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# THE EFFECT OF STIGMATIZATION OF HUMANS WITH ERECTILE DYSFUNCTION DUE TO UNHEALTHY LIFESTYLE USING A MATHEMATICAL MODEL

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**Abstract.** In this study, we present a novel model for understanding the dynamics of Erectile Dysfunction (ED) in various male populations, using ordinary differential equations with stochastic perturbations. Our model takes into account the impact of both unhealthy lifestyle choices and genetic factors, as well as the reluctance of some individuals to disclose their ED status due to fear of stigmatization. We establish the existence, uniqueness, and global positive solution of the stochastic model, demonstrating its well-posed nature. By examining the drift and diffusion components and utilizing the Eulers–Maruyama (E-M) numerical method, we derive closed-form solutions and compare the stochastic model's behavior with deterministic model solutions. Through simulations, we reveal that the number of individuals unwilling to disclose their ED status increases over time without interventions to reduce ED prevalence.

## 1. Introduction

Erectile Dysfunction (ED), or male impotence, refers to a man's inability to maintain an erection during sexual intercourse. ED complications include unsatisfactory sex life, stress, anxiety, embarrassment, self-doubt, marital problems, and infertility. Globally, the ED prevalence ranges from 2% to 86% in men under and over 40 years of age, respectively, with age-adjusted incidence rates in Nigeria, Egypt, and Pakistan at 57%, 63.6%, and 80% [21]. Recent findings suggest genetic factors also contribute to the development of ED [14, 24]. Treatment can be achieved by abstaining from unhealthy lifestyles such as smoking, excessive intake of drugs, alcohol and junk foods, getting rid of emotional problems of trauma, depression, and addressing emotional issues [25].

Mathematical modeling has been widely employed in physical, biological, and social research. Several models have been developed to describe ED, including linear and nonlinear models accounting for age and depression, penile and physiological problems, penile vascular dysfunction, and leak area in impotent men. Both deterministic and stochastic models have been formulated to describe biological and epidemiological processes, with stochastic models proving particularly relevant due to their incorporation of random perturbations found in many biological scenarios. Wald et al. [23], studied the linear and nonlinear models of moderate ED with age and depression factor, which gives room for future investigation in age-related ED. Ng, Ng and Chia [16] studied penile and physiological problems using mathematical modeling. Their results showed that a difference of 0.1–0.3$^oC$ can be observed, and hence used to develop database of smart diagnosis for ED using the machine artificial intelligence. Mulhall and Damaser [15], studied the area of leak using ED data from healthy men and men with impotence, and found that leakage area in impotent men is higher than that in healthy men.

Barnea, Hayun and Gillon [6], derived a mathematical model to describe penile vascular dysfunction. They showed through simulations that leakage should be avoided by applying control measures. Several deterministic and stochastic models have been formulated to describe biological and epidemiological process [1–5]. It is therefore pertinent to consider the stochastic model formulation, since random perturbations are involved in the evolution of many biological scenarios [17, 22]. The dynamic system can practically undergo random disturbances, otherwise known as Brownian motion or white noise. A deterministic model with a random term is a stochastic model [7, 9, 11]. Several iterative schemes have been used by the authors to get the closed form solutions of the stochastic model, but

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the numerical method of interest in this work is the Euler-Meyer method. This method is an extended version of the Euler method for ordinary equations to stochastic differential equations with strong convergence [10, 18, 19].

In this study, we propose a population-based ED model using linear ordinary differential equations with stochastic perturbations. The model accounts for individuals at risk of ED due to unhealthy lifestyle and genetic factors, considering treatment availability and the fear of stigmatization for those who disclose or conceal their ED status. To our knowledge, this research problem has not been previously explored.

The paper is organized as follows. Section 2 presents the mathematical formulation of the method with stochastic perturbations. Section 3 deals with the qualitative analysis of the stochastic model using the Lipschitz continuity theorem and an analysis of global positive solutions. Section 4 employs the Euler–Maruyama (E-M) numerical scheme to obtain approximate stochastic model solutions via numerical simulations. Lastly, Section 5 concludes the work.

## 2. The Mathematical Model Formulation

In order to formulate the model, we made some assumptions as follows:

- Erectile dysfunction is a non-communicable disease and humans with ED who engage in unhealthy practices like eating sugary substance, intake of alcohol, smoking, are much more that humans with ED due to genetic predisposition [14, 24].
- Humans with ED may or may not disclose their ED status due to the fear of stigmatization, but humans with ED who disclose their status are considered for treatment [20].

In the model, $H_e(t)$ denotes the number of humans who are at the risk of having ED, $E_l(t)$ denotes the number of humans with ED due to unhealthy lifestyle, $E_g(t)$ denotes the number of humans with ED due to genetic factors, $E_d(t)$ denotes the number of humans who disclosed their ED status without the fear of stigmatization, $E_n(t)$ denotes the number of humans who did not disclose their ED status due to the fear of stigmatization and $T_d(t)$ denotes the number of humans treated for ED, these variables collectively give the deterministic model given by

$$\left.\begin{aligned}
\dot{H}_e &= \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e,\\
\dot{E}_l &= k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l,\\
\dot{E}_g &= \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g,\\
\dot{E}_d &= k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d,\\
\dot{E}_n &= \beta_2 E_l + \beta_4 E_g - \mu_p E_n,\\
\dot{T}_d &= \sigma E_d - \mu_p T_d,
\end{aligned}\right\} \tag{2.1}$$

under the initial conditions $H_e \geq 0, E_l \geq 0, E_g \geq 0, E_d \geq 0, E_n \geq 0$ and $T_d \geq 0$. In (2.1), the recruitment rate of humans who are at the risk of ED is denoted by $\psi$ and the incidence rates of the ED in humans with unhealthy practices and genetic factors are denoted by $\delta_1$ and $\delta_2$, while $k_o$ represents the modification factor that accounts for the increased infection in humans who engage in unhealthy practices relative to humans with ED due to genetic factors. Moreover, $\beta_i (i = 1-4)$ denotes the progression rates of humans who engage in unhealthy lifestyle and genetically pre-disposed to ED, while $k_1$ and $k_2$ represent the modification factors that describe the increasing rate of disclosure of ED status relative to humans who did not disclose their ED status. Finally, $\sigma$ represents the recovery rate of humans treated from ED due to disclosure of their ED status.

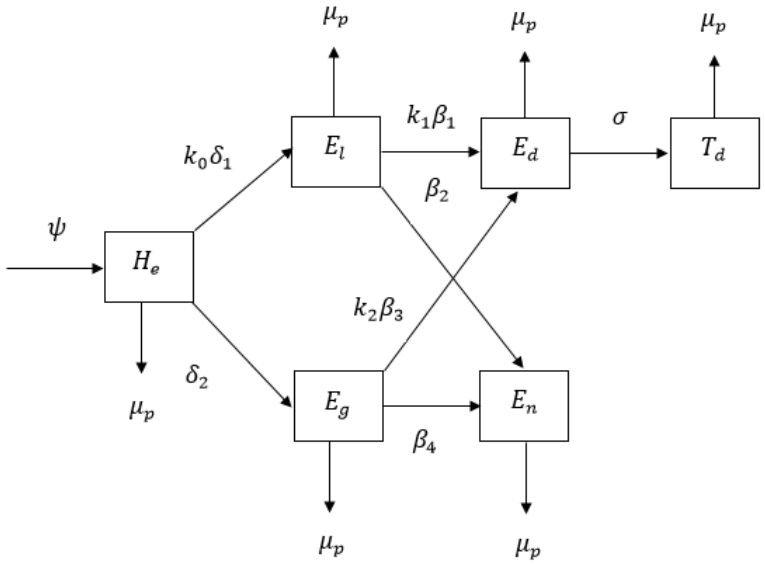


FIGURE 1. Diagrammatic description of the ED incidence progression in different categories of humans

Taking the environmental flunctuations into consideration, we set $\chi_i(t)(i = 1{-}6)$ with $\chi(0) = 0$, $\forall\chi_i$ as the standard Brownian motion, and $\rho_i(i = 1-6)$ denotes the white noise, so that (2.1) becomes a model based on the stochastic differential equation given by

$$\left.\begin{aligned}
\dot{H_e} &= \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e + \rho_1 H_e(t)d\chi_1(t),\\
\dot{E_l} &= k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l + \rho_2 E_l(t)d\chi_2(t),\\
\dot{E_g} &= \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g + \rho_3 E_g(t)d\chi_3(t),\\
\dot{E_d} &= k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d + \rho_4 E_d(t)d\chi_4(t),\\
\dot{E_n} &= \beta_2 E_l + \beta_4 E_g - \mu_p E_n + \rho_5 E_n(t)d\chi_5(t),\\
\dot{T_d} &= \sigma E_d - \mu_p T_d + \rho_6 T_d(t)d\chi_6(t).
\end{aligned}\right\} \tag{2.2}$$

**Theorem 2.1.** *The stochastic ED model* (2.2) *is presented in the compact form given by*

$$\frac{dX(t)}{dt} = f(t, X(t)) + Q(t, X(t))\frac{d\chi}{dt}, \tag{2.3}$$

*where* $\chi = (\chi_j)^T_{j=1,\ldots,6}$ *represents a Brownian motion, and* $f(t, X(t))$ *is a variation speed vector given by*

$$f(t, X(t)) = \begin{pmatrix}
\psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e,\\
k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l,\\
\delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g,\\
k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d,\\
\beta_2 E_l + \beta_4 E_g - \mu_p E_n,\\
\sigma E_d - \mu_p T_d.
\end{pmatrix},$$

*while the noise matrix* $G = G(t, X(t))$ *is given by*

$$G = \begin{pmatrix} G_1 & -G_2 & -G_3 & -G_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & -G_5 & -G_6 & -G_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4 & 0 & 0 & 0 & -G_8 & -G_9 & -G_{10} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 & 0 & 0 & G_9 & 0 & -G_{11} & -G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_7 & 0 & 0 & G_{10} & 0 & 0 & -G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_{12} & 0 & -G_{14} \end{pmatrix},$$

*where the following representations are defined;* $G_1 = \sqrt{\psi}$, $G_2 = \sqrt{\mu_p H_e}$, $G_3 = \sqrt{k_0\delta_1 H_e}$, $G_4 = \sqrt{\delta_2 H_e}$, $G_5 = \sqrt{\mu_p E_l}$, $G_6 = \sqrt{k_1\beta_1 E_l}$, $G_7 = \sqrt{\beta_2 E_l}$, $G_8 = \sqrt{\mu_p E_g}$, $G_9 = \sqrt{k_2\beta_3 E_g}$, $G_{10} = \sqrt{\beta_4 E_g}$, $G_{11} = \sqrt{\mu_p E_d}$, $G_{12} = \sqrt{\sigma E_d}$, $G_{13} = \sqrt{\mu_p E_n}$, $G_{14} = \sqrt{\mu_p T_d}$. *The matrix* $G$ *possesses the following properties such that the singular value decomposition of* $G$ *is* $G = PDQ$. *Furthermore,* $P$ *and* $Q$ *are the orthogonal matrices of dimensions* $(6 \times 6)$ *and* $(14 \times 14)$, *respectively, and* $D$ *denotes the* $(6 \times 14)$ *dimensional matrix with* $r(r \leq 6)$ *positive diagonal entries. It follows that* $V = GG^T = PDQ(PDQ)^T = PD(QQ^T)D^T P^T = P(DD^T)P^T$, *where*

$$\sqrt{V} = P(DD^T)^{\frac{1}{2}} P^T.$$

*Proof.* Let the random variables be denoted by $\Delta X = (\Delta H_e, \Delta E_l, \Delta E_g, \Delta E_d, \Delta E_n, \Delta T_d)$ over the variation time $\Delta t$, so that $P_j$ denotes the probability of state changes and suppose that there exist $m$ changes of possible states, $j = 1, 2, \ldots, m$, we need to consider two states $X^j$ and $X^{j+1}$ which represent both the initial and intermediate states at time $t$ and $t + \Delta t$, respectively. Therefore, taking $\Delta X^j = X^{j+1} - X^j$, we get $P_j = Prob(\Delta X^j) = Prob\left\{X^{j+1}/X^j\right\}$. To this effect, there exist 15 possible state changes for model (2.1), which are summarized below in Table 1.

TABLE 1. Possible transitions in the stochastic model process

| Possibility Change of state $(\Delta X^j)$ | Probability $P_j = Prob(\Delta X^j)$ |
|---|---|
| $\Delta X^1 = [1, 0, 0, 0, 0, 0]^T$ | $P_1 = \Psi \Delta t$ |
| $\Delta X^2 = [-1, 0, 0, 0, 0, 0]^T$ | $P_2 = \mu_p H_e \Delta t$ |
| $\Delta X^3 = [-1, 1, 0, 0, 0, 0]^T$ | $P_3 = k_0\delta_1 H_e \Delta t$ |
| $\Delta X^4 = [-1, 0, 1, 0, 0, 0]^T$ | $P_4 = \delta_2 H_e \Delta t$ |
| $\Delta X^5 = [0, -1, 0, 0, 0, 0]^T$ | $P_5 = \mu_p E_l \Delta t$ |
| $\Delta X^6 = [0, -1, 0, 1, 0, 0]^T$ | $P_6 = k_1\beta_1 E_l \Delta t$ |
| $\Delta X^7 = [0, -1, 0, 0, 1, 0]^T$ | $P_7 = \beta_2 E_l \Delta t$ |
| $\Delta X^8 = [0, 0, -1, 0, 0, 0]^T$ | $P_8 = \mu_p E_g \Delta t$ |
| $\Delta X^9 = [0, 0, -1, 1, 0, 0]^T$ | $P_9 = k_2\beta_3 E_g \Delta t$ |
| $\Delta X^{10} = [0, 0, -1, 0, 1, 0]^T$ | $P_{10} = \beta_4 E_g \Delta t$ |
| $\Delta X^{11} = [0, 0, 0, -1, 0, 0]^T$ | $P_{11} = \mu_p E_d \Delta t$ |
| $\Delta X^{12} = [0, 0, 0, -1, 0, 1]^T$ | $P_{12} = \sigma E_d \Delta t$ |
| $\Delta X^{13} = [0, 0, 0, 0, -1, 0]^T$ | $P_{13} = \mu_p E_n \Delta t$ |
| $\Delta X^{14} = [0, 0, 0, 0, 0, -1]^T$ | $P_{14} = \mu_p T_d \Delta t$ |
| $\Delta X^{15} = [0, 0, 0, 0, 0, 0]^T$ | $P_{15} = 1 - \sum_{j=1}^{14} P_j$ |
| Otherwise: $\Delta X^i, i \neq 1, 2, \ldots, 15$ | $P_i = 0$ |

Therefore the mean and variance transitions of $\Delta X$ are then given by

$$\mathbf{E}(\Delta X) = \sum_{j=1}^{m} P_j \Delta X^j = f(t, X)\Delta t$$

and

$$V(\Delta X) = \mathbf{E}((\Delta X)(\Delta X)^T - \mathbf{E}(\Delta X) \cdot \mathbf{E}(\Delta X)^T = \sum_{j=1}^{m} P_j (\Delta X^j)(\Delta X^j)^T = V(t, X)\Delta t,$$

where

$$\mathbf{E}(\Delta X) = \begin{pmatrix} \psi - (k_0\delta_1 + \delta_2 + \mu_p)H_e \\ k_0\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l \\ \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g \\ k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d \\ \beta_2 E_l + \beta_4 E_g - \mu_p E_n \\ \sigma E_d - \mu_p T_d \end{pmatrix} \Delta t$$

and

$$\mathbf{E}((\Delta X)(\Delta X)^T) = \begin{pmatrix} c_1 & -P_3 - P_4 & 0 & 0 & 0 & 0 \\ -P_3 & c_2 & -P_6 - P_7 & 0 & 0 & 0 \\ -P_4 & 0 & c_3 & -P_9 - P_{10} & 0 & 0 \\ 0 & -P_6 & -P_9 & c_4 & -P_{12} & 0 \\ 0 & -P_7 & -P_{10} & 0 & c_5 & 0 \\ 0 & 0 & 0 & -P_{12} & 0 & c_6 \end{pmatrix} \Delta t$$

where

$$\left.\begin{aligned} &P_j = \frac{P_6}{\Delta t}, j = 1, 2, \ldots, 14. \\ &c_1 = P_1 + P_2 + P_3 + P_4 = \psi + (k_0\delta_1 + \delta_2 + \mu_p)H_e, \\ &c_2 = P_3 + P_5 + P_6 + P_7 = k_0\delta_1 H_e + (k_1\beta_1 + \beta_2 + \mu_p)E_l, \\ &c_3 = P_4 + P_8 + P_9 + P_{10} = \delta_2 H_e + (k_2\beta_3 + \beta_4 + \mu_p)E_g, \\ &c_4 = P_6 + P_9 + P_{11} + P_{12} = k_1\beta_1 E_l + k_2\beta_3 E_g + (\sigma + \mu_p)E_d, \\ &c_5 = P_7 + P_{10} + P_{13} = \beta_2 E_l + \beta_4 E_g + \mu_p E_n, \\ &c_6 = P_{12} + P_{14} = \sigma E_d + \mu_p T_d, \end{aligned}\right\} \tag{2.4}$$

which imply that

$$V(t, X) = \begin{pmatrix} c_1 & -(k_0\delta_1 + \delta_2)H_e & 0 & 0 & 0 & 0 \\ -k_0\delta_1 H_e & c_2 & -(k_1\beta_1 + \beta_2)E_l & 0 & 0 & 0 \\ -\delta_2 H_e & 0 & c_3 & -(k_1\beta_1 + \beta_2)E_l & 0 & 0 \\ 0 & -k_1\beta_1 E_l & -k_2\beta_3 E_g & c_4 & -\sigma E_d & 0 \\ 0 & -\beta_2 E_l & -\beta_4 E_g & 0 & c_5 & 0 \\ 0 & 0 & 0 & -\sigma E_d & 0 & c_6 \end{pmatrix}.$$

It is clear that $X(t + \Delta t) = X(t) + \Delta X$ is represented in a discrete form given by:

$$X^{j+1} = X^j + \Delta X^j,\ j = 1, \ldots, n.$$

Note that for $j = 1, \ldots, n$, a normal distribution with the mean $E(\Delta X^j)$ and variance $V(\Delta X^j)$ exist. Using the Gaussian approximation, we obtain

$$\Delta X^j = \mathbf{E}(\Delta X^j) + \sqrt{V(\Delta X^j)\Delta \bar{W}^j},$$

where $\Delta \bar{W}^j \sim N(0, 1)$ and, using the central limit theorem, we get

$$\Delta X = f(t, X(t))\Delta t + \sqrt{(V(t, X(t)))}\Delta t\bar{\omega},$$

where the random noise $\bar{\omega} = (\bar{\omega_1}, \bar{\omega_2}, \bar{\omega_3}, \bar{\omega_4}, \bar{\omega_5}, \bar{\omega_6})$ is such that $\bar{\omega_1} \sim N(0, 1)$ $(i = 1, 2, 3, 4, 5, 6)$, and it is observed that as $\Delta t \to 0$, $X(t)$ converges strongly towards model (2.2). In order to reduce model (2), we introduce the following lemma (see [2]).

**Lemma 2.1.** *The Itô stochastic differential equations $(EDS_*)$ and $(EDS_{**})$ below are shown to be equivalent,*

$$\left.\begin{aligned} &\frac{dX}{dt} = f(t, X) + \sqrt{(V(t, X))}\frac{dW}{dt}, \quad (EDS_*), \\ &\frac{dX^*}{dt} = f(t, X^*) + G(t, X^*)\frac{dW^*}{dt}, (EDS_{**}), \end{aligned}\right\} \tag{2.5}$$

*where $f : \mathbf{T} \times \mathbf{R}^d \to \mathbf{R}^d$, $G : \mathbf{T} \times \mathbf{R}^d \to R^{d\times m}$ and $V : \mathbf{T} \times \mathbf{R}^d \to R^{d\times d}$ are such that $V = GG^T$. Also, the sample solution paths of $(EDS_*)$ and $(EDS_{**})$ are the same, with the probability axioms*

*satisfying the Lipschitzian condition of* $(EDS_*)$ *and* $(EDS_{**})$*. Moreover, the solutions* $X$ *and* $X^*$ *of* $(EDS_*)$ *and* $(EDS_{**})$ *have the same random distribution.*

By Lemma 2.1 we can easily find that model (2.5) is equivalent to (2.3). Given

$$\frac{dX^*}{dt} = f(t, X^*) + G(t, X^*)\frac{dW^*}{dt},$$

we can observe different events of state changes of every component of the vector $\Delta X(\Delta X = \Delta H_e, \Delta E_l, \Delta E_g, \Delta E_d, \Delta E_n, \Delta T_d)$ to determine the matrix $G$ as $GG^T = V$.

Let the Poisson probability be denoted by $\mathbb{P}^*$, so that

$$\left.\begin{aligned} \Delta H_e &= k_1 - k_2 - k_3 - k_4, \\ \Delta E_l &= k_3 - k_5 - k_6 - k_7, \\ \Delta E_g &= k_4 - k_8 - k_9 + k_{10}, \\ \Delta E_d &= k_6 + k_9 - k_{11} - k_{12}, \\ \Delta E_n &= k_7 + k_{10} - k_{13}, \\ \Delta T_d &= k_{12} - k_{14}, \end{aligned}\right\} \tag{2.6}$$

with $k_1 \sim \mathbb{P}(\psi\Delta t)$, $k_2 \sim \mathbb{P}(\mu_p H_e \Delta t)$, $k_3 \sim \mathbb{P}(k_0\delta_1 H_e\Delta t)$, $k_4 \sim \mathbb{P}(\delta_2 H_e\Delta t)$, $k_5 \sim \mathbb{P}(\mu_p E_l\Delta t)$, $k_6 \sim \mathbb{P}(k_1\beta_1 E_l\Delta t)$, $k_7 \sim \mathbb{P}(\beta_2 E_l\Delta t)$, $k_8 \sim \mathbb{P}(\mu_p E_g\Delta t)$, $k_9 \sim \mathbb{P}(k_2\beta_3 E_g\Delta t)$, $k_{10} \sim \mathbb{P}(\beta_4 E_g\Delta t)$, $k_{11} \sim \mathbb{P}(\mu_p E_d\Delta t)$, $k_{12} \sim \mathbb{P}(\sigma E_d\Delta t)$, $k_{13} \sim \mathbb{P}(\mu_p E_n\Delta t)$, $k_{12} \sim \mathbb{P}(\mu_p T_d\Delta t)$.

Then system (2.6) becomes

$$\left.\begin{aligned} \Delta H_e &= \psi\Delta t + \sqrt{\psi\Delta t\omega_1} - \mu_p H_e\Delta t - \sqrt{(\mu_p H_e\Delta t)}\omega_2 - k_0\delta_1 H_e\Delta t \\ &\quad - \sqrt{(k_0\delta_1 H_e\Delta t)}\omega_2 - \delta_2 H_e\Delta t - \sqrt{(\delta_2 H_e\Delta t)\omega_4}, \\ \Delta E_l &= k_0\delta_1 H_e\Delta t + \sqrt{(k_0\delta_1 H_e\Delta t)}\omega_2 - \mu_p E_l\Delta t - \sqrt{(\mu_p E_l\Delta t)\omega_5} - k_1\beta_1 E_l\Delta t \\ &\quad - \sqrt{(k_1\beta_1 E_l\Delta t)\omega_6} - \beta_2 E_l\Delta t - \sqrt{(\beta_2 E_l\Delta t)\omega_7}, \\ \Delta E_g &= \delta_2 H_e\Delta t + \sqrt{(\delta_2 H_e\Delta t)}\omega_4 - \mu_p E_g\Delta t - \sqrt{(\mu_p E_g\Delta t)\omega_8} - k_2\beta_3 E_g\Delta t \\ &\quad - \sqrt{(k_2\beta_3 E_g\Delta t)\omega_9} - \beta_4 E_g\Delta t - \sqrt{(\beta_4 E_g\Delta t)\omega_{10}}, \\ \Delta E_d &= k_1\beta_1 E_l\Delta t + \sqrt{(k_1\beta_1 E_l\Delta t)}\omega_6 + k_2\beta_3 E_g\Delta t + \sqrt{(k_2\beta_3 E_g\Delta t)\omega_9} - \mu_p E_d\Delta t \\ &\quad - \sqrt{(\mu_p E_d\Delta t)\omega_{11}} - \sigma E_d\Delta t - \sqrt{(\sigma E_d\Delta t)\omega_{12}}, \\ \Delta E_n &= \beta_2 E_l\Delta t + \sqrt{(\beta_2 E_l\Delta t)\omega_7} + \beta_4 E_g\Delta t + \sqrt{(\beta_4 E_g\Delta t)\omega_{10}} - \mu_p E_n\Delta t \\ &\quad - \sqrt{(\mu_p E_n\Delta t)\omega_{13}}, \\ \Delta T_d &= \sigma E_d\Delta t + \sqrt{(\sigma E_d\Delta t)\omega_{12}} - \mu_p T_d\Delta t - \sqrt{(\mu_p T_d\Delta t)\omega_{14}}, \end{aligned}\right\} \tag{2.7}$$

that is, $u_j \sim N(0,1)$ for $j = 1, 2, \ldots, 14$. As $\Delta t \to 0$, (2.7) converges to the stochastic differential equation (2.3), and we have $\frac{dX(t)}{dt} = f(t, X(t)) + G(t, X(t))\frac{d\chi(t)}{dt}$, which ends the proof. □

## 3. Qualitative Analysis of the Stochastic Model

### 3.1. Existence and uniqueness results of the stochastic model.

**Theorem 3.1.** *For* $\frac{dX(t)}{dt} = f(t, X(t)) + G(t, X(t))\frac{d\chi(t)}{dt}$*, we have the following properties:*

- *Both* $f(X(t))$ *and* $G(X(t))$ *are continuous on* $t \in [t_o T]$*,* $T > 0$*.*
- *The coefficient functions* $f$ *and* $G$ *satisfy the Lipschitz condition* $|f(x,t) - f(y,t)| + |G(x,t) - G(x,t)| \leq K|x - y|$ *for some constant* $K$ *and all* $t \in [t_o, T]$*,* $T > 0$*.*

- *The coefficient functions $f$ and $G$ satisfying the growth condition $|f(y,t)|^2 + |G(x,t)|^2 \leq K^2(1+|x|^2)$ for some constant $K^2$ and for all $t \in [t_o, T]$, $T > 0$, are continuous with probability* 1.

*Assume the coefficients in the system satisfy the following conditions:*

$$dX_t^i = a_i(t, X_t)dt + \sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij}(t, X_t)dW_t^j,$$

*where*

$$\left.\begin{aligned} X_t &= (X_t^1, X_t^2, \ldots, X_t^n)^T, \\ W_t &= (W_t^1, W_t^2, \ldots, W_t^m)^T, \end{aligned}\right\}$$

*where $a_i(t, X_t)$ denotes the drift coefficient vector and $b_{ij}(t, X_t)$ are entries of an $n \times m$ dimensional diffusion matrix. The coefficients $a_i(t, X_t)$ and $b_{ij}(t, x)$ satisfy the Lipschitz and growth conditions for some constant $k < \infty$ and for all $t \in \Re$ and $x$, $x, y \in \Re^n$, such that*

$$\left.\begin{aligned} &||a_i(t,x) - a_i(t,y) \leq k||x-y||, \\ &||b_{ij}(t,x) - b_{ij}(t,y)|| \leq k||x-y||, \\ &||a_i(t,x)|| \leq k||x||, \\ &||b_{ij}(t,x)|| \leq k||x||, \\ &||b|| = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij}(x)^2}, \\ &||a|| = \sqrt{\sum_{i=1}^{n} a_i(x)^2}, \end{aligned}\right\}$$

*Then for each $x_o \in \Re^n$, there exists a unique solution to system* (2.3) *with initial condition $X_o = x_o$.*

*Proof.* Let

$$\left.\begin{aligned} f_1 &= \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e, \\ f_2 &= k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l, \\ f_3 &= \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g, \\ f_4 &= k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d, \\ f_5 &= \beta_2 E_l + \beta_4 E_g - \mu_p E_n, \\ f_6 &= \sigma E_d - \mu_p T_d, \end{aligned}\right\}$$

$$\left.\begin{array}{l}
\left|\frac{\partial f_1}{\partial H_e}\right| = |k_o\delta_1 + \delta_2 + \mu_p| \leq M, \ \left|\frac{\partial f_1}{\partial E_l}\right| = \left|\frac{\partial f_1}{\partial E_g}\right| = \left|\frac{\partial f_1}{\partial E_d}\right| = \left|\frac{\partial f_1}{\partial E_n}\right| = \left|\frac{\partial f_1}{\partial T_d}\right| = 0,\\
\left|\frac{\partial f_2}{\partial H_e}\right| = |k_o\delta_1| \leq M, \ \left|\frac{\partial f_2}{\partial E_l}\right| = |k_1\beta_1 + \beta_2 + \mu_p| \leq M,\\
\left|\frac{\partial f_2}{\partial E_g}\right| = \left|\frac{\partial f_2}{\partial E_d}\right| = \left|\frac{\partial f_2}{\partial E_n}\right| = \left|\frac{\partial f_2}{\partial T_d}\right| = 0,\\
\left|\frac{\partial f_3}{\partial H_e}\right| = |\delta_2| \leq M, \ \left|\frac{\partial f_3}{\partial E_l}\right| = |k_2\beta_3 + \beta_4 + \mu_p| \leq M,\\
\left|\frac{\partial f_3}{\partial E_g}\right| = \left|\frac{\partial f_3}{\partial E_d}\right| = \left|\frac{\partial f_3}{\partial E_n}\right| = \left|\frac{\partial f_3}{\partial T_d}\right| = 0,\\
\left|\frac{\partial f_4}{\partial H_e}\right| = \left|\frac{\partial f_4}{\partial T_d}\right| = \left|\frac{\partial f_4}{\partial E_n}\right| = 0, \ \left|\frac{\partial f_4}{\partial E_l}\right| = |k_1\beta_1| \leq M, \ \left|\frac{\partial f_4}{\partial E_g}\right| = |k_2\beta_3| \leq M,\\
\left|\frac{\partial f_4}{\partial E_d}\right| = |\sigma + \mu_p| \leq M,\\
\left|\frac{\partial f_5}{\partial H_e}\right| = \left|\frac{\partial f_5}{\partial E_d}\right| = \left|\frac{\partial f_1}{\partial T_d}\right| = 0, \ \left|\frac{\partial f_5}{\partial E_l}\right| = |\beta_2| \leq M, \ \left|\frac{\partial f_5}{\partial E_g}\right| = |\beta_4| \leq M,\\
\left|\frac{\partial f_5}{\partial E_n}\right| = |\mu_p| \leq M,\\
\left|\frac{\partial f_6}{\partial H_e}\right| = \left|\frac{\partial f_6}{\partial E_l}\right| = \left|\frac{\partial f_6}{\partial E_g}\right| = \left|\frac{\partial f_6}{\partial E_n}\right| = 0, \ \left|\frac{\partial f_6}{\partial E_d}\right| = |\sigma| \leq M,\\
\left|\frac{\partial f_6}{\partial T_d}\right| = |\mu_p| \leq M.
\end{array}\right\}$$

It is observed in (3.4) that the elements of the diffusion/or drift part of the model is continuously differentiable. Also, for the stochastic/diffusion part of (2.3), we obtain $||f|| = \sqrt{\sum_{i=1}^{6} f_i(x)^2}$ and $||G|| = \sqrt{\sum_{i=1}^{6}\sum_{j=1}^{14} b_{ij}(x)^2}$, which imply that both $f_i$ and $g_{ij}$ are continuously differentiable, bounded, and satisfy the Lipschitz condition. Hence, this implies that the model exists and is unique. □

3.2. **Positive global bounded solutions of the stochastic model.** We define the following set:

$$\Theta = \left\{(H_e, E_l, E_g, E_d, E_n, T_d) \in \Re^{+6} : N_e = H_e + E_l + E_g + E_d + E_n + T_d \leq \frac{\psi}{\mu_p}\right\},$$

then we obtain the following analytical result.

**Theorem 3.2.** *Let* $X_o = (H_e(0), E_l(0), E_g(0), E_g(0), E_d(0), E_n(0), T_d(0))$ *and*

$$X(t) = (H_e(t), E_l(t), E_g(t), E_d(t), E_n(0), T_d(0))$$

*for* $t \geq 0$*, where* $X(t)$ *is the solution curve of model* (2.3) *passing through* $X_o$*, then* $X(t)$ *remains in* $\Theta$ *for all* $t \geq 0$*, with probability* 1*.*

*Proof.* Let the random variable $N_d(t) = H_e(t) + E_l(t) + E_g(t) + E_d(t) + E_n(t) + T_d(t)$ be the total number of human host population, so that

$$\begin{aligned}\frac{dN_d(t)}{dt} = \psi - \mu_p N_d(t) - \Big(&D_1\frac{d\chi_1(t)}{dt} + D_2\frac{d\chi_2(t)}{dt} + D_5\frac{d\chi_5(t)}{dt}\\ &+D_8\frac{d\chi_8(t)}{dt} + D_{11}\frac{d\chi_{11}(t)}{dt} + D_{13}\frac{d\chi_{13}(t)}{dt} + D_{14}\frac{d\chi_{14}(t)}{dt}\Big),\end{aligned}$$

since

$$\xi\Big(X(t),\frac{d\chi(t)}{dt}\Big)=\Big(D_1\frac{d\chi_1(t)}{dt}+D_2\frac{d\chi_2(t)}{dt}+D_5\frac{d\chi_5(t)}{dt}$$
$$+D_8\frac{d\chi_8(t)}{dt}+D_{11}\frac{d\chi_{11}(t)}{dt}+D_{13}\frac{d\chi_{13}(t)}{dt}+D_{14}\frac{d\chi_{14}(t)}{dt}\Big).$$

If $X(c)\in R_+^6$ for all $0\le c\le t$ almost surely, we obtain the following inequality:

$$\frac{dN_d(t)}{dt}\le\psi-\mu_pN_d(c),$$

almost surely. Therefore,

$$N_d(c)\le\frac{\psi}{\mu_p},$$

so that $H_e(c)$, $E_l(c)$, $E_g(c)$, $E_d(c)$, $E_n(c)$, $T_d(c)\in\left[0,\frac{\psi}{\mu_p}\right]$ for all $c\in[0,t]$.

Clearly, for $f$ and $G$ that are locally Lipschitz continuous, there exists a unique locally Lipschitz solution $X(c)$ on $[0,t]$ for any given initial value $X_0$, to show that the solution $X(t)$ is global for $t\in[0,x_e]$, where $x_e$ denotes the explosion time.

Let $\gamma_o>0$ such that $(H_e(0),E_l(0),E_g(0),E_d(0),E_n(0),T_d(0))>\gamma_o$ and for $\gamma\le 0$, the stopping time is defined by

$$x_\gamma=\inf\{t\in[0,x_e],H_e(t)<\gamma,E_l(t)<\gamma,E_g(t)<\gamma,E_d(t)<\gamma,E_n(t)<\gamma,T_d(t)<\gamma,\}$$

and we have $\lim_{\gamma\to 0}x_\gamma=x_o$ with

$$x_o=\{t\in[0,x_e],H_e(t)<\gamma,E_l(t)<\gamma,E_g(t)<\gamma,E_d(t)<\gamma,E_n(t)<\gamma,T_d(t)<0.\}$$

Consider the function $C$ defined for $X=(H_e,E_l,E_g,E_d,E_n,T_d)$ by

$$C=-\ln\Big(\frac{\psi}{\mu_p}H_e\Big)-\ln\Big(\frac{\psi}{\mu_p}E_l\Big)-\ln\Big(\frac{\psi}{\mu_p}E_g\Big)-\ln\Big(\frac{\psi}{\mu_p}E_d\Big)-\ln\Big(\frac{\psi}{\mu_p}E_n\Big)-\ln\Big(\frac{\psi}{\mu_p}T_d\Big).$$

Using the Itô formula for multidimensional problems on the interval $[0,\min(t,x_\gamma)]$, for all z, we have

$$dC(X(z))=\Big(\frac{\partial C(X(z))}{\partial z}+\sum_{i=1}^{6}f_i(z,X(z))\frac{\partial C(X(z))}{\partial X_i}$$
$$+\frac{1}{2}\sum_{i,j=1}^{6}(GG^T)_{ij}\frac{\partial^2C(X(z))}{\partial X_i\partial X_j}\Big)dz+\sum_{i=1}^{6}\sum_{j=1}^{14}G_{ij}dW_j(z)\frac{\partial C(X(z))}{\partial X_i}$$

with

$$G=(G_{ij})_{i=1,\dots,6,\ j=1,2,\dots,14}$$

and

$$(GG^T)_{ij}=\sum_{k=1}^{14}G_{ik}G_{kj},$$

that is,

$$\left.\begin{aligned} dC(X) &= [6\mu_p + k_o\delta_1 + \delta_2 + k_1\beta_1 + \beta_2 + k_2\beta_3 + \beta_4 + \sigma]dz \\ &- \Big[\psi\frac{(2H_e-1)}{2H_e^2} + (k_o\delta_1 H_e)\frac{(2E_l-1)}{2E_l^2} + (\delta_2 H_e)\frac{(2E_g-1)}{2E_g^2} \\ &+ (k_1\beta_1 E_l + k_2\beta_3 E_g)\frac{(2E_d-1)}{2E_d^2} + (\beta_2 E_l + \beta_4 E_g)\frac{(2E_n-1)}{2E_n^2} \\ &+ (\sigma E_d)\frac{(2T_d-1)}{2T_d^2}\Big]dz + \Big[(k_o\delta_1 + \delta_2 + \mu_p)\frac{1}{2H_e} \\ &+ (k_1\beta_1 + \beta_2 + \mu_p)\frac{1}{2E_l} + (k_2\beta_3 + \beta_4 + \mu_p)\frac{1}{2E_g} \\ &+ (\sigma + \mu_p)\frac{1}{2E_d} + \mu_p\frac{1}{2E_n} + \mu_p\frac{1}{2T_d}\Big]dz \\ &- \frac{1}{H_e}(G_1dW_1 + G_2dW_2 + G_3dW_3 + G_4dW_4) \\ &- \frac{1}{E_l}(G_3dW_3 - G_5dW_5 - G_6dW_6 - G_7dW_7) \\ &- \frac{1}{E_g}(G_4dW_4 - G_8dW_8 - G_9dW_9 - G_{10}dW_{10}) \\ &- \frac{1}{E_d}(G_7dW_7 + G_9dW_9 - G_{11}dW_{11} - G_{12}dW_{12}) \\ &- \frac{1}{E_n}(G_7dW_7 + G_{10}dW_{10} - G_{13}dW_{13}) \\ &+ \frac{1}{T_d}(G_{12}dW_{12} - G_{14}dW_{14}). \end{aligned}\right\}$$

Also, for almost surely $z \in [0,t]$,

$$\left.\begin{aligned} dC(X(z))t &\leq [6\mu_p + 2k_o\delta_1 + 2delta_2 + 2k_1\beta_1 + 2\beta_2 + 2k_2\beta_3 + 2\beta_4 + 2\sigma]dz \\ &+ \frac{1}{H_e}(G_1dW_1) + \frac{1}{E_l}(G_3dW_3) \\ &+ \frac{1}{E_g}(G_4dW_4) + \frac{1}{E_d}(G_7dW_7 + G_9dW_9) \\ &+ \frac{1}{E_n}(G_7dW_7 + G_{10}dW_{10}) + \frac{1}{T_d}(G_{12}dW_{12}). \end{aligned}\right\} \tag{3.1}$$

Integrating the inequality in (3.1), we obtain

$$\left.\begin{aligned} dC(X(z))t &\leq Qt + \int_0^t \frac{1}{H_e}(G_1dW_1(z)) + \int_0^t \frac{1}{E_l}(G_3dW_3(z)) \\ &+ \int_0^t \frac{1}{E_g}(G_4dW_4(z)) + \int_0^t \frac{1}{E_d}(G_7dW_7(z) + G_9dW_9(z)) \\ &+ \int_0^t \frac{1}{E_n}(G_7dW_7(z) + G_{10}dW_{10}(z)) + \int_0^t \frac{1}{T_d}(G_{12}dW_{12}(z)), \end{aligned}\right\}$$

where $Q = 6\mu_p + 2k_o\delta_1 + 2delta_2 + 2k_1\beta_1 + 2\beta_2 + 2k_2\beta_3 + 2\beta_4 + 2\sigma$. Taking the mathematical expectation $\mathbf{E}(.)$ in (3.1), we get

$$E[C(Q(t))] \leq Qt + \Big(E\Big[\int_0^t \frac{1}{H_e}(G_1dW_1(z))\Big] + E\Big[\int_0^t \frac{1}{E_l}(G_3dW_3(z))\Big]$$

$$+E\Big[\int_0^t \frac{1}{E_g}(G_4dW_4(z))\Big]+E\Big[\int_0^t \frac{1}{E_d}(G_7dW_7(z)+G_9dW_9(z))\Big]$$
$$+E\Big[\int_0^t \frac{1}{E_n}(G_7dW_7(z)+G_{10}dW_{10}(z))\Big]+E\Big[\int_0^t \frac{1}{T_d}(G_{12}dW_{12}(z))\Big]. \tag{3.2}$$

Therefore (3.2) leads to the following Lemma below (see [13]).

**Lemma 3.1.** *The stochastic integral $\int_0^t B(z)dW(z)$ yields its quadratic variation given by $\int_0^t B^2(z)dz \leq Qt$, while the local martingales for the strong law imply that*

$$\lim_{t\to\infty}\int_0^t B(z)dW(z)=0,$$

*almost surely. Thus, we have*

$$\left.\begin{aligned}
&\mathbf{E}\Big[\int_0^t \frac{1}{H_e}(G_1dW_1(z))\Big]=0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_l}(G_3dW_3(z))\Big]=0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_g}(G_4dW_4(z))\Big]=0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_d}(G_7dW_7(z)+G_9dW_9(z))\Big]=0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_n}(G_7dW_7(z)+G_{10}dW_{10}(z))\Big]=0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{T_d}(G_{12}dW_{12}(z))\Big]=0.
\end{aligned}\right\}$$

Then for all $t\geq 0$, we have

$$\mathbf{E}[C(X(\min(t,x_\gamma)))]\leq Q\min(t,x_\gamma)\leq Qt.$$

Since $V(Q(\min(t,x_\gamma)))>0$, we have

$$\mathbf{E}[V(Q(\min(t,x_\gamma)))]=\mathbf{E}\Big[V(Q(\min(t,x_\gamma)))\mathbf{1}_{(x_\gamma\leq t)}\Big]+\mathbf{E}\Big[V(Q(\min(t,x_\gamma)))\mathbf{1}_{(x_\gamma\leq t)}\Big]$$
$$\geq \mathbf{E}\Big[V(Q(\min(t,x_\gamma)))\mathbf{1}_{(x_\gamma\leq t)}\Big],$$

where $\mathbf{1_A}$ is the characteristic function of A, and there exists some component of $Q(x_\gamma)$ equal to $\gamma$, then $C(Q(x_\gamma))\geq -\ln\left(\frac{\mu_p\gamma}{\psi}\right)$. Hence

$$\mathbf{E}[C(Q(\min(t,x_\gamma)))]\geq -\ln\left(\frac{\mu_p\gamma}{\psi}\right)\mathbf{P}^*(x_\gamma\leq t). \tag{3.3}$$

Taking $\gamma\to 0$ in (3.3), for all $t\geq 0$, we obtain

$$\mathbf{P}^*(x_\gamma\leq t)=0. \tag{3.4}$$

Hence $\mathbf{P}^*(x=\infty)=1$. As $x_\gamma\geq x$, then $x_\gamma=x=\infty$ almost surely. This ends the proof to the theorem. □

## 4. Numerical Method of Solution and Simulations

4.1. **Numerical method of solution.** Here, we employ the Euler–Maruyama numerical scheme [10] given by $X(t_{n+1}) = X(t_n) + fX(t_n)\Delta t + GX(t_n)\Delta\frac{d\chi}{dt}$, where $n = i = 0, 1, 2, \ldots, N$–1, with $X(0) = X_o$ as the initial condition, and $0 < t < T$, where $\chi(t)$ is the Brownian motion. We apply the numerical scheme to model (2.3) to obtain

$$\begin{pmatrix} H_{e_{n+1}} \\ E_{l_{n+1}} \\ E_{g_{n+1}} \\ E_{d_{n+1}} \\ E_{n_{n+1}} \\ T_{d_{n+1}} \end{pmatrix} = \begin{pmatrix} H_{e_n} \\ E_{l_n} \\ E_{g_n} \\ E_{d_n} \\ E_{n_n} \\ T_{d_n} \end{pmatrix} + \begin{bmatrix} \psi - (k_0\delta_1 + \delta_2 + \mu_p)H_e \\ k_0\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l \\ \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g \\ k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d \\ \beta_2 E_l + \beta_4 E_g - \mu_p E_n \\ \sigma E_d - \mu_p T_d \end{bmatrix} \Delta t$$

$$+ \sqrt{\left[\begin{bmatrix} c_1 & -P_3 - P_4 & 0 & 0 & 0 & 0 \\ -P_3 & c_2 & -P_6 - P_7 & 0 & 0 & 0 \\ -P_4 & 0 & c_3 & -P_9 - P_{10} & 0 & 0 \\ 0 & -P_6 & -P_9 & c_4 & -P_{12} & 0 \\ 0 & -P_7 & -P_{10} & 0 & c_5 & 0 \\ 0 & 0 & 0 & -P_1 2 & 0 & c_6 \end{bmatrix} \Delta t\right] \Delta\chi_n}, \tag{4.1}$$

where $\Delta t$ is a discretized parameter and $c_i's$ are defined in (2.4). We implement (4.1), using computational software MATLAB to obtain the numerical simulations below.

4.2. **Numerical simulations.** Here, we estimate parameters describing the ED incidence in Nigeria [20, 21]. The recruitment rate $\psi$ which determines the population size since the asymptotic carrying capacity of the population is $\frac{\psi}{\mu} \approx 10^6$, but for the simulation purposes, we put $\psi$ to be 500 per week. We also assume $\mu$ to be inversely proportional to the life expectancy at birth which in Nigeria is approximately 45 years, that is, $\frac{1}{\mu} = \frac{1}{45} = 0.222$ per year, while we take the recovery rate $\sigma$ to be the inverse of the time between ED activation and recovery by treatment. Furthermore, we consider the treatment period as 2–10 weeks, and the recovery period as 10 weeks, that is, $\frac{1}{10} = 0.1$ per week. The prevalence of ED in Nigeria is 60.5% per week [20], while we assume that the prevalence of ED is due to unhealthy lifestyle and genetic factors, and the prevalence modification coefficients of the disease are in the range between 0 and 1. Figures 2–5 present the stochastic and deterministic behavior of the model solutions (2.2). We observe that the wavy curves (stochastic solutions) evolve slower than the smooth curves (deterministic solutions), since the latter account for randomness or environmental fluctuations, which implies that the wavy curves are closer to the real solution of the ED model than the smooth curves.

Figure 2 shows that individuals at risk for developing Erectile Dysfunction (ED), represented as $H_e$, exhibit a declining trend over time due to the lack of proactive health measures. Meanwhile, those with ED resulting from an unhealthy lifestyle, denoted as $E_l$, decrease over time as individuals disclose their ED status $E_d$ and receive appropriate treatments. Cases of ED attributed to genetic factors $E_g$ experience a slight increase and peak within the first two weeks, but gradually decline with proper consultation and treatment. Conversely, the population of individuals not disclosing their ED status $E_n$ increases over time, reflecting a lack of healthy interventions due to the fear of stigmatization. Those who do disclose their status and receive treatment $T_d$ gradually increase over a 10-week period.

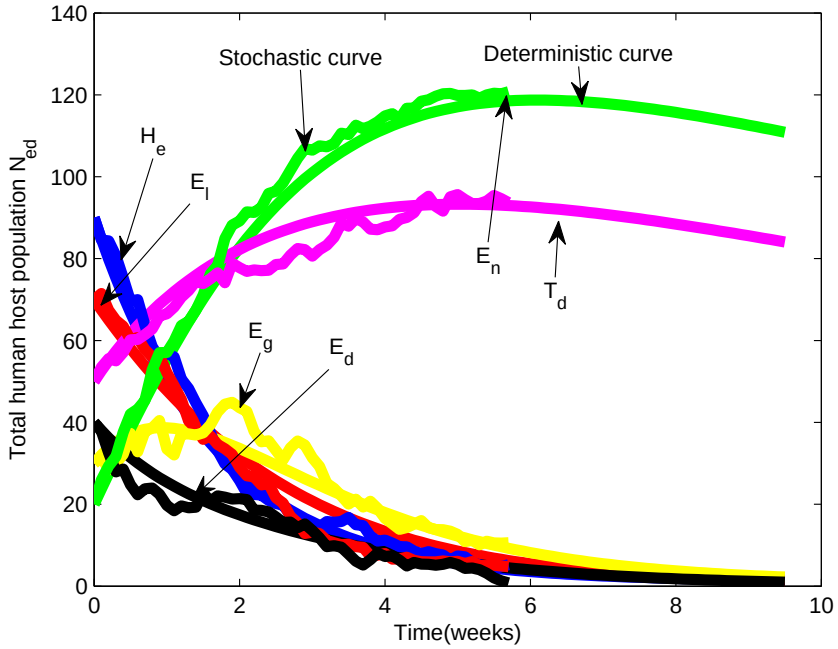


FIGURE 2. Behavior of the stochastic and deterministic model solutions with the the fixed parameters from Table 1 together with the initial values $H_e = 90$, $E_l = 70$, $E_g = 35$, $E_d = 40$, $E_n = 20$ and $T_d = 55$ for ten weeks

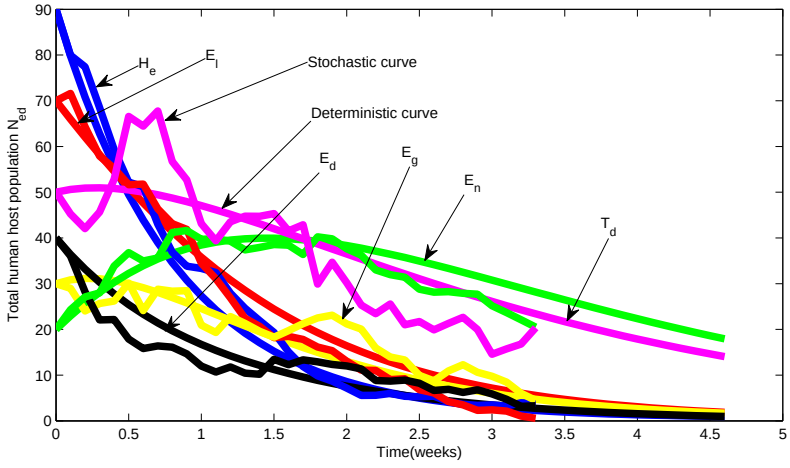


FIGURE 3. Behavior of the stochastic and deterministic model solutions with the varying parameters from Table 1 together with the initial values $H_e = 90$, $E_l = 70$, $E_g = 35$, $E_d = 40$, $E_n = 20$ and $T_d = 55$ for five weeks

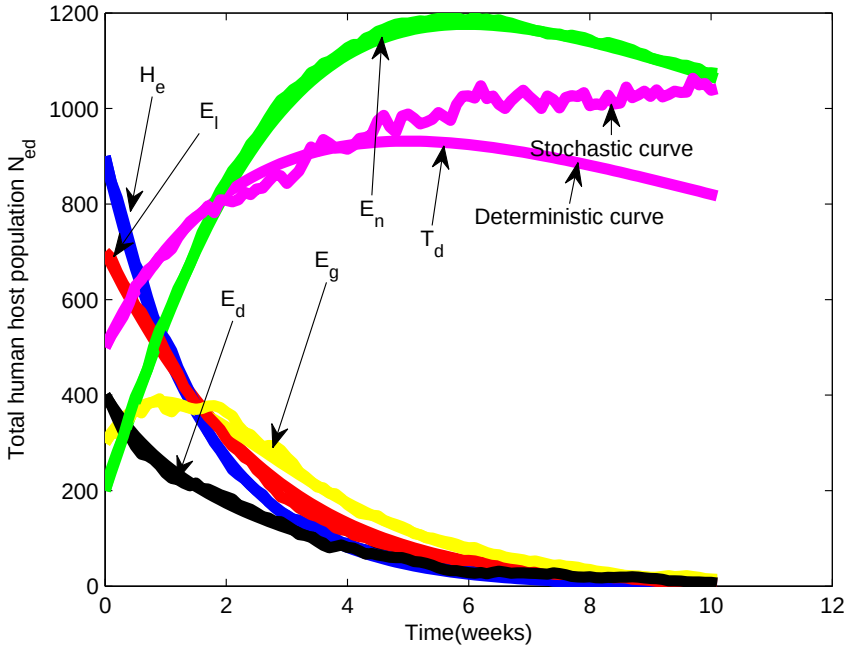


FIGURE 4. Behavior of the stochastic and deterministic model solutions with the fixed parameters from Table 1 together with the initial values $H_e = 900$, $E_l = 700$, $E_g = 350$, $E_d = 400$, $E_n = 200$ and $T_d = 550$

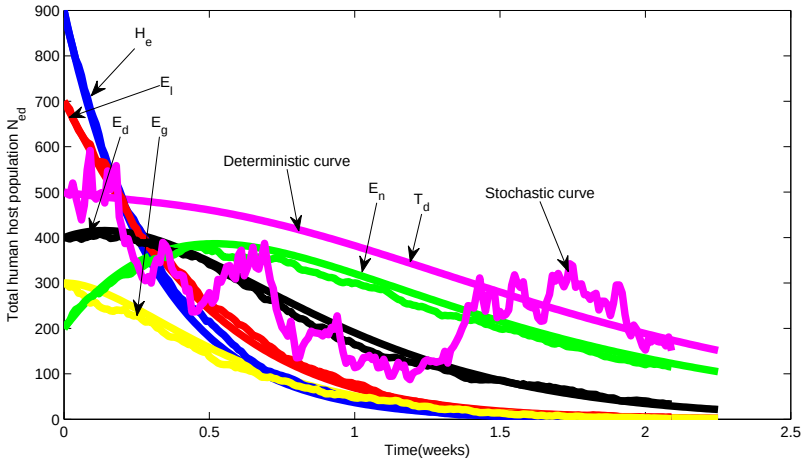


FIGURE 5. Behavior of the stochastic and deterministic model solutions with the varying parameters from Table 1 together with the initial values $H_e = 900$, $E_l = 700$, $E_g = 350$, $E_d = 400$, $E_n = 200$ and $T_d = 550$

In Figure 3, increasing the initial condition parameter values leads to a rapid convergence of the stochastic curve within 4.5 weeks, highlighting the swift manifestation of ED within a short timeframe compared to the deterministic curve. Figure 4 illustrates that incresing the initial conditions and parameters results in a sudden increase in individuals with ED who choose not to disclose their status

due to stigmatization fears. Simultaneously, an increase in the stochastic behavior of those who disclose their status and undergo treatment is observed over time. Figure 5 demonstrates that decreasing the parameter values and increasing the initial conditions leads to a rapid biological response, similar to Figure 2. Both stochastic and deterministic curves converge rapidly within 2.5 weeks.

## 5. Conclusion

In conclusion, our study has developed a stochastic model employing ordinary differential equations to elucidate the incidence of Erectile Dysfunction (ED) in men, considering both unhealthy lifestyles and genetic factors. The model not only establishes the existence, uniqueness, and positive global solutions, but also includes transition probabilities, drift, and diffusion matrices. Through the application of the E-M method, we have obtained approximate solutions, revealing the better performance of stochastic curves compared to deterministic ones, particularly in the presence of environmental noise. Our simulations provide valuable insights into the dynamics of ED, emphasizing the importance of timely interventions. The simulations illustrate the declining trend of individuals at risk $H_e$, the decreasing prevalence of ED due to unhealthy lifestyles $E_l$ upon disclosure and treatment, and the transient increase followed by a decline in cases associated with genetic factors $E_g$. Concurrently, there is an alarming rise in those withholding their ED status $E_n$ due to the fear of stigmatization, underscoring the need for targeted strategies to address this trend. Further simulations support our conclusions, demonstrating the rapid convergence of stochastic curves, especially with increased parameter values for initial conditions, and by highlighting the pronounced impact of stigmatization fears on individuals choosing not to disclose their ED status. Overall, our findings contribute to a comprehensive understanding of ED disease dynamics and provide insights into potential strategies for its effective treatment. Overcoming the psychological barriers associated with stigmatization emerges as a crucial aspect of enhancing interventions and minimizing the long-term prevalence of undisclosed ED cases.

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