

Professor Gigla Djanashia
Letter of Recommendation

I have known professor Gigla Djanashia for many years and have been learning his excellent scientific works, particularly the ones related to factorization of matrix functions, because of its importance for theory of stationary stochastic processes. I know "Djanashia Reduction Algorithm" for factorization of positive defined matrix functions and can compare it with other known algorithms. I consider Djanashia's Algorithm as the best among known algorithms.

The problem of factorization of matrix functions was formulated by N.Wiener and A. Kolmogorov in 1940-1941. The main theoretical results were obtained by N.Wiener in 1955. He proposed the first algorithm for calculation. This work was extended by professor Masani and his pupils.

Algorithms elaborated by Masani and his students were based on functional analysis.

Professor Djanashia proposed Algorithm based on the complex analysis. His algorithm can be applied to all matrix functions with no restrictions. It is possible to estimate the size of calculation for any given accuracy. To solve the problem for matrices of any order we have to consider only matrices of second order.

I consider "Djanashia Reduction Algorithm" as the highest achievement in this field at the present time.

Below is a short description of the algorithm.

Djanashia Reduction Algorithm

Let C be the unit circle in the complex plane. We consider the complex matrix-valued functions $x(t)$ that are defined on C . The problem is to find the representation: $x(t) = y(t) y'(t)$ where the matrix $y(t)$ is analytic in the circle C and y' is the conjugate matrix for y . N. Wiener proved that such a representation can be constructed iff the functions $x(t)$ and $\log \det x(t)$ are integrable on C .

Masani constructed an algorithm for the calculation of matrix $y(t)$ if $x(t)$ has the integrable inverse matrix and satisfies some additional conditions which were improved by other authors.

Djanasha proposed a new algorithm that needs no restrictions on $x(t)$ but those that were considered by Wiener. The algorithm consists of several steps.

Step 1. The matrix $x(t)$ is represented as a product $A(t) A'(t)$ where $A(t)$ is a triangle matrix. Using the well known algorithm for the factorization of complex-valued functions one can construct a new representation: $x(t) = a(t) a'(t)$, where $a(t)$ is a triangle matrix function diagonal elements of which are analytic in the circle C , so $\det a(t)$ is the same function.

Step 2. Matrices of order 2 are considered. For these matrices an algorithm for $y(t)$ is proposed, it is based on the solution of a system of boundary problems for some auxiliary analytical function $g(t)$.

Step 3. For matrices of order 3 the function $y(t)$ can be constructed in the following way. Using steps 1 and 2 we can obtain the representation: $x(t) = b(t) b'(t)$ where the matrix $b(t)$ has analytic in the circle C elements that are on the crossing of two first rows and two first columns. After this the construction of $y(t)$ can be accomplished using the algorithm of step 2. In the same way the case of the matrices of the order 4 can be reduced to the case of the matrices of order 3 and so on.

Step 4. Since the solution of the problem is a polynomial function that approximates $y(t)$ we have to calculate the accuracy of the approximation. It directly depends on the accuracy of the approximation of the function $g(t)$ which is considered in step 2 in the square mean metric. For the matrices of order more than 2 we have a several such functions. We can calculate the size of the calculation which is needed for given accuracy.



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