: t_buchukuri@yahoo.com, Phone: +995.32.303278 , Fax: +995.32.364086
[**] eMail: dudu@num.uni-sb.de, Phone: +995.32.251619 , Fax: +995.32.364086
[***] eMail: sigua@rmi.acnet.ge, Phone: +995.32.663741

$$\varepsilon^{(j)} = \begin{bmatrix} \varepsilon^{(j)}_{xx} & \varepsilon^{(j)}_{xy} & \varepsilon^{(j)}_{xz} \\ \varepsilon^{(j)}_{yx} & \varepsilon^{(j)}_{yy} & \varepsilon^{(j)}_{yz} \\ \varepsilon^{(j)}_{zx} & \varepsilon^{(j)}_{zy} & \varepsilon^{(j)}_{zz} \end{bmatrix} \text{–relative dielectric permittivity,}$$

$$\mu^{(j)} = \begin{bmatrix} \mu^{(j)}_{xx} & \mu^{(j)}_{xy} & \mu^{(j)}_{xz} \\ \mu^{(j)}_{yx} & \mu^{(j)}_{yy} & \mu^{(j)}_{yz} \\ \mu^{(j)}_{zx} & \mu^{(j)}_{zy} & \mu^{(j)}_{zz} \end{bmatrix} \text{–relative magnetic permeability,}$$

$$\xi^{(j)} = \begin{bmatrix} \xi^{(j)}_{xx} & \xi^{(j)}_{xy} & \xi^{(j)}_{xz} \\ \xi^{(j)}_{yx} & \xi^{(j)}_{yy} & \xi^{(j)}_{yz} \\ \xi^{(j)}_{zx} & \xi^{(j)}_{zy} & \xi^{(j)}_{zz} \end{bmatrix} \text{ and } \eta^{(j)} = \begin{bmatrix} \eta^{(j)}_{xx} & \eta^{(j)}_{xy} & \eta^{(j)}_{xz} \\ \eta^{(j)}_{yx} & \eta^{(j)}_{yy} & \eta^{(j)}_{yz} \\ \eta^{(j)}_{zx} & \eta^{(j)}_{zy} & \eta^{(j)}_{zz} \end{bmatrix} \text{–cross coupling tensors.}$$

Interaction of electromagnetic waves with the above described system of bianisotropic slabs are governed by the Maxwell equations. For time harmonic fields with angular frequency ω we get the following equation system for the slab Ω_j with respect to electric and magnetic field vectors E end H:

$$\operatorname{rot} E = -i\omega\mu_0(\zeta_0^{-1}\eta^{(j)}E + \mu^{(j)}H)\,, \tag{0.1}$$

$$\operatorname{rot} H = i\omega\varepsilon_0(\varepsilon^{(j)}E + \zeta_0\xi^{(j)}H)\,, \quad j = 1, \dots, n\,, \tag{0.2}$$

$$\zeta_0 := \left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{2}} \tag{0.3}$$

In the domains Ω' and Ω'' the equations acquire the form

$$\operatorname{rot} E = -i\omega\mu_0\mu H\,, \tag{0.4}$$

$$\operatorname{rot} H = i\omega\varepsilon_0\varepsilon E\,. \tag{0.5}$$

Here

$$E := \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}, \quad H := \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix},$$

ε_0 is the permittivity and μ_0 is the permeability of a vacuum, ε and μ are permittivity and permeability of the particular domain (Ω' or Ω'').

We assume that the domain Ω' is illuminated with plane waves and describe the electromagnetic field in each of the domains $\Omega_j,\ j = 1, \dots, n$, Ω'. Moreover, in the case when the slab is not grounded and Ω'' is present, we also describe the electromagnetic field in Ω''.

The problem was treated by J.Tsalamengas in [5]. The solution found in the present paper (1.6) is simpler and more general. Moreover, (1.6) is valid in more general situation when the matrix P_j has multiple eigenvalues, but is still diagonalizable. (cf. [5, (22b)-(26)]). Other important case of multiple eigenvalues is the case of isotropic slab. Then the eigenvalues are $\pm i$ and each eigenvalue has multiplicity 2. This case can not be covered by the approach suggested in [5].

In § § 2–3 a general grounded bianisotropic slab, consisting of a single layer, is illuminated by the plane wave (E^{inc}, H^{inc}) incident from the domain Ω' along the unit vector k^{inc} ([5]). The solution obtained below improves the corresponding results in ([5]).

In the concluding § 4 we treat the problem of reflection and transmission of a plane wave through a general bianisotropic slab between two half spaces Ω' and Ω'' filled by isotropic material.

1 Solution of Maxwell's equations for a slab

Applying the partial Fourier transform with respect to the variables (x, y)

$$\mathcal{K}(k_x, k_y, z) := \mathcal{F}_{(x,y)\to(k_x,k_y)}[K(x,y,z)](k_x,k_y,z)$$
$$:= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ik_x x + ik_y y} K(x,y,z)\,dx\,dy$$

(the dual variables are k_x and k_y, respectively) we get the following system of ordinary differential equations with respect to the Fourier images of E and H in Ω_j:

$$\begin{cases} -G\,\dfrac{d}{dz}\,\mathcal{E}_t + A_1^{(j)}\mathcal{E}_t + B_1^{(j)}\mathcal{H}_t + C_1^{(j)}\mathcal{N}_z = 0, \\ G\,\dfrac{d}{dz}\,\mathcal{H}_t + A_2^{(j)}\mathcal{E}_t + B_2^{(j)}\mathcal{H}_t + C_2^{(j)}\mathcal{N}_z = 0, \\ A_3^{(j)}\mathcal{E}_t + B_3^{(j)}\mathcal{H}_t + C_3^{(j)}\mathcal{N}_z = 0. \end{cases} \tag{1.1}$$

Here

$$\mathcal{E} := \begin{bmatrix} \mathcal{E}_x \\ \mathcal{E}_y \\ \mathcal{E}_z \end{bmatrix} = \mathcal{F}E\,, \quad \mathcal{H} := \begin{bmatrix} \mathcal{H}_x \\ \mathcal{H}_y \\ \mathcal{H}_z \end{bmatrix} = \mathcal{F}H$$

$$\mathcal{E}_t := \begin{bmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{bmatrix}, \quad \mathcal{H}_t := \begin{bmatrix} \mathcal{H}_x \\ \mathcal{H}_y \end{bmatrix}, \quad \mathcal{N}_z := \begin{bmatrix} \mathcal{E}_z \\ \mathcal{H}_z \end{bmatrix}$$

$$A_1^{(j)} := -ik_0\eta_t^{(j)}\,, \quad B_1^{(j)} := -ik_0\zeta_0\mu_t^{(j)}\,, \quad C_1^{(j)} := -i\begin{bmatrix} k_0\eta_{xz}^{(j)} - k_y & k_0\zeta_0\mu_{xz}^{(j)} \\ k_0\eta_{yz}^{(j)} + k_x & k_0\zeta_0\mu_{yz}^{(j)} \end{bmatrix},$$

$$A_2^{(j)} := -ik_0\zeta_0^{-1}\varepsilon_t^{(j)}\,, \quad B_2^{(j)} := -ik_0\xi_t^{(j)}\,, \quad C_2^{(j)} := -i\begin{bmatrix} k_0\zeta_0^{-1}\varepsilon^{(j)} & k_0\xi_{xz}^{(j)} + k_y \\ k_0\zeta_0^{-1}\varepsilon_{yz}^{(j)} & k_0\xi_{yz}^{(j)} - k_x \end{bmatrix},$$

$$k_0 := \zeta_0\omega\varepsilon_0 = \omega\sqrt{\mu_0\varepsilon_0}\,,$$

$$A_3^{(j)} := -i\begin{bmatrix} k_0\zeta_0^{-1}\varepsilon_{zx}^{(j)} & k_0\zeta_0^{-1}\varepsilon_{zy}^{(j)} \\ k_0\eta_{zx}^{(j)} + k_y & k_0\eta_{zy}^{(j)} - k_x \end{bmatrix},$$

$$B_3^{(j)} := -i\begin{bmatrix} -k_y + k_0\xi_{zx}^{(j)} & k_x + k_0\xi_{zy}^{(j)} \\ k_0\zeta_0\mu_{zx}^{(j)} & k_0\zeta_0\mu_{zy}^{(j)} \end{bmatrix},$$

$$C_3^{(j)} := -ik_0\begin{bmatrix} \zeta_0^{-1}\varepsilon_{zz}^{(j)} & \xi_{zz}^{(j)} \\ \eta_{zz}^{(j)} & \zeta_0\mu_{zz}^{(j)} \end{bmatrix}, \quad G := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

where $M_t = \big[M_{jk}\big]_{2\times 2}$ denotes the upper left 2×2 block of the initial 3×3 matrix $M = \big[M_{jk}\big]_{3\times 3}$.

The total stored energy U of the dynamical electromagnetic field $\begin{bmatrix} E \\ H \end{bmatrix}$ in bianisotropic medium is given by the formula (see [3, § 1.3]):

$$2U = \varepsilon_0\varepsilon_{uv}E_uE_v + (\mu_0\varepsilon_0)^{\frac{1}{2}}(\xi_{uv}E_uH_v + \eta_{vu}E_vH_u) + \mu_0\mu_{uv}H_uH_v \tag{1.2}$$

(we accept the standard Einstein's convention that with respect to repeated indices u, v the sum over $u, v = x, y, z$ is taken automatically).

If the electromagnetic field is excited ($(E, H) \neq 0$), the corresponding stored electromagnetic energy should be positive $U > 0$ and the energy vanishes if and only if $E = H = 0$. To enforce the formulated property, we have to accept that the quadratic form U in (1.2) is positive definite. The condition (1.2) implies in particular

$$\varepsilon_{uu} > 0\,, \quad \mu_{uu} > 0\,, \quad \mathcal{D}_u := \varepsilon_{uu}\mu_{uu} - \eta_{uu}\xi_{uu} > 0 \quad \text{for all} \quad u = x, y, z\,,$$

and, therefore,

$$\det C_3^{(j)} = -k_0^2(\varepsilon_{zz}^{(j)}\mu_{zz}^{(j)} - \xi_{zz}^{(j)}\eta_{zz}^{(j)}) \neq 0 \quad \forall j = 1, \ldots, n. \tag{1.3}$$

If we solve for $\overline{\mathcal{N}}_z$ the third equation in (1.1) and insert it into the first two equations we obtain

$$\frac{d}{dz}\begin{bmatrix} \mathcal{E}_t \\ \mathcal{H}_t \end{bmatrix} = -ik_0 P^{(j)} \begin{bmatrix} \mathcal{E}_t \\ \mathcal{H}_t \end{bmatrix}, \quad \text{in } \Omega_j, \tag{1.4}$$

where

$$P^{(j)} := \frac{i}{k_0}\begin{bmatrix} G\,C_1^{(j)}(C_3^{(j)})^{-1}A_3^{(j)} - G\,A_1^{(j)} & G\,C_1^{(j)}(C_3^{(j)})^{-1}B_3^{(j)} - G\,B_1^{(j)} \\ G\,A_2^{(j)} - G\,C_2^{(j)}(C_3^{(j)})^{-1}A_3^{(j)} & G\,B_2^{(j)} - G\,C_2^{(j)}(C_3^{(j)})^{-1}B_3^{(j)} \end{bmatrix}. \tag{1.5}$$

General solution to (1.4) in Ω_j has the form

$$\begin{bmatrix} \mathcal{E}_t(z) \\ \mathcal{H}_t(z) \end{bmatrix} = \exp[-ik_0(d_{j-1}+z)P^{(j)}]\begin{bmatrix} \mathcal{E}_t(-d_{j-1}) \\ \mathcal{H}_t(-d_{j-1}) \end{bmatrix} = \cdots \tag{1.6}$$

$$= T^{(j)}(d_{j-1}+z)T^{(j-1)}(d_{j-2}-d_{j-1})\dots T^{(1)}(-d_1)\begin{bmatrix} \mathcal{E}_t(0) \\ \mathcal{H}_t(0) \end{bmatrix}, \quad -d_j < z < -d_{j-1},$$

$$T^{(k)}(\zeta) := \exp[-ik_0\zeta P^{(k)}] = \begin{bmatrix} T_1^{(k)}(\zeta) & T_2^{(k)}(\zeta) \\ T_3^{(k)}(\zeta) & T_4^{(k)}(\zeta) \end{bmatrix}, \qquad j,k = 1,\dots,n \tag{1.7}$$

and $T_1^{(k)}(\zeta),\dots,T_4^{(k)}(\zeta)$ are 2×2 matrix functions. From the structure of the matrix $T^{(k)}$ it follows that $T^{(k)}(-\zeta) = (T^{(k)})^{-1}(\zeta)$.

We can express $T^{(j)}$ as a polynomial of the matrix $P^{(j)}$. If the matrix $P^{(j)}$ is diagonalizable (i.e., $P^{(j)}$ has a simple Jordan structure) and $\lambda_1^{(j)}(k_x,k_y),\dots,\lambda_m^{(j)}(k_x,k_y)$, $1 \le m \le 4$ of the matrix $P^{(j)}(k_x,k_y)$ are all different eigenvalues:

$$\lambda_k^{(j)}(k_x,k_y) \neq \lambda_l^{(j)}(k_x,k_y) \qquad \forall k_x,k_y \in \mathbb{R}, \quad k,l = 1,\dots,m, \quad k \neq l,$$

then due to the Lagrange interpolation

$$T^{(j)}(z) := \exp(-ik_0 z P^{(j)}) = \sum_{k=1}^{m}\prod_{l\neq k}\frac{P^{(j)} - \lambda_l^{(j)} I}{\lambda_k^{(j)} - \lambda_l^{(j)}} e^{-ik_0\lambda_k^{(j)} z}, \tag{1.8}$$

where I is the unit matrix (see [1]). In particular, (1.8) holds, if all eigenvalues of the matrix $P^{(j)}(k_x,k_y)$ are distinct (i.e. $m = 4$).

Other cases can be treated similarly. For example, if eigenvalues coincide pairwise $\lambda_1^{(j)} = \lambda_2^{(j)} \neq \lambda_3^{(j)} = \lambda_4^{(j)}$; then

$$\begin{aligned} T^{(j)}(z) = {} & \frac{\exp(-ik_0 z\lambda_1^{(j)})}{(\lambda_1^{(j)} - \lambda_3^{(j)})^2}(P^{(j)} - \lambda_3^{(j)} I)^2 - 2\frac{\exp(-ik_0 z\lambda_1^{(j)})}{(\lambda_1^{(j)} - \lambda_3^{(j)})^3}(P^{(j)} - \lambda_1^{(j)} I)(P^{(j)} - \lambda_3^{(j)} I)^2 \\ & -ik_0 z\frac{\exp(-ik_0 z\lambda_1^{(j)})}{(\lambda_1^{(j)} - \lambda_3^{(j)})^2}(P^{(j)} - \lambda_1^{(j)} I)(P^{(j)} - \lambda_3^{(j)} I)^2 + \frac{\exp(-ik_0 z\lambda_3^{(j)})}{(\lambda_3^{(j)} - \lambda_1^{(j)})^2}(P^{(j)} - \lambda_1^{(j)} I)^2 \\ & -2\frac{\exp(-ik_0 z\lambda_3^{(j)})}{(\lambda_3^{(j)} - \lambda_1^{(j)})^3}(P^{(j)} - \lambda_1^{(j)} I)^2(P^{(j)} - \lambda_3^{(j)} I) \\ & -ik_0 z\frac{\exp(-ik_0 z\lambda_3^{(j)})}{(\lambda_1^{(j)} - \lambda_3^{(j)})^2}(P^{(j)} - \lambda_1^{(j)} I)^2(P^{(j)} - \lambda_3^{(j)} I). \end{aligned} \tag{1.9}$$

Let us note that for an isotropic layer $\Omega^{(j)}$ we have to deal just with this case.

2 Interaction of plane waves with bianisotropic grounded slab

In the present section we shall consider reflection of waves by a general grounded bianisotropic slab, consisting of a single layer. The slab is illuminated by the plane wave (E^{inc}, H^{inc}) incident from the domain Ω' along the unit vector k^{inc} ([5]):

$$\begin{aligned} E^{inc}(r) &= E_0 \exp\big[-ik_0 k^{inc}\cdot r\big], \\ H^{inc}(r) &= H_0 \exp\big[-ik_0 k^{inc}\cdot r\big], \end{aligned} \tag{2.1}$$

where

$$k^{inc} = \overline{x}\cdot\sin\psi - \overline{z}\cos\psi, \quad r = x\cdot\overline{x} + y\cdot\overline{y} + z\cdot\overline{z},$$

$\overline{x}, \overline{y}, \overline{z}$ are the basis vectors of coordinate system, ψ is the angle of incidence of the plane wave, and E_0, H_0 are constant vectors. They are expressed by means of constant quantities E^{TM}_{inc} and E^{TE}_{inc} as follows

$$\begin{aligned} E_0 &= -E^{TM}_{inc}\overline{y} - E^{TE}_{inc}(k^{inc}\times\overline{y}), \\ H_0 &= \Big[-E^{TM}_{inc}(k^{inc}\times\overline{y}) + E^{TE}_{inc}\overline{y}\Big]\Big/\zeta_0. \end{aligned} \tag{2.2}$$

Since the excitation wave is independent of y-coordinate, it is rather natural to assume that also a solution is independent of y. Then, applying the Fourier transform $\mathcal{F}_{x\to k_x}$ to (2.1), we get

$$\mathcal{E}^{inc}(k_x; z) = E_0 \exp(izk_0\cos\psi)\delta(k_x - k_0\sin\psi), \tag{2.3}$$

$$\mathcal{H}^{inc}(k_x; z) = H_0 \exp(izk_0\cos\psi)\delta(k_x - k_0\sin\psi). \tag{2.4}$$

where δ denotes the Dirac's distribution.

In the domain Ω' occupied by the air, besides the excited wave, propagates the reflected wave, which in fact is a solution of Maxwell homogeneous equation in the half-space $z > 0$ satisfying radiation condition at infinity. It can be expressed by means of **TM** and **TE** waves as follows:

$$\mathcal{E}^{(r)}(k_x; z) = \Big[-E^{TM}_0(k_x)\overline{y} - E^{TE}_0(k_x)\big(i\gamma_0\overline{x} + k_x\overline{z}\big)/k_0\Big]e^{-\gamma_0 z}, \tag{2.5}$$

$$\mathcal{H}^{(r)}(k_x; z) = \frac{1}{\zeta_0}\Big[-E^{TM}_0(k_x)\big(i\gamma_0\overline{x} + k_x\overline{z}\big)/k_0 + E^{TE}_0(k_x)\overline{y}\Big]e^{-\gamma_0 z} \tag{2.6}$$

Here $\mathcal{E}^{(r)}$ and $\mathcal{H}^{(r)}$ denote the Fourier transforms of reflected electric and magnetic fields, respectively, E^{TE}_0 and E^{TM}_0 are unknown temperate distributions, $\gamma_0 = (k_x^2 - k_0^2)^{1/2}$ and is chosen so that $-\dfrac{\pi}{2} < \arg\gamma_0 \le \dfrac{\pi}{2}$, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, $\zeta_0 = \sqrt{\mu_0/\varepsilon_0}$. (see [5]).

On the plane $z = 0$, separating the air and the slab boundary conditions have the form:

$$\left\{\begin{array}{c} \mathcal{E}_t{}'(0) \\ \mathcal{H}_t{}'(0) \end{array}\right\} = \left\{\begin{array}{c} \mathcal{E}_t{}^{(1)}(0) \\ \mathcal{H}_t{}^{(1)}(0) \end{array}\right\}; \tag{2.7}$$

where $\left\{\begin{array}{c} \mathcal{E}_t{}' \\ \mathcal{H}_t{}' \end{array}\right\}$ and $\left\{\begin{array}{c} \mathcal{E}_t{}^{(1)} \\ \mathcal{H}_t{}^{(1)} \end{array}\right\}$ represent value of $\left\{\begin{array}{c} \mathcal{E}_t \\ \mathcal{H}_t \end{array}\right\}$ in Ω' and $\Omega^{(1)}$ respectively. On the plane $z = -d$, separating the slab and the ground

$$\mathcal{E}_t{}^{(1)}(-d) = 0, \tag{2.8}$$

because the slab is grounded.

Thus the problem of interaction of plane waves with grounded bianisotropic slab can be formulated in the following way:

In the half-space $\{z > -d\}$ find electric and magnetic fields $E(x, z)$ and $H(x, z)$ belonging to the space of temperate distributions $S'(\mathbb{R})$ for each $z > -d$, satisfying Maxwell equations for isotropic medium together with radiation conditions in the half-space $\{z > 0\}$, Maxwell equations for bianisotropic medium in the slab $\{-d < z < 0\}$ and boundary conditions (2.7) and (2.8).

An electromagnetic field appears in the domain Ω' as the superposition of the initiated and the reflected waves:

$$\left\{ \begin{array}{c} \mathcal{E}_t{}'(0) \\ \mathcal{H}_t{}'(0) \end{array} \right\} = \left\{ \begin{array}{c} E_{0t} \\ H_{0t} \end{array} \right\} \delta(k_x - k_0 \sin\psi) + \left\{ \begin{array}{c} \mathcal{E}_t^{(r)}(k_x, 0) \\ \mathcal{H}_t^{(r)}(k_x, 0) \end{array} \right\}. \tag{2.9}$$

Here

$$E_{0t} = \left(\begin{array}{c} E_{0x} \\ E_{0y} \end{array} \right), \quad H_{0t} = \left(\begin{array}{c} H_{0x} \\ H_{0y} \end{array} \right). \tag{2.10}$$

In the domain $\Omega^{(1)}$ we have

$$\left(\begin{array}{c} \mathcal{E}_t^{(1)}(z) \\ \mathcal{H}_t^{(1)}(z) \end{array} \right) = T(k_x, 0; z) \left(\begin{array}{c} \mathcal{E}_t^{(1)}(0) \\ \mathcal{H}_t^{(1)}(0) \end{array} \right), \tag{2.11}$$

$$T(k_x, 0; z) = \exp\big(- ik_0 z P(k_x, 0)\big), \quad T = \left(\begin{array}{cc} T_1 & T_2 \\ T_3 & T_4 \end{array} \right), \tag{2.12}$$

where $T_1, \ldots, T_4$ are 2×2 matrices (see (1.6)–(1.7)) and P is defined as $P^{(j)}$ in (1.5) by replacing $A_k^{(j)}$, $B_k^{(j)}$, $C_k^{(j)}$ with A_k, B_k, C_k, respectively.

From (2.7)–(2.12) we get

$$T_2(d)\mathcal{H}_t{}^{(1)}(-d) = E_{0t}\delta(k_x - k_0 \sin\psi) + \mathcal{E}_t^{(r)}(k_x, 0), \tag{2.13}$$

$$T_4(d)\mathcal{H}_t{}^{(1)}(-d) = H_{0t}\delta(k_x - k_0 \sin\psi) + \mathcal{H}_t^{(r)}(k_x, 0). \tag{2.14}$$

Involving 2.3–2.6 we can eliminate E_{0t} and H_{0t} from (2.13)–(2.14) and obtain an equation with respect to the unknown $\mathcal{H}_t{}^{(1)}(-d)$:

$$\Big[\zeta_0 N\, T_4(d) - M\, T_2(d)\Big]\mathcal{H}_t{}^{(1)}(-d) = \Big[N\, V + M\, U\Big] \left(\begin{array}{c} E_{inc}^{TE} \\ E_{inc}^{TM} \end{array} \right) \delta(k_x - k_0 \sin\psi)\,. \tag{2.15}$$

where M, N, U and V are the following dimensionless matrices

$$M = - \left(\begin{array}{cc} k_0 & 0 \\ 0 & i\gamma_0 \end{array} \right), \quad N = \left(\begin{array}{cc} 0 & -i\gamma_0 \\ k_0 & 0 \end{array} \right).$$

$$U = \left(\begin{array}{cc} \cos\psi & 0 \\ 0 & 1 \end{array} \right), \quad V = \left(\begin{array}{cc} 0 & -\cos\psi \\ 1 & 0 \end{array} \right).$$

By solving $\mathcal{H}_t(-d)$ from (2.15) and inserting it back into (2.13), (2.14), we get the equations for the vector $\big(E_0^{TE},\, E_0^{TM}\big)$:

$$\left(\begin{array}{c} E_0^{TE} \\ E_0^{TM} \end{array} \right) = K(d)\mathcal{H}_t(-d) - \left(\begin{array}{c} E_{inc}^{TE} \\ E_{inc}^{TM} \end{array} \right) \delta(k_x - k_0 \sin\psi)\,, \tag{2.16}$$

where

$$K(z) := \left(\begin{array}{cc} \zeta_0 T_{43}(z) & \zeta_0 T_{44}(z) \\ T_{23}(z) & T_{24}(z) \end{array} \right),$$

As a consequence we can detect the reflected wave $\big(\mathcal{E}^{(r)}, \mathcal{H}^{(r)}\big)$ from (2.5), (2.6).

As for the electromagnetic field inside the bianisotropic slab, we can find it from (2.11), (2.12):

$$\left(\begin{array}{c} \mathcal{E}_t(z) \\ \mathcal{H}_t(z) \end{array} \right) = T(k_x, 0; z) \left(\begin{array}{c} T_2(k_x, 0; d) \\ T_4(k_x, 0; d) \end{array} \right) \mathcal{H}_t(-d). \tag{2.17}$$

3 Solution of Equation (2.15)

Under the notation

$$A(k_x) = \zeta_0 N\, T_4(d) - M\, T_2(d),$$
$$B(k_x) = N\, V + M\, U = -(i\gamma_0 + k_0 \cos\psi) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

Equation (2.15) can be rewritten as follows

$$A(k_x)\mathcal{H}_t(-d) = B(k_x) \begin{pmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{pmatrix} \delta(k_x - k_0 \sin\psi). \tag{3.1}$$

Entries of the matrices $T_2(d)$ and $T_4(d)$ are analytic functions with respect to k_x. Elements of the matrices M and N are analytic for $k_x \neq \pm k_0$ and are only continuous at these points. Therefore, the determinant $\det\, A(k_x)$ is analytic also for $k_x \neq \pm k_0$ and has either finite or infinite number of zeroes say, only at $\{k_x = a_j\}_{j\in\sigma}$, where either $\sigma = \{1, 2, \ldots, n\}$ or $\sigma = \{1, 2, \ldots\}$. They are isolated if $a_j \neq \pm k_0$.

Let us prove now, that if some a_j coincides to k_0 or $-k_0$, then it is isolated zero as well.

Consider

$$\Phi(k_x, \gamma_0) = \zeta_0 N(\gamma_0) T_4(k_x, d) - M(\gamma_0) T_2(k_x, d),$$

which coincides with $A(k_x)$, when $\gamma_0 = (k_x^2 - k_0^2)^{1/2}$ (cf. the notation after (2.6)). If $k_x = a_j$ is a zero of $A(k_x)$, then $\gamma_0 = (a_j^2 - k_0^2)^{1/2}$ is a zero of either $\Phi((\gamma_0^2 + k_0^2)^{1/2}, \gamma_0)$, or $\Phi(-(\gamma_0^2 + k_0^2)^{1/2}, \gamma_0)$. Both functions are analytic with respect to the variable γ_0 everywhere except the points $\gamma_0 = \pm ik_0$; therefore, if $k_0 \neq 0$ then $\gamma_0 = 0$ (i.e., $k_x = \pm k_0$) can only be an isolated zero.

If $k_0 = 0$ then a_j for each j is a zero of either $\Phi(k_x, k_x)$, or $\Phi(k_x, -k_x)$. Both functions are analytic and therefore have only isolated roots.

Employing partition of unity we can construct a sequence $\{\eta_j(x)\}$ of functions $\eta_j \in C^\infty(\mathbb{R})$ possessing the following properties:

1) $\eta_j = 1$ in some neighborhood of a_j.

2) $\operatorname{supp} \eta_j \subset (p_j, q_j)$, and $a_k \notin (p_j, q_j)$, if $k \neq j$. Here p_j, q_j are either finite numbers or $\pm\infty$.

3) $$\sum_{j\in\sigma} \eta_j(x) = 1. \tag{3.2}$$

If u is a solution of the system

$$A(k_x)u = 0, \quad k_x \in \mathbb{R}, \tag{3.3}$$

then $u^{(j)} = \eta_j u$ solves the equation

$$A(k_x)u^{(j)}(k_x) = 0, \quad k_x \in (p_j, q_j). \tag{3.4}$$

Inside the interval (p_j, q_j) the matrix A only degenerates at the point a_j. We have to treat the following two cases separately:

1) $a_j \neq \pm k_0$,

2) A degenerates at the point $a_j = +k_0$ or $a_j = -k_0$.

In the first case entries of A are analytic functions in (p_j, q_j) and $\det A = 0$ only at one point of (p_j, q_j), namely at a_j. Then the matrix A can be represented as follows ([4, § 8.1])

$$A(k_x) = A_0^{(j)}(k_x) D^{(j)}(k_x) R^{(j)}(k_x), \quad k_x \in (p_j, q_j), \tag{3.5}$$

where A_0 is invertible with analytic entries in the interval $p_j < k_x < q_j$. The matrix $D^{(j)}(k_x)$ is diagonal

$$D^{(j)}(k_x) = \begin{pmatrix} (k_x - a_j)^{\mu_1^{(j)}} & 0 \\ 0 & (k_x - a_j)^{\mu_2^{(j)}} \end{pmatrix}, \quad \mu_1^{(j)},\, \mu_2^{(j)} \in \mathbb{N}_0, \tag{3.6}$$

where $\mathbb{N}_0$ denotes the set of nonnegative integers and $R^{(j)}(k_x)$ is a polynomial matrix-function with the constant non vanishing determinant. In [4, § 8.1] is described an algorithm of finding $A_0^{(j)}$, $D^{(j)}$ and $R^{(j)}$.

Denoting

$$v^{(j)} = R^{(j)} u^{(j)} \tag{3.7}$$

from (3.4) we get the scalar equations

$$(k_x - a_j)^{\mu_p^{(j)}} v_p^{(j)} = 0, \quad p = 1, 2. \tag{3.8}$$

A solution of (3.8) in the space $D'((p_j, q_j))$ of distributions on (p_j, q_j) has the form

$$v_p^{(j)}(k_x) = \sum_{\ell=0}^{\mu_p^{(j)}-1} C_{p\ell}^{(j)} \delta^{(\ell)}(k_x - a_j), \quad p = 1, 2, \tag{3.9}$$

where $C_k^{(j)}$ are arbitrary constants (see, e.g., [6, Ch.II, § 6.4]). To verify (3.9) it is sufficient to recall that a solution of (3.8) has pointwise support $\operatorname{supp} v_p^{(j)} = \{a_j\}$ and, therefore, represents a finite linear combination of derivatives $\delta^{(\ell)}(k_x - a_j)$ (see [2, Ch.2, § 2.3]). Now it is sufficient to note that $(k_x - a_j)^m \delta^{(\ell)}(k_x - a_j) \neq 0$ iff $\ell \geq m$.

Inserting $v_p^{(j)}(k_x)$ from (3.9) into (3.7) and solving (3.7) we find

$$u_k^{(j)} = \sum_{p=1}^{2} \sum_{\ell=0}^{\mu_p^{(j)}-1} C_{p\ell}^{(j)} L_{kp}^{(j)}(k_x) \delta^{(\ell)}(k_x - a_j), \quad k = 1, 2 \tag{3.10}$$

where

$$L^{(j)} = \begin{pmatrix} L_{11}^{(j)} & L_{12}^{(j)} \\ L_{21}^{(j)} & L_{22}^{(j)} \end{pmatrix} = \big(R^{(j)}\big)^{-1}.$$

Due to the properties of $R^{(j)}$ entries of $L_{ik}^{(j)}(k_x)$ are analytic in the neighborhood of (p_j, q_j).

Let $f \in C^\ell((p, q))$, $a \in (p, q)$, then

$$f(x)\delta^{(\ell)}(x - a) = \sum_{m=0}^{\ell} (-1)^{m+\ell} C_m^\ell f^{(\ell-m)}(a) \delta^{(m)}(x - a), \tag{3.11}$$

where $C_m^\ell = \dfrac{\ell!}{m!(\ell - m)!}$ are the binomial coefficients.

Using (3.11) we can rewrite (3.10) as

$$u_k^{(j)}(k_x) = \sum_{p=1}^{2} \sum_{\ell=0}^{\mu_p^{(j)}-1} \sum_{m=0}^{\ell} (-1)^{m+\ell} C_{p\ell}^{(j)} C_m^\ell \frac{d^{\ell-m}}{dk_x^{\ell-m}} L_{kp}^{(j)}(a_j) \delta^{(m)}(k_x - a_j) \tag{3.12}$$

and obtain a solution of (3.4) when $a_j \neq \pm k_0$.

Representation (3.5) also holds when a_j coincides with one of the points $\pm k_0$, say, with k_0. If that's the case, the matrix A_0 in the representation (3.5) has continuous entries in the interval $p_j < k_x < q_j$ and is invertible. The entries of $R^{(j)}(k_x)$, participating in the expression (3.6), are polynomials of $(k_x - a_j)^{1/2}$ and det $R^{(j)}(k_x) = \text{const} \neq 0$. And, finally, $2\mu_1^{(j)}$, $2\mu_2^{(j)} \in \mathbb{N}_0$, where $\mu_1^{(j)}, \mu_2^{(j)}$ are the numbers participating in the expression $D^{(j)}(k_x)$ (see (3.6)).

Inserting the factorization (3.5) into Equation (3.4) we obtain the following replacement to the system (3.8):

$$(k_x - a_j)^{\mu_p^{(j)}} v_p^{(j)} = 0, \quad 2\mu \in \mathbb{N}_0, \quad x \in (p_j, q_j), \quad p = 1, 2. \tag{3.13}$$

Solution to (3.13) in the space $D'((p_j, q_j))$ has the form

$$v_p^{(j)} = \sum_{\ell=0}^{n_p^{(j)}} C_{p\ell}^{(j)} \delta^{(\ell)}(k_x - a_j)$$

with arbitrary constants $C_{p\ell}^{(j)}$ and with

$$n_p^{(j)} := \begin{cases} [\mu_p^{(j)}], & \mu_p^{(j)} \notin \mathbb{N}_0, \\ \mu_p^{(j)} - 1, & \mu_p^{(j)} \in \mathbb{N}_0, \end{cases} \tag{3.14}$$

where $[a]$ denotes the integer part of a. Note, that in this case solution $v_p^{(j)}$ in fact is in the space $D'_{[\mu]}((p_j, q_j))$ of distributions of order $[\mu]$, i.e. in the space of linear continuous functionals over the space $C_0^{[\mu]}((p_j, q_j))$ of functions with $[\mu]$-continuous derivatives and compact supports in (p_j, q_j). In this case the product $(x - a)^\mu v_p^{(j)}$ is defined correctly (see [2, Ch.2]). Now, if we repeat the foregoing reasoning, we obtain instead of (3.12):

$$u_k^{(j)}(k_x) = \sum_{p=1}^{2} \sum_{\ell=0}^{n_p^{(j)}} \sum_{m=0}^{\ell} (-1)^{m+\ell} C_{p\ell}^{(j)} C_m^{(\ell)} \frac{d^{\ell-m}}{dk_x^{\ell-m}} L_{kp}^{(j)}(a_j) \delta^{(m)}(k_x - a_j).$$

Due to (3.2) the sum

$$u_k(k_x) = \sum_{j\in\sigma} u_k^{(j)}(k_x) = \sum_{j\in\sigma} \sum_{p=1}^{2} \sum_{\ell=0}^{n_p^{(j)}} \sum_{m=0}^{\ell} (-1)^{m+\ell} C_{p\ell}^{(j)} C_m^{(\ell)} \frac{d^{\ell-m}}{dk_x^{\ell-m}} L_{kp}^{(j)}(a_j) \delta^{(m)}(k_x - a_j)\,. \tag{3.15}$$

represents a solution to (3.3).

To solve (3.1) we should find prior a solution of inhomogeneous equation

$$A(k_x)u = \delta(k_x - b)F(k_x). \tag{3.16}$$

Consider the following cases:

1) The matrix A is invertible at the point $k_x = b$, i.e., $b \neq a_j$, $j \in \sigma$, then

$$u = A^{-1}(b)F(b)\delta(k_x - b) \tag{3.17}$$

obviously solves (3.16).

2) For some $j \in \sigma$, $b = a_j$ and $a_j \neq \pm k_0$. It can easily be observed that in this case

$$u_k^{(j)} = \sum_{p=1}^{2} \sum_{m=0}^{\mu_p^{(j)}} \frac{(-1)^{\mu_p^{(j)}+m}}{\mu_p^{(j)}!} \Big[(A_0^{(j)}(a_j))^{-1} F(a_j)\Big]_p C_m^{\mu_p^{(j)}} \frac{d^{\mu_p^{(j)}-m}}{dk_x^{\mu_p^{(j)}-m}} L_{kp}^{(j)}(a_j) \delta^{(m)}(k_x - a_j) \tag{3.18}$$

solves Equation (3.16).

3) If $b = a_j$ and a_j equals to one of the points $\pm k_0$, then from (3.5), (3.7), (3.16) follows that:

$$(k_x - a_j)^{\mu_p^{(j)}} v_p^{(j)} = \Big[(A_0^{(j)}(a_j))^{-1} F(a_j)\Big]_p \delta(k_x - a_j). \tag{3.19}$$

If $\Big[(A_0^{(j)}(a_j))^{-1} F(a_j)\Big]_p \neq 0$ and $\mu_p^{(j)}$ is not an integer for some j, then (3.19) has no solution; otherwise the case 3) converts into the case 2).

Thus we have obtained a general solution of Equation (3.16) in the following form:

$$u_k(x) = \sum_{j\in\sigma}\sum_{p=1}^{\ell}\sum_{\ell=0}^{n_p^{(j)}}\sum_{m=0}^{\ell}(-1)^{m+\ell}C_{p\ell}^{(j)}C_m^\ell \frac{d^{\ell-m}}{dk_x^{\ell-m}}L_{kp}^{(j)}(a_j)\delta^{(m)}(k_x - a_j) + u_k^{(1)}(k_x), \tag{3.20}$$

where $n_p^{(j)}$ is defined from (3.14) and $u_k^{(1)}(x)$ is a solution of (3.16).

If we apply obtained results to Equation (3.1) we get the following:

If $\psi \neq \pm\dfrac{\pi}{2}$, then Equation (3.1) has the solution

$$\mathcal{H}_t(-d) = \sum_{j\in\sigma}\sum_{p=1}^{2}\sum_{\ell=0}^{n_p^{(j)}}\sum_{m=0}^{\ell}(-1)^{m+\ell}C_{p\ell}^{(j)}C_m^\ell \frac{d^{\ell-m}}{dk_x^{\ell-m}} L_p^{(j)}(a_j)\delta^{(m)}(k_x - a_j) + \mathcal{H}^{(1)}(k_x), \tag{3.21}$$

where $L_p^{(j)} = \begin{pmatrix} L_{1p}^{(j)} \\ L_{2p}^{(j)} \end{pmatrix}$, $p = 1, 2$, $j \in \sigma$, and

$$\mathcal{H}^{(1)}(k_x) = -2k_0\cos\psi A^{-1}(k_0\sin\psi)\begin{pmatrix} E_{inc}^{TE} \\ -E_{inc}^{TM}\end{pmatrix}\delta(k_x - k_0\sin\psi) \quad \text{for} \quad \det A(k_0\sin\psi) \neq 0\,, \tag{3.22}$$

$$\begin{aligned}\mathcal{H}^{(1)}(k_x) = &\sum_{p=1}^{2}\sum_{m=0}^{\mu_p^{(j)}}\frac{(-1)^{\mu_p^{(j)}+m+1}}{\mu_p^{(j)}!}\,2k_0\cos\psi\left[\left(A_0^{(j)}(a_j)\right)^{-1}\begin{pmatrix} E_{inc}^{TE} \\ -E_{inc}^{TM}\end{pmatrix}\right]_p \\ &\times C_m^{\mu_p^{(j)}}\frac{d^{\mu_p^{(j)}-m}}{dk_x^{\mu_p^{(j)}-m}}\,\overline{L}_p^{(j)}(a_j)\delta^{(m)}(k_x - a_j) \qquad \text{for} \quad \det A(k_0\sin\psi) = 0\,,\end{aligned} \tag{3.23}$$

where $k_0\sin\psi = a_j$. For the limiting cases $\psi = \pm\dfrac{\pi}{2}$

$$F(k_x)\delta(k_x - k_0\sin\psi) = B(k_0\sin\psi)\delta(k_x - k_0\sin\psi) = B(k_0)\delta(k_x - k_0) = 0$$

and (3.1) becomes homogeneous. The solutions is then given by (3.21) with $\mathcal{H}^{(1)} = 0$.

The functions E_0^{TE} and E_0^{TM} can be found from (2.16) by inserting there the known solution $\mathcal{H}_t(-d)$ and performing the multiplication according to (3.11):

$$\begin{aligned}\begin{pmatrix} E_0^{TE} \\ E_0^{TM}\end{pmatrix} = &\sum_{j\in\sigma}\sum_{p=1}^{2}\sum_{\ell=0}^{n_p^{(j)}}\sum_{m=0}^{\ell}\sum_{n=0}^{m}(-1)^{\ell+n}C_{p\ell}^{(j)}C_m^\ell C_n^m\frac{d^{\ell-m}}{dk_x^{\ell-m}}\,L_p^{(j)}(a_j) \\ &\times\frac{d^{m-n}}{dk_x^{m-n}}\,K(a_j, 0, d)\delta^{(n)}(k_x - a_j) + S(k_x) - \begin{pmatrix} E_{inc}^{TE} \\ E_{inc}^{TM}\end{pmatrix}\delta(k_x - k_0\sin\psi),\end{aligned} \tag{3.24}$$

where

$$S(k_x) = -2k_0\cos\psi S(k_0\sin\psi, 0, d)A^{-1}(k_0\sin\psi)\begin{pmatrix} E_{inc}^{TE} \\ -E_{inc}^{TM}\end{pmatrix}\delta(k_x - k_0\sin\psi) \tag{3.25}$$

for $\det A(k_0 \sin\psi) \neq 0$ and

$$S(k_x) = \sum_{p=1}^{2} \sum_{m=0}^{\mu_p^{(j)}} \sum_{n=0}^{m} \frac{(-1)^{\mu_p^{(j)}+n+1}}{\mu_p^{(j)}!} 2k_0 \cos\psi \left[\left(A_0^{(j)}(k_0 \sin\psi) \right)^{-1} \begin{pmatrix} E_{inc}^{TE} \\ -E_{inc}^{TM} \end{pmatrix} \right]_p$$
$$\times C_m^{\mu_p^{(j)}} C_n^m \frac{d^{m-n}}{dk_x^{m-n}} K(k_0 \sin\psi, 0, d) \frac{d^{\mu_p^{(j)}-m}}{dk_x^{\mu_p^{(j)}-m}} L_p(k_0 \sin\psi) \delta^{(n)}(k_x - a_j) \tag{3.26}$$

for $\det A(k_0 \sin\psi) = 0$, with $k_0 \sin\psi = a_j$. The vectors $\mathcal{E}_t(z)$ and $\mathcal{H}_t(z)$ can be found from (2.17) similarly. In conclusion we perform inverse Fourier transform in obtained expressions simply replacing
$\delta^{(n)}(k_x - a)$ by $\dfrac{(ix)^n}{2\pi} \exp(-iax)$.

Summarizing the above considerations we can formulate the following theorem:

Theorem 3.1 *If the matrix* $A(k_x) = \zeta_0 N(k_x) T_4(k_x, 0; d) - M(k_x) T_2(k_x, 0; d)$ *degenerates at some point* a_j*, then the homogeneous equation* $A(k_x)\mathcal{H}_t(-d) = 0$ *has then nontrivial solutions given by (3.15).*

A general solution of inhomogeneous Equation (3.1) is given by formulae (2.5),(2.6),(2.17), (3.24)–(3.26) and depends on arbitrary constants $C_{p\ell}^{(j)}$*. Functions* E_0^{TE}*,* E_0^{TM}*,* $\mathcal{E}_t(z)$*,* $\mathcal{H}_t(z)$*, depend on these constants as well.*

Finally note, that expressions obtained in the present section for bianisotropic slab consisting of a single layer also remain valid for multilayered slab if in our considerations transition matrix $T(d)$ is replaced by a product of transition matrices of component layers (see (1.6), (1.7)):

$$T(d) = \begin{pmatrix} T_1(d) & T_2(d) \\ T_3(d) & T_4(d) \end{pmatrix} = T^{(1)}(d_1)\, T^{(2)}(d_2) \cdots T^{(n)}(d_n). \tag{3.27}$$

4 Reflection and transmission through a general bianisotropic slab

Consider reflection and transmission of a plane wave through a general bianisotropic slab between two half spaces Ω' and Ω'' filled by isotropic material (cf. Fig. 1). In this case in addition to fields into the domains Ω' and $\Omega_j,\ j = 1, \ldots, n$ we have an electromagnetic field $(\mathcal{E}^{(t)}, \mathcal{H}^{(t)})$ transmitted into the domain Ω'' ([5]):

$$\mathcal{E}^{(t)}(k_x; z) = \Big[-E_2^{TM} \overline{y} - E_2^{TE} \big(-i\gamma_2 \overline{x} + k_x \overline{z} \big) / k_2 \Big] e^{\gamma_2 (z+d)}, \tag{4.1}$$

$$\mathcal{H}^{(t)}(k_x; z) = \frac{1}{\zeta_2} \Big[-E_2^{TM} \big(-i\gamma_2 \overline{x} + k_x \overline{z} \big) / k_2 + E_2^{TE} \overline{y} \Big] e^{\gamma_2 (z+d)} \tag{4.2}$$

From continuity of an electromagnetic field on the hyperplane $z = 0$

$$\left\{ \begin{array}{c} \mathcal{E}_t{}'(0) \\ \mathcal{H}_t{}'(0) \end{array} \right\} = \left\{ \begin{array}{c} \mathcal{E}_t{}^{(1)}(0) \\ \mathcal{H}_t{}^{(1)}(0) \end{array} \right\}; \tag{4.3}$$

(see (2.7)) and on the hyperplane $z = -d$

$$\left\{ \begin{array}{c} \mathcal{E}_t{}^{(1)}(0) \\ \mathcal{H}_t{}^{(1)}(0) \end{array} \right\} = \left\{ \begin{array}{c} \mathcal{E}_t{}^{(t)}(0) \\ \mathcal{H}_t{}^{(t)}(0) \end{array} \right\}; \tag{4.4}$$

similarly as in section 2 we derive the following equation

$$\begin{pmatrix} W & T_1(d)\Theta - T_2(d)\Lambda \\ Q & T_3(d)\Theta - T_4(d)\Lambda \end{pmatrix} \begin{bmatrix} E_0^{TE} \\ E_0^{TM} \\ E_2^{TE} \\ E_2^{TM} \end{bmatrix} = \begin{pmatrix} U \\ -V \end{pmatrix} \begin{pmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{pmatrix} \delta(k_x - k_0 \sin\psi) \tag{4.5}$$

for an unknown vector field $(E_0^{TE},\, E_0^{TM},\, E_2^{TE},\, E_2^{TM})$. Here

$$\begin{aligned} W &= -\begin{pmatrix} \dfrac{i\gamma_0}{k_0} & 0 \\ 0 & 1 \end{pmatrix}, \qquad Q = \frac{1}{\zeta_0}\begin{pmatrix} 0 & 1 \\ \dfrac{-i\gamma_0}{k_0} & 0 \end{pmatrix}, \\ \Theta &= \begin{pmatrix} \dfrac{-i\gamma_2}{k_2} & 0 \\ 0 & 1 \end{pmatrix}, \qquad \Lambda = \frac{1}{\zeta_2}\begin{pmatrix} 0 & 1 \\ \dfrac{i\gamma_2}{k_2} & 0 \end{pmatrix} \end{aligned} \tag{4.6}$$

and $\gamma_p = (k_x^2 - k_p^2)^{\frac{1}{2}}$, $-\dfrac{\pi}{2} < \arg \gamma_p \le \dfrac{\pi}{2}$ $(p = 0, 2)$.

Applying the method of solution developed in the previous section we get

$$\begin{pmatrix} E_0^{TE} \\ E_0^{TM} \\ E_2^{TE} \\ E_2^{TM} \end{pmatrix} = \sum_{j\in\sigma}\sum_{p=1}^{4}\sum_{\ell=0}^{n_p^{(j)}}\sum_{m=0}^{\ell} (-1)^{m+\ell} C_{p\ell}^{(j)} C_m^{\ell} \frac{d^{\ell-m}}{dk_x^{\ell-m}} L_{kp}^{(j)}(a_j)\delta^{(m)}(k_x - a_j) + U^{(1)}(k_x), \tag{4.7}$$

where $\{a_j\}$ are all zeros of $\det A(k_x)$ and

$$A = \begin{pmatrix} W & T_1(d)\Theta - T_2(d)\Lambda \\ Q & T_3(d)\Theta - T_4(d)\Lambda \end{pmatrix}. \tag{4.8}$$

Note, that the matrix $A(k_x)$ is analytic in the complex place $\mathbb{C}$ except the following four points $k_x = \pm k_p, p = 0, 2$ and the set σ contains these points; $L^{(j)} = (L_{kp}^{(j)})_{4\times 4} = {R^{(j)}}^{-1}$ and the matrix $R^{(j)}$ participates in the decomposition of A at the point k_x (see(3.5)); C_m^{ℓ} are binomial coefficients and $C_{p\ell}^{(j)}$ are arbitrary constants; $n_p^{(j)}$ is defined in (3.14); the vector $U^{(1)}$ is a solution of inhomogeneous Equation (4.5):

$$U^{(1)}(k_x) = A^{-1}(k_0\sin\psi)F(k_0\sin\psi)\delta(k_x - k_0\sin\psi) \qquad \text{if} \quad \det A(k_0\sin\psi) \neq 0\,, \tag{4.9}$$

$$\begin{aligned} U_k^{(1)}(k_x) = \sum_{p=1}^{4}\sum_{m=0}^{\mu_p} \frac{(-1)^{\mu_p+m}}{\mu_p!} \Big[(A_0(k_0\sin\psi))^{-1}\,\overline{F}(k_0\sin\psi)\Big]_p \\ + C_m^{\mu_p} \frac{d^{\mu_p-m}}{dk_x^{\mu_p-m}} L_{kp}^{(0)}(k_0\sin\psi)\delta^{(m)}(k_x - k_0\sin\psi), \qquad \text{if} \quad \det A(k_0\sin\psi) = 0\,. \end{aligned} \tag{4.10}$$

In (4.9), (4.10) $F = \begin{pmatrix} U \\ -V \end{pmatrix}$, $L^0 = (R^0)^{-1}$; A_0, R^0 are the matrices participating in decomposition of A at the point $a_0 = k_0\sin\psi$, and μ_p are the indices of the decomposition.

Theorem 4.1 *A solution to the problem of reflection and transmission of plane waves through a general bianisotropic slab is given by formulae (4.7)–(4.10).*

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