Transactions of A. Razmadze
Mathematical Institute
Vol. **179** (2025), issue 1, 1–10

ON SOME GEOMETRIC RESULTS OF PROFESSOR ALEXANDER KHARAZISHVILI

ALEKS KIRTADZE[1,2] AND TENGIZ TETUNASHVILI[3,4]

Dedicated to Professor Alexander Kharazishvili on the occasion of his 75th birthday

Abstract. A survey is given of some results of Professor A. Kharazishvili in the field of convex and combinatorial geometry.

We begin our presentation with some data concerning scientific biography of prof. A. Kharazishvili.

In 1972, he graduated from I. Javakhishvili Tbilisi State University, Faculty of Mechanics and Mathematics.

In 1972–1983, he worked at I. Vekua Institute of Applied Mathematics, as a scientific researcher.

Since 1983, he holds the position of Head of Department of Discrete Mathematics at the same institute.

Since 2006 to the present, A. Kharazishvili is also a Chief Researcher of the Department of Mathematical Analysis at A. Razmadze Mathematical Institute.

In 1974, A. Kharazishvili defended his Ph.D thesis in specialty–Geometry and Topology. The title of his Ph.D thesis is: "On Some Combinatorial Properties of Subsets of Euclidean Spaces". This thesis was carried out without a scientific supervisor.

In 1982, he defended his Dr. Sci. thesis in specialty–Mathematical Analysis. The title of his Dr. Sci. thesis is: "On Invariant Extensions of the Lebesgue Measure".

In 2009, he became a corresponding member of the Georgian National Academy of Sciences, and since 2015, he is a member of this academy.

In 2017, he was elected as an Honorary member of Bulgarian Geometrical Society.

Scientific interests of Prof. A. Kharazishvili are rather wide and cover several areas in mathematics: convex and combinatorial geometry, real analysis and measure theory, set theory and general topology.

A survey of his results concerning the structure of various singular real-valued functions was given in [108]. Here, we wish to present some (in fact, randomly selected) results of A. Kharazishvili in the area of combinatorial and convex geometry.

During a long-term period, different geometric topics were (and still remain) in the sphere of his scientific interests. From the very beginning, these interests were inspired and stimulated by the series of popular geometry text-books written by professor V. G. Boltyanskii (see, for instance, [10,16]), who is widely known as an outstanding geometer-topologist and one of the leading specialists in modern optimization theory. We would like to add to the said above that A. Kharazishvili turned his attention to various themes and questions of combinatorial geometry when he was a student of Tbilisi State University. Moreover, being a young student of this university, he was able to infer a solution of Sylvester's beautiful problem on collinear points as a consequence of Euler formula for connected planar graphs. Kharazishvili also proved that the dual version of Sylvester's problem is valid not only for a finite family of straight lines in the plane $\mathbf{R}^2$, but also for finite families of much more general curves in $\mathbf{R}^2$. Unfortunately, he did not publish his first nontrivial result in combinatorial geometry. Afterwards, an extensive series of his publications in this area of mathematics followed. References [51–105] of the present survey reflect his long-term research in convex and combinatorial geometry.

2020 *Mathematics Subject Classification.* 52A05, 52A10, 52A15, 52B11, 52C10.

Key words and phrases. Convex set; Euclidean space; Dissection of a polyhedron; Parallelepiped; *at*-set; *ot*-set; *rt*-set; *k*-homogeneous covering; Ramsey theorem; Sperner lemma; Rainbow simplex.

1. A. Kharazishvili's earliest paper [54] gives formulation of the following somewhat unexpected characterization property of parallelepipeds in the Euclidean space:

A nonempty compact convex set P in the space $\mathbf{R}^m$ is an n-dimensional parallelepiped (where $n \leq m$) if and only if there exists a real $t \in]0,1[$ such that the sets $P \cap (tP + h)$ are centrally symmetric for all vectors $h \in \mathbf{R}^m$.

In the proof of this statement the main difficulty lies in establishing the fact that such a P is necessarily a polyhedron, and then the argument can be continued by using induction on the dimension m. The above-mentioned result was cited by the experts in convex geometry (see, e.g., [21, 33–35, 37, 120, 142]). In particular, it is interesting to note that, by starting from Kharazishvili's result, P. Gruber [33] was able to demonstrate that the intersection of any decreasing family of parallelepipeds in $\mathbf{R}^m$ is again a parallelepiped. For simplexes, the analogous statement, proven also some time earlier gives a solution to one problem posed by A. Kolmogorov. It should be noted that in the case of simplexes, their certain characterization property in the family of all nonempty compact convex subsets of $\mathbf{R}^m$ leads to the required result (the mentioned characteristic property of simplexes is due to Rogers and Shephard [133]; it easily follows from one general statement of Kharazishvili [58] concerning locally conical sets).

In connection with the result obtained in [54], Kharazishvili posed the following problem.

Let $\mathcal{K}$ be a class of all nonempty compact convex sets in $\mathbf{R}^m$ satisfying the following conditions:

(1) $\mathcal{K}$ is invariant with respect to the group of all affine transformations of $\mathbf{R}^m$;

(2) $\mathcal{K}$ is upper semicontinuous, i.e., for any decreasing by inclusion sequence $\{K_n : n \in \mathbf{N}\}$ of members of $\mathcal{K}$, the set $\cap\{K_n : n \in \mathbf{N}\}$ is also a member of $\mathcal{K}$.

Give a geometric characterization of all such classes $\mathcal{K}$.

As far as we know, the above problem remains still unsolved.

2. The short note by Kharazishvili [53] concerning the illumination problem of multi-dimensional compact convex bodies deserves to be pointed out. In [53], for $m \geq 4$, he gives an example of an m-dimensional compact convex body in the space $\mathbf{R}^m$ which has exactly $m+1$ singular boundary points, but cannot be illuminated by any $m+1$ rays in $\mathbf{R}^n$.

This result gives an answer to a problem posed by Boltyanskii and Gokhberg in [16]. In the same paper [53], Kharazishvili shows that every compact convex body in $\mathbf{R}^3$ having exactly 4 singular boundary points can be illuminated by 4 rays.

Kharazishvili's example in $\mathbf{R}^4$ may be considered as a counterpart of Boltianskii's theorem stating that any compact convex body in $\mathbf{R}^m$ with at most m singular boundary points can be illuminated by using $m+1$ rays in general position.

The note [53] was cited in a number of works (see, e.g., [11–15, 17–20, 35, 120, 126, 139]).

It should be especially mentioned that for constructing his example in the space $\mathbf{R}^4$, Kharazishvili starts with a 5-point set X in $\mathbf{R}^3$ which has the property that every three-element subset of X forms an acute-angled triangle. The sets with this property are called *at*-sets (more generally, a subset Z of the space $\mathbf{R}^m$, where $m \geq 2$, is an *at*-set if any three distinct points of Z are the vertices of an acute-angled triangle). Note that if $Y \subset \mathbf{R}^3$ is an arbitrary *at*-set, then $\text{card}(Y) \leq 5$. The question concerning finding more or less precise estimates of the cardinalities of *at*-sets in multi-dimensional Euclidean spaces is of interest for discrete and combinatorial geometry. It was shown in [26] (see also [2]) that there exist *at*-sets in $\mathbf{R}^m$ whose cardinalities are of exponential order with respect to the dimension m (this result answers one question posed by L. Danzer and B. Grünbaum). For obtaining their result, the authors of [26] use probabilistic methods.

In [94] and [99], Kharazishvili gives a deterministic proof of the same result. He applies the method of induction on m and obtains a nice formula

$$r_m = 2^m((3^m + 1)/2 - 2^m),$$

where r_m denotes the total number of those right-angled triangles whose vertices belong to the set V_m of all vertices of the unit m-cube $[0,1]^m$. It follows from the above formula that

$$(2^m)!/(3!(2^m - 3)!) - 2^m((3^m + 1)/2 - 2^m)$$

is precisely the total number of those acute-angled triangles whose vertices belong to V_m. Using appropriate estimates from below for the last number, Kharazishvili concludes that the cardinality of a maximal *at*-subset of V_m is of an exponential order with respect to m.

3. In [72], Kharazishvili proved that any finite point set in the space $\mathbf{R}^m$ $(m \geq 2)$, which is not contained in an affine hyperplane of $\mathbf{R}^m$, determines a sphere-like polyhedral hypersurface, and an appropriate algorithm to obtain such a hypersurface was described. Moreover, in the same work it is indicated that the polyhedron bounded by this hypersurface admits a triangulation without adding new vertices and has maximally simple combinatorial structure. Much later, for the three-space $\mathbf{R}^3$, the analogous question was studied by some other authors. Among them, let us refer to:

F. Hurtado, G. T. Toussaint, J. Trias, *On polyhedra induced by point sets in space*, Proceedings of 15th Canadian Conference on Computational Geometry, Canada, August 11–13, 2003, pp. 107–110;

P. K. Agarwal, F. Hurtado, G. T. Toussaint, J. Trias, *On polyhedra induced by point sets in space*, Discrete Applied Mathematics, v. 156, issue 1, 2008, pp. 42–54.

These authors were not aware of the above-mentioned paper [72]. In addition, it was observed by A. Kharazishvili that the Erdös length minimizing algorithm for finding a simple polygonal cycle, determined by a given finite non-collinear set of points in $\mathbf{R}^2$, successfully works in the case of points in general position, but sometimes is not applicable if the property of general position does not hold.

4. The famous problem posed by Erdös and Szekeres in their joint article [27] is formulated as follows:

Find the least natural number $c(n)$ such that any set $X \subset \mathbf{R}^2$ of points in general position in $\mathbf{R}^2$ with $\mathrm{card}(X) = c(n)$ contains n convexly independent points.

Erdös and Szekeres conjectured that $c(n) = 2^{n-2} + 1$. Up to now, this conjecture has not been confirmed, but it also has not been disproved either.

By using the countable version of Ramsey's theorem, it can be proved that any infinite set of points in general position in $\mathbf{R}^2$ contains an infinite convexly independent subset.

In the article by Kharazishvili [81], the same result is established directly, without appealing to Ramsey's theory. Actually, in [81], some algorithm of finding such a subset is described. On the other hand, it is shown in [81] that for uncountable sets in $\mathbf{R}^2$ the analogous result fails to be true, i.e., there exists an uncountable set $Z \subset \mathbf{R}^2$ of points in general position such that no uncountable subset of Z can be convexly independent. The analogous result also takes place for the space $\mathbf{R}^m$, where $m > 2$.

5. In combinatorial geometry of Euclidean space, various point sets with a prescribed property of all their three-point subsets are extensively studied. For instance, an *ot*-set (respectively, *at*-set, *rt*-set) in $\mathbf{R}^m$ is defined as a point set, all three-element subsets of which form obtuse-angled (respectively, acute-angled, right-angled) triangles. To indicate one of *ot*-sets, it suffices to take arbitrarily many points on the moment curve in $\mathbf{R}^m$ defined by the mapping

$$t \to (t, t^2, t^3, \ldots, t^m) \ \ (t \in [0,1]).$$

It is well-known that, for $m \geq 4$, this curve produces some remarkable geometric objects, and one of them is the so-called Carathéodory–Gale polyhedron.

It can easily be seen that in the plane $\mathbf{R}^2$ the half-open semicircle is a maximal (with respect to the inclusion relation) *ot*-subset of $\mathbf{R}^2$, and it is not hard to verify that no finite *ot*-set in $\mathbf{R}^2$ can be maximal.

In [82], Kharazishvili gives a delicate combinatorial construction of an *ot*-set D in $\mathbf{R}^2$ which is discrete and, simultaneously, maximal with respect to inclusion. In view of the discreteness of D, this set is countably infinite.

Similar results concerning maximal *ot*-sets were established in [82] for the space $\mathbf{R}^m$, where $m \geq 3$. In this connection, it should be remarked that a geometric characterization of all maximal *ot*-sets in $\mathbf{R}^m$ is still unknown.

The analogous question for *rt*-sets was considered by Kharazishvili in [61] and it was shown by him that if a given *rt*-set is not a rectangle, then it coincides with the set of vertices of some orthogonal simplex (ortho-scheme). In [62], Kharazishvili considers the Hadwiger problem of polyhedra dissections into finitely many orthogonal simplexes and gives its solution for the space $\mathbf{R}^4$. These results of Kharazishvili were cited in [9, 23, 44–47, 146].

6. In two papers [87] and [96] Kharazishvili returns to Ramsey's combinatorial theorem and demonstrates its usefulness in several questions of geometric flavor. In particular, he proves that, for the space $\mathbf{R}^m$ and for a given natural number n, there exists a natural number $k(m,n)$ such that any set in the space $\mathbf{R}^m$ having cardinality greater than $k(m,n)$ contains an *ot*-subset with cardinality greater than n.

In the process of proving this result a finite version of Ramsey's theorem is essentially used. On the other hand, using infinite version of Ramsey's theorem, Kharazishvili proves that in a real pre-Hilbert space H all *ot*-sets and all *rt*-sets are separable, so their cardinality is at most continuum ($= \mathbf{c}$). However, the cardinalities of *at*-sets in an infinite-dimensional H can be arbitrarily large and, more precisely, can be equal to $\text{card}(H)$.

In this context, it should also be mentioned that for the space $\mathbf{R}^3$, Kharazishvili in his paper [102] considers a very simple geometric statement of Ramsey type concerning uncountable families of straight lines in this space, and shows that the validity of the statement heavily depends on the set-theoretical nature of $\mathbf{c}$, i.e., on the regularity or singularity of $\mathbf{c}$.

7. In the article by Kharazishvili [75], the notion of a quasi-polygon in the Euclidean plane $\mathbf{R}^2$ was introduced and examined from the topological and measure-theoretical view-points. By definition, a quasi-polygon in $\mathbf{R}^2$ is any subset of $\mathbf{R}^2$, homeomorphic to the unit disc $\mathbf{B}_2 \subset \mathbf{R}^2$ and such that its boundary is representable as the union of a singleton with countably many non-degenerate line segments. Starting with the existence of a Jordan curve in $\mathbf{R}^2$ whose boundary is of strictly positive two-dimensional Lebesgue measure (see, e.g., [4]; such curves were first constructed by Osgood), it was shown in [75] that the union of an uncountable family of quasi-polygons may be non-measurable in the Lebesgue sense. The study of quasi-polygons was continued by Kharazishvili in his works [104] and [105] (co-author T. Tetunashvili), in connection with combinatorial realizations of abstract families of sets as families of compact convex sets in $\mathbf{R}^2$. In particular, it was proved in the above-mentioned works that in $\mathbf{R}^2$ there exists an uncountable family of convex quasi-polygons which is independent in the set-theoretical sense. In this context, it makes sense to underline that any independent family of two-dimensional polygons in $\mathbf{R}^2$ (which are not assumed to be convex) is at most countable.

8. In [84], the notion of a k-homogeneous covering of the Euclidean space $\mathbf{R}^m$ was introduced and examined. Let $k > 0$ be a natural number and let $\mathcal{F}$ be a family of geometric figures in $\mathbf{R}^m$. This family is called a k-homogeneous covering of $\mathbf{R}^m$ if each point of $\mathbf{R}^m$ belongs to exactly k members of $\mathcal{F}$.

Let $\mathcal{G}$ be a family of subsets of an abstract infinite space E. In [84], a general statement was established in terms of k and $\mathcal{G}$, which enables one to claim that $\mathcal{G}$ contains a subfamily $\mathcal{F}$ forming a k-homogeneous covering of E. This general statement (proved by using the method of transfinite induction) is then applied to special families of figures in $E = \mathbf{R}^m$. Many interesting examples of such subfamilies $\mathcal{F}$ were given in [84].

To illustrate the general situation, let us consider a more concrete example. It is not hard to show that, for any even number $k > 0$, there exists a k-homogeneous covering of the plane $\mathbf{R}^2$ with pairwise congruent circles. This result can be obtained effectively (i.e., within **ZF** set theory) by presenting an individual construction of the required k-homogeneous covering of $\mathbf{R}^2$ (see, e.g., [88]). For $k = 1$, there exists no k-homogeneous covering of $\mathbf{R}^2$ with homeomorphic images of $\mathbf{S}_1$. But for any odd $k > 1$, it can be proved that there exists a k-homogeneous covering of $\mathbf{R}^2$ with circles which are all congruent to $\mathbf{S}_1$. The proof is based on the Axiom of Choice and it is unknown whether the existence of such a covering can be established within the **ZF** theory.

Also, for the space $\mathbf{R}^3$, the following nice result takes place: there exists a partition of $\mathbf{R}^3$ into circles congruent to $\mathbf{S}_1$ (cf., [74, 79]). The proof of this result again essentially relies on the Axiom of Choice. Using an analogue of the Osgood curve in $\mathbf{R}^3$, Kharazishvili gave an example of a homeomorphic image C of $\mathbf{S}_1$ in $\mathbf{R}^3$ such that there exists no partition of $\mathbf{R}^3$ into curves congruent to C. In this connection, he posed the problem of finding a characterization of all those curves $L \subset \mathbf{R}^3$ which are homeomorphic to $\mathbf{S}_1$ and are divisors of $\mathbf{R}^3$ (i.e., $\mathbf{R}^3$ admits a partition into curves congruent to L).

As far as we know, this problem remains unsolved. The works of A. Kharazishvili in this direction are cited, e.g., in [24, 39, 43, 134, 138, 151].

9. Many interesting and intriguing geometric problems are formulated in terms of colorings of points of the Euclidean space (see, for example, [2]). One of such problems was considered by Kharazishvili in [97]. A three-coloring of the Euclidean plane is defined as any surjective mapping

$$g : \mathbf{R}^2 \to \{1, 2, 3\},$$

where the set $\{1, 2, 3\}$ plays the role of three distinct colors (say, blue, green, and red). A triangle in $\mathbf{R}^2$ is called rainbow if its vertices carry all three colors.

The following natural question arises: how many rainbow triangles of a prescribed type (e.g., right-angled, acute-angled, obtuse-angled, isosceles) are there?

In [97], it was proved that for any three-coloring of the plane there are continuum many rainbow acute-angled (respectively, right-angled, obtuse-angled, isosceles) triangles. In this connection, it should be mentioned that the proof of the existence of sufficiently many rainbow acute-angled triangles is based on the special case of Sperner's combinatorial lemma.

The same work also shows that for equilateral triangles the answer is negative, namely, one can constructively define a three-coloring of $\mathbf{R}^2$ such that no equilateral triangle in $\mathbf{R}^2$ is rainbow.

Since the cardinality of a set is not an object of first-order logic and there are axiomatic systems of plane geometry within first-order logic, the question of the existence of rainbow triangles of a prescribed type remains actual in the weakened axiomatic of the plane. In this direction, the work by V. Pambuccian [128] is noteworthy where the question of the existence of rainbow triangles is thoroughly examined for the so-called weak geometries of the plane.

A multi-dimensional variant of the above problem is discussed by Kharazishvili in [101] and [103]. In this case, the notion of $(m+1)$-coloring of the space $\mathbf{R}^m$ is introduced analogously, as any surjective mapping

$$g : \mathbf{R}^m \to \{1, 2, \dots, m+1\}.$$

The question of the existence of rainbow m-simplexes of a prescribed type is discussed, but it turns out that for many $(m+1)$-colorings, the answer is negative. Taking this circumstance into account the notion of an admissible $(m+1)$-coloring is introduced in [101, 103] and it is proved that for such colorings there are continuum many rainbow isosceles m-simplexes. Again, in the process of proving this fact, Sperner's lemma is essentially used. It makes sense to remark that a lot of admissible $(m+1)$-colorings of $\mathbf{R}^m$ can be constructed constructively. However, there are admissible colorings which are obtained with the aid of the Axiom of Choice by starting with valuations on the ring of reals.

10. Some works of Kharazishvili were devoted to the equidecomposability theory of polyhedra in the Euclidean spaces (see [57, 62, 63, 66, 67, 85, 89]) and to the problems of equidecomposability of more general subsets of a ground space E endowed with a transformation group G. For example, he proved in [85] that the decomposition numbers of any Euclidean m-dimensional cube constitute a co-countable subset of the set $\mathbf{N}$ of all natural numbers. More generally, if P is an m-dimensional right right-angled parallelepiped in $\mathbf{R}^m$, then its decomposition numbers (with respect to the family of m-cubes) either do not exist or constitute a co-countable subset of $\mathbf{N}$. Kharazishvili also observed in [85] that, for $m \leq 3$, any m-dimensional parallelepiped is primitive (for this notion of a primitive polyhedron see [85]). At the same time, he proved that, for $m \geq 4$, the unit m-dimensional cube is not primitive.

Among other publications of A. Kharazishvili devoted to geometric topics, let us point out the works [59, 60, 68, 71, 77] on some combinatorial properties of motions (i.e., isometries) of the Euclidean space $\mathbf{R}^m$, the work [65] in which a solution to Grünbaum's problem on affine diameters of convex bodies is given, the papers [87, 90] in which non-elementary and set-theoretical methods in elementary geometry are discussed, the article [92] on approximation of convex plane curves of constant width by algebraic convex curves of constant width, and the work [95] concerning properties of external bisectors of triangles.

We especially would like to indicate several books written by prof. A. Kharazishvili that are dedicated to various themes in convex and combinatorial geometry and, accordingly, to applications of geometric methods in other fields of mathematics.

(a) *Selected Topics in the Geometry of Euclidean Spaces*, Izd. Tbil. Gos. Univ., Tbilisi, 1978 (in Russian).

(b) *Introduction to Combinatorial Geometry*, Izd. Tbil. Gos. Univ., Tbilisi, 1985 (in Russian).

(c) *Brunn-Minkowski Inequality and its Applications*, Izd. Naukova Dumka, Kiev, 1985 (in Russian, co-author V. V. Buldygin; a revised and expanded version of this book was translated into English, see (d)).

(d) *Geometric Aspects of Probability Theory and Mathematical Statistics*, Kluwer Academic Publishers, Dordrecht, 2000 (co-author V. V. Buldygin).

(e) *Elements of Combinatorial Geometry*, Part I, The Publishing House of Georgian National Academy of Sciences, Tbilisi, 2016.

(f) *Elements of Combinatorial Geometry*, Part II, The Publishing House of Georgian National Academy of Sciences, Tbilisi, 2020.

The results of A. Kharazishvili listed above were cited in [1–8, 11–14] and those which are included in his books (a)–(f) were cited by various authors (see, for instance, [1, 3, 5–8, 22, 28–32, 36, 38, 40–42, 49, 50, 106, 107, 109–132, 134–152]).

In conclusion of our survey, we sincerely wish Prof. A. Kharazishvili (who for many years ago was our scientific supervisor and currently is our colleague and friend) good health, happiness, activity in his further research work and obtaining many attractive results in mathematics.

References

1. V. A. Aguilar-Arteaga, R. Ayala-Figueroa, I. González-Garcia, J. Jerónimo-Castro, On evolutoids of planar convex curves II. *Aequationes Math.* **89** (2015), no. 6, 1433–1447.
2. M. Aigner, G. Ziegler, *Proofs From the Book.* Springer-Verlag, Berlin-New York, 2009.
3. I. A. Aljazaery, Signal encryption using random functions and wavelet transform. Journal of Babylon University–Pure and Applied Sciences, **23** (2015), no. 4, 1709–1718.
4. M. Balcerzak, A. Kharazishvili, On uncountable unions and intersections of measurable sets. *Georgian Math. J.* **6** (1999), no. 3, 201–212.
5. Sh. Beriashvili, On some properties of primitive polyhedrons. *Trans. A. Razmadze Math. Inst.* **174** (2020), no. 1, 23–28.
6. Sh. Beriashvili, Some classes of polyhedrons in $\mathbf{R}^3$ which are not primitive. In: *The XI International Conference of the Georgian Mathematical Union, Batumi*, p. 58, 2021.
7. M. Beriashvili, R. Schindler, Mazurkiewicz sets with no well-ordering of the reals. *Georgian Math. J.* **29** (2022), no. 3, 343–345.
8. I. Blazhievska, V. Zaiats, On cross-correlogram IRF's estimators of two-output systems in spaces of continuous functions. *Comm. Statist. Theory Methods* **50** (2021), no. 24, 6024–6048.
9. J. Böhm, E. Hertel, *Polyedergeometrie in n-dimensionalen Räumen Konstanter Krümmung.* Birkhäuser Verlag, Basel-Boston, Mass., 1981.
10. V. G. Boltyanskii, *Hilbert's Third Problem.* (Russian) Nauka, Moscow, 1977.
11. V. G. Boltyanskii, Combinatorial geometry. (Russian) In: *Algebra. Topology. Geometry.* vol. 19, pp. 209–274, 276, Itogi Nauki i Tekhniki, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1981.
12. V. G. Boltyanskii, Solution to the lighting problem for zone bodies. *Mathematical Notes* **58** (1995), no. 4 : 1029–1032.
13. V. G. Boltyanski, Solution of the illumination problem for bodies with md $M = 2$. *Discrete Comput. Geom.* **26** (2001), no. 4, 527–541.
14. V. G. Boltyanskii, A counterexample to an illumination conjecture. (Russian) *Dokl. Akad. Nauk* **403** (2005), no. 2, 151–154.
15. V. G. Boltyanskii, E. D. Baladze, The Szökefalvi-Nagy problem in combinatorial geometry. (Russian) Nauka, Moscow, 1997.
16. V. G. Boltyanskii, I. Ts. Gokhberg, *Theorems and Problems of Combinatorial Geometry.* (Russian) Nauka, Moscow, 1965.
17. V. Boltyanskii, H. Martini, Illumination of direct vector sums of convex bodies. *Studia Sci. Math. Hungar.* **44** (2007), no. 3, 367–376.
18. V. Boltyanskii, H. Martini, P. S. Soltan, *Excursions into Combinatorial Geometry.* Universitext. Springer-Verlag, Berlin, 1997.

19. V. Boltyanskii, A. Soifer, *Geometric Études in Combinatorial Mathematics.* With introductions by Paul Erdős, Branko Grünbaum and Cecil Rousseau. Center for Excellence in Mathematical Education, Colorado Springs, CO, 1991.
20. V. Boltyanskii, P. S. Soltan, *Combinatorial Geometry of Various Classes of Convex Sets.* (Russian)"Štiinca", Kishinev, 1978.
21. K. Boroczky, V. Soltan, Translational and homothetic clouds for a convex body. *Studia Sci. Math. Hungar.* **32** (1996), no. 1-2, 93–102.
22. J. Brandts, S. Korotov, M. Křížek, O triangulacích bez tupých úhlů.(Czech) *Pokroky matematiky, fyziky a astronomie* **50** (2005), no. 3, 193–207.
23. J. Brandts, S. Korotov, M. Křížek, J. Šolc, On nonobtuse simplicial partitions. *SIAM Rev.* **51** (2009), no. 2, 317–335.
24. J. Cobb, Nice decompositions of $\mathbf{R}^n$ entirely into nice sets are mostly impossible. *Geom. Dedicata* **62** (1996), no. 1, 107–114.
25. Ch. Davis, B. Grünbaum, F.A. Sherk, *The Geometric Vein.* Springer-Verlag, Berlin, 1981.
26. P. Erdös, Z. Füredi, The greatest angle among n points in the d-dimensional Euclidean space. *Ann. Discrete Math.* **17** (1983): 275–283.
27. P. Erdös, G. Szekeres, A combinatorial problem in geometry. *Compositio Math.* **2** (1935), 463–470.
28. A. A. Gabrichidze, On symmetric sets. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **100** (1980), no. 3, 553–555.
29. A. A. Gabrichidze, A class of symmetric sets. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **114** (1984), no. 1, 41–43.
30. A. Gabrichidze, On some questions of combinatorial geometry in finite-dimensional normed vector spaces. (Russian) In: *The First Soviet Conference in Combinatorial Geometry, Batumi*, 1985.
31. G. A. Gal'perin, A solution of Proizvolov's problem of covering a multidimensional polyhedron by pyramids, and its generalizations. In: *Doklady Akademii Nauk* vol. 293, no. 2, pp. 283–288. Russian Academy of Sciences, 1987.
32. R. J. Gardner, The Brunn-Minkowski inequality. *Bull. Amer. Math. Soc. (N.S.)* **39** (2002), no. 3, 355–405.
33. P. Gruber, Über den Durchschnitt einer abnehmenden Folge von Parallelepipeden. *Elemente Math.* **32** (1977), 13–15.
34. P. Gruber, J. M. Wills, *Convexity and its Applications.* Birkhauser, Basel, 1983.
35. P. Gruber, J. M. Wills, *Handbook of Convex Geometry.* vol. A, vol. B North-Holland, Amsterdam, 1993.
36. Z. Guerrero-Zarazua, J. Jeronimo Castro, Some comments on floating and centroid bodies in the plane. *Aequationes Mathematicae* **92** (2018), 211–222.
37. H. Guggenheimer, E. Lutwak, A characterization of the n-dimensional parallelotope. *Amer. Math. Monthly* **83** (1976), no. 6, 475–478.
38. Q. Guo, *Minkowski Measure of Asymmetry and Minkowski Distance for Convex Bodies.* Thesis (Ph.D.)–Uppsala Universitet (Sweden). ProQuest LLC, Ann Arbor, MI, 2004.
39. R. K. Guy, R. E. Woodrow, The lighter side of mathematics. In: *Proc. E. Strens Memorial Conf. on Recr. Math. and its History, Calgary*, 1986.
40. E. Hertel, *Some Generalizations of Equidecomposability Theory of Polyhedra.* (Russian) Forschungsergebnisse, Universitat Jena, 1982.
41. E. Hertel, Decomposition similarity of polygons. (German) *Elem. Math.* **41** (1986), no. 6, 139–143.
42. G. M. Jerlitsin, A. S. Matveev, Optimal codifying in sensor networks with limited channel capacities. (Russian) *Vestnik S-P Univ.* **1** (2012), no. 3, 24–33.
43. M. Jonsson, J. Wästlund, Partitions of $\mathbf{R}^3$ into curves. *Math. Scand.* **83** (1998), no. 2, 192–204.
44. H. Kaiser, Mehrfach-orthogonale Simplexe in Räumen konstanter Krümmung. (German) [Multiply-orthogonal simplexes in spaces of constant curvature] *Beiträge Algebra Geom.* no. 19, (1985), 131–143.
45. H. Kaiser, To the question on decomposability of simplices into k-orthogonal simplices. (Russian) In: *The First Soviet Conference in Combinatorial Geometry, Batumi*, 1985.
46. H. Kaiser, Zum Problem der Zerlegbarkeit von Simplexen in Orthoscheme. (German) [On the problem of decomposing simplices into orthoschemes] *Studia Sci. Math. Hungar.* **21** (1986), no. 1-2, 227–242.
47. H. Kaizer, On a classification of n-dimensional simplexes. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **125** (1987), no. 1, 29–31.
48. T. Kasrashvili, Discrete point systems in Euclidean spaces and invariants of associated continuous images. (Georgian) *Journal of Science and Technologies*, no. 1–3, 2012, 7–16.
49. T. Kasrashvili, A. Kirtadze, On some combinatorial properties of Diophantine sets in Euclidean spaces. In: *Abstracts of the 10th International Conference on Geometry and Applications*, pp. 34–35, September 03–09, Varna, 2011.
50. T. Kasrashvili, A. Kirtadze, Elementary volume and measurability properties of additive functions. *Georgian Math. J.* **23** (2016), no. 1, 69–73.
51. A. B. Kharazishvili, *Dispositions on a Straight Line, on a Plane and in the Space.* (Georgian) Metsniereba da Tekhnika, 1972.
52. A. B. Kharazishvili, A certain property of convex hulls. (Russian) *Bull. Acad. Sci. GSSR* **71** (1973), 25–27.
53. A. B. Kharazishvili, On the problem of illumination. (Russian) *Bull. Acad. Sci. GSSR* **71** (1973), 289–291.
54. A. B. Kharazishvili, Characteristic properties of a parallelepiped. (Russian) *Bull. Acad. Sci. GSSR* **72** (1973), 17–19.
55. A. B. Kharazishvili, *Some Combinatorial Properties of Subsets of the Euclidean Spaces.* Thesis, Izd. Tbil. Gos. Univ., Tbilisi, 1974.

56. A. B. Kharazishvili, Realization of colored graphs in Hilbert space. (Russian) *Bull. Acad. Sci. GSSR* **76** (1974), 53–56.
57. A. B. Kharazishvili, Decompositions of polytopes. (Russian) *Bull. Acad. Sci. GSSR* **77** (1975), 37–39.
58. A. B. Kharazishvili, Locally conical sets. (Russian) *Bull. Acad. Sci. GSSR* **80** (1975), no. 1, 29–32.
59. A. B. Kharazishvili, Intransitivity classes of groups of motions of Euclidean spaces. (Russian) *Dokl. Akad. Nauk SSSR* **226** (1976), no. 3, 520–522.
60. A. B. Kharazishvili, Fully symmetric configurations. *Bull. Acad. Sci. GSSR*. (Russian) **85** (1977) no. 2, 297–300.
61. A. B. Kharazishvili, A characterization of *rt*-sets in Euclidean spaces. (Russian) *Bull. Acad. Sci. GSSR* **86** (1977), no. 3, 537–540.
62. A. B. Kharazishvili, Orthogonal simplexes in four-dimensional space. (Russian) *Bull. Acad. Sci. GSSR* **88** (1977), no. 1, 33–36.
63. A. B. Kharazishvili, Equidissectability of polyhedra relative to the group of homotheties and translations. (Russian) *Dokl. Akad. Nauk SSSR* **236** (1977), no. 3, 552–555.
64. A. B. Kharazishvili, Some properties of the center of symmetry. (Russian) *Bull. Acad. Sci. GSSR* **89** (1978), no. 1, 21–24.
65. A. B. Kharazishvili, Affine diameters of convex bodies. (Russian) *Bull. Acad. Sci. GSSR* **90** (1978), no. 3, 541–544.
66. A. B. Kharazishvili, An invariant defined on the class of three-dimensional polytopes. (Russian) *Bull. Acad. Sci. GSSR* **91** (1978), no. 1, 33–36.
67. A. B. Kharazishvili, Equidecomposability of three-dimensional polyhedra with respect to the group of parallel translations and central symmetries. (Russian) *Dokl. Akad. Nauk SSSR* **243** (1978), no. 6, 1410–1413.
68. A. B. Kharazishvili, The index of an isometric imbedding. (Russian) *Tbiliss. Gos. Univ. Inst. Prikl. Mat. Trudy* **5/6** (1978), 259–264.
69. A. B. Kharazishvili, On a property of families of convex figures. (Russian) *Bull. Acad. Sci. GSSR* **102** (1981), no. 1, 25–28.
70. A. B. Kharazishvili, To the theory of equidecomposability. (Russian) *Bull. Acad. Sci. GSSR*, **103** (1981), no. 3, 537–540.
71. A. B. Kharazishvili, On the general theory of volume. *Bull. Acad. Sci. GSSR* **108** (1982), no. 3, 485–488.
72. A. B. Kharazishvili, Simple polyhedra. (Russian) *Sem. Inst. Prikl. Mat. Dokl.* **18** (1984), 34–38, 83.
73. A. B. Kharazishvili, A problem of combinatorial geometry. (Russian) *Bull. Acad. Sci. GSSR* **118** (1985), no. 1, 37–40.
74. A. B. Kharazishvili, Partition of a three-dimensional space into congruent circles. (Russian) *Bull. Acad. Sci. GSSR* **119** (1985), no. 1, 57–60.
75. A. B. Kharazishvili, Quasipolygons and their uncountable unions. (Russian) *Bull. Acad. Sci. GSSR* **124** (1986), no. 3, 465–468.
76. A. B. Kharazishvili, Some properties of Jordan curves. (Russian) Soobshch. *Bull. Acad. Sci. GSSR* **129** (1988), no. 1, 29–32.
77. A. B. Kharazishvili, Groups of motions and the uniqueness of the Lebesgue measure. (Russian) *Bull. Acad. Sci. GSSR* **130** (1988), no. 1, 29–32.
78. A. B. Kharazishvili, Realizations of graphs in the three-dimensional Euclidean space. In: *IX Soviet Union Conference in Geometry, Theses*, Kishinev, 1988.
79. A. B. Kharazishvili, *Applications of Set Theory.* (Russian) Tbilis. Gos. Univ., Tbilisi, 1989.
80. A. Kharazishvili, Some partitions consisting of Jordan curves. *Georgian Math. J.* **3** (1996), no. 3, 233–238.
81. A. Kharazishvili, A note on convexly independent subsets of an infinite set of points. *Georgian Math. J.* **9** (2002), no. 2, 303–307.
82. A. Kharazishvili, On maximal ot-subsets of the Euclidean plane. *Georgian Math. J.* **10** (2003), no. 1, 127–131.
83. A. Kharazishvili, On faces of a convex polyhedron in $\mathbf{R}^3$ with a small number of sides. *Georgian Math. J.* **10** (2003), no. 4, 709–715.
84. A. Kharazishvili, On homogeneous coverings of Euclidean spaces. *Georgian Math. J.* **11** (2004), no. 1, 105–109.
85. A. Kharazishvili, On decompositions of a cube into cubes and simplexes. *Georgian Math. J.* **13** (2006), no. 2, 285–290
86. A. Kharazishvili, Some combinatorial properties of finite line-systems in the Euclidean plane. *Georgian Math. J.* **14** (2007), no. 4, 681–686.
87. A. Kharazishvili, On some applications of set-theoretical methods in Euclidean geometry. *Bull. TICMI* **13** (2009), 12–20.
88. A. Kharazishvili, T. Sh. Tetunashvili, On some coverings of the Euclidean plane with pairwise congruent circles. *Amer. Math. Monthly* **117** (2010), no. 5, 414–423.
89. A. Kharazishvili, Piecewise affine approximations of continuous functions on several variables and Gale polyhedra. *Proc. A. Razmadze Math. Inst.* **152** (2010), 133–140.
90. A. Kharazishvili, On non-elementary methods in elementary geometry. In: *Abstracts of the 9th International Conference on Geometry and Applications*, 16–17, September 05-10, Varna, 2009.
91. A. Kharazishvili, Some discrete geometric structures and associated algorithms. *Proc. A. Razmadze Math. Inst.* **155** (2011), 138–144.
92. A. Kharazishvili, A note on algebraic convex curves of constant width. *Georgian Math. J.* **18** (2011), no. 4, 727–733.

93. A. Kharazishvili, On finite and infinite *OT*-sets. *Bull. TICMI* **15** (2011), 27–34.
94. A. Kharazishvili, Some properties of *at*-sets and *ot*-sets in a Hilbert space. *Proc. A. Razmadze Math. Inst.* **159** (2012), 147–151.
95. A. Kharazishvili, Some topologic-geometrical properties of external bisectors of a triangle. *Georgian Math. J.* **19** (2012), no. 4, 697–704.
96. A. Kharazishvili, Some geometric consequences of Ramsey's combinatorial theorem. *Bull. TICMI* **16** (2012), no. 1, 34–42.
97. A. Kharazishvili, On three-colorings of the Euclidean plane and associated triangles of a prescribed type. In: *Abstracts of the 11th International Conference on Geometry and Applications*, Varna, September 3-7, 2013.
98. A. Kharazishvili, On inscribed and circumscribed convex polyhedra. *Proc. A. Razmadze Math. Inst.* **167** (2015), 123–129.
99. A. Kharazishvili, On the cardinalities of *at*-sets in a real Hilbert space. *Georgian Math. J.* **22** (2015), no. 2, 259–263.
100. A. Kharazishvili, Acute triangles in the context of the illumination problem. *God. Sofiĭ. Univ. "Sv. Kliment Okhridski." Fac. Mat. Inform.* **103** (2016), 39–44.
101. A. Kharazishvili, On $(n+1)$-colorings of the n-space and associated isosceles simplexes. In: *Abstracts of the 14th International Conference on Geometry and Applications*, Varna, August 26-31, 2019.
102. A. Kharazishvili, On a geometric statement of Ramsey type. *Georgian Math. J.* **29** (2022), no. 2, 229–232.
103. A. Kharazishvili, On rainbow isosceles n-simplexes. *Georgian Math. J.* **29** (2022), no. 4, 543–549.
104. A. Kharazishvili, T. Tetunashvili, Combinatorial properties of families of sets and Euler-Venn diagrams. *Proc. A. Razmadze Math. Inst.* **146** (2008), 115–119.
105. A. Kharazishvili, T. Tetunashvili, On some combinatorial problems concerning geometrical realizations of finite and infinite families of sets. *Georgian Math. J.* **15** (2008), no. 4, 665–675.
106. H. A. Kim, S. Puchinger, A. Wachter-Zeh, Error correction for partially stuck memory cells. (Russia) In: *XVI International Symposium "roblems of Redundancy in Information and Control Systems"*, 87–92, Moscow, 2019.
107. A. Kirtadze, On volume type functionals in Euclidean geometry. *God. Sofiĭ. Univ. "Sv. Kliment Okhridski." Fac. Mat. Inform.* **103** (2016), 45–51.
108. A. Kirtadze, G. Pantsulaia, A. Kharazishvili's some results of on the structure of pathological functions. *Trans. A. Razmadze Math. Inst.* **174** (2020), no. 1, 1–8.
109. S. Kwapień, W. Woyczynski, *Random Series and Stochastic Integrals: Single and Multiple*. Probability and its Applications. Birkhäuser Boston, Inc., Boston, MA, 1992.
110. V. Labute, *On the Illumination of Three Dimensional Convex Bodies with Affine Plane Symmetry*. PhD diss., University of Calgary, 2015.
111. R. Latala, K. Oleszkiewicz, A note on sums of independent uniformly distributed random variables. *Colloq. Math.* **68** (1995), no. 2, 197–206.
112. M. A. Lifshits, *Gaussian Random Functions*. Mathematics and its Applications, 322. Kluwer Academic Publishers, Dordrecht, 1995.
113. S. Liu, W. Fu, R. Guo, T. Liu, *An effective teaching method of the course "Probability Theory and Mathematical Statistics" in higher education by formative evaluation*, In: *2015 International Conference on Mechatronics, Electronic, Industrial and Control Engineering (MEIC-15)*, 1088–1091, Atlantis Press, 2015.
114. A. Liu, B. Shawyer (eds), *A Taste of Mathematics*. Canadian Math. Society, Ontario, 2005.
115. A. Liu, B. Shawyer (eds), *Problems from Murray Klamkin: The Canadian Collection*. Mathematical Association of America, 2009.
116. V. V. Makeev, Intersections of affine diameters of a convex body. (Russian) *translated from Ukrain. Geom. Sb.* **33** (1990), 70–73, iii *J. Soviet Math.* 53 (1991), no. 5, 511–513.
117. F. M. Malyshev, Proofs of Brunn–Minkowski inequality by using elementary methods. (Russian) *Itogi Nauki i Techniki* **182** (2020), 70–94.
118. F. M. Malyshev, Proof of the Brunn-Minkowski theorem by Brunn cuts. (Russian) *translated from Mat. Zametki* **111** (2022), no. 1, 80–92 *Math. Notes* **111** (2022), no. 1-2, 82–92.
119. H. Martini, L. Montejano, D. Oliveros, *Bodies of Constant Width*. An introduction to convex geometry with applications. Birkhäuser/Springer, Cham, 2019.
120. H. Martini, V. Soltan, Combinatorial problems on the illumination of convex bodies. *Aequationes Math.* **57** (1999), no. 2-3, 121–152.
121. H. Martini, V. Soltan, A theorem on affine diameters of convex polytopes. *Acta Sci. Math. (Szeged)* **69** (2003), no. 1-2, 431–440.
122. J. Mioduszewski, *Topologia Przestrzeni Euklidesowych.* (Polish) WUS, Warszawa-Katowice, 1994.
123. K. Mönius, Algebraische Kurven konstanter Breite. Julius-Maximilians-Universität Würzburg, Institut fur Mathematik, January 2016, 1–38.
124. K. Nikodem, Sz. Wasowicz, *A sandwich theorem and Hyers-Ulam stability of affine functions*, Aequationes Mathematicae, v. 49, n. 1, 1995.
125. G. Nizharadze, On some geometric questions of measure and volume theory. *Proc. A. Razmadze Math. Inst.* (Russian) **85** (1987), 21–33.
126. J. Pach (eds), *New Trends in Discrete and Computational Geometry*. Springer-Verlag, Berlin, 1993.

127. V. Pambuccian, Negation-free and contradiction-free proof of the Steiner-Lehmus theorem. *Notre Dame J. Form. Log.* **59** (2018), no. 1, 75–90.
128. V. Pambuccian, Existence of special rainbow triangles in weak geometries. *Georgian Math. J.* **26** (2019), no. 4, 489–498.
129. V. Pambuccian, H. Struve, R. Struve, The Steiner-Lehmus theorem and "triangles with congruent medians are isosceles" hold in weak geometries. *Beitr. Algebra Geom.* **57** (2016), no. 2, 483–497.
130. G. Pantsulaia, On Measurability of Unions of Plane Disks. *Georg. Inter. J. Sci. Tech.* **2** (2010), no. 2, 125–129.
131. V. Pokrovskii, Sections of n-dimensional polyhedra and the Zylev-Debrunner theorem. (Russian) **85** (1987), 34–36.
132. P. B. Ramkumar, Higher dimensional image analysis using Brunn-Minkowski theorem, convexity and mathematical morphology. Journal of Information Engineering and Applications, **2** (2012), no. 4, 1–8.
133. C. A. Rogers, G. C. Shephard, The difference body of a convex body. *Arch. Math. (Basel)* **8** (1957), 220–233.
134. J. Rosen, How to tee a hyperplane. *Amer. Math. Monthly* **129** (2022), no. 8, 781–784.
135. M. G. Rozinas, A criterion for the nonlinear homogeneity of plane figures. (Russian) *Izv. Vyssh. Uchebn. Zaved. Mat.* **1986**, no. 11, 51–59, 86.
136. T. Sazhen'juk, On combinatorial properties of nets in Euclidean spaces. (Russian) In: *Soviet Geometry Conference in Honour of A. D. Alexandrov, Novosibirsk*, 1987.
137. T. Sazhen'juk, On combinatorial properties of finite sets in the Euclidean plane. (Russian) *Proc. A. Razmadze Math. Inst.* **85** (1987), 37–39.
138. T. Sazhen'juk, On some inductive constructions in combinatorial geometry. (Russian) *Proc. I. Vekua Inst. Appl. Math.* **20** (1987), 32-50.
139. P. Schmitt, Problems in discrete and combinatorial geometry. In: *Handbook of convex geometry*, vol. A, B, 449–483, North-Holland, Amsterdam, 1993.
140. R. Schneider, *Convex Bodies: the Brunn-Minkowski Theory.* Encyclopedia of Mathematics and its Applications, 44. Cambridge University Press, Cambridge, 1993.
141. V. Soltan, Affine diameters of convex-bodies—a survey. *Expo. Math.* **23** (2005), no. 1, 47–63.
142. V. Soltan, Pairs of convex bodies with centrally symmetric intersections of translates. *Discrete Comput. Geom.* **33** (2005), no. 4, 605–616.
143. V. Soltan, N. M. Hung, On the Grünbaum problem on affine diameters. (Russian) *Bull. Acad. Sci. GSSR* **132** (1988), no. 1, 33–35.
144. V. I. Subbotin, Strongly Symmetric Polyhedra. (Russian) Dr. Thesis, Novocherkassk, 2004.
145. T. Tetunashvili, On the structure of constituents of finite independent families of convex bodies in R^2 and R^3 spaces. *Trans. A. Razmadze Math. Inst.* **172** (2018), no. 1, 115–125.
146. T. Tetunashvili, On combinatorial and set-theoretical aspects of some finite and infinite point sets. *Georgian Math. J.* **26** (2019), no. 4, 583–590.
147. T. Tetunashvili, On some versions of Sylvester's problem. In: *XI International Conference of the Georgian Mathematical Union, Batumi, Book of Abstracts*, p. 170, 2021.
148. G. Toth, *Measures of Symmetry for Convex Sets and Stability.* Universitext. Springer, Cham, 2015.
149. Yu. Trokhimchuk, *Deletable Singularities of Analytic Functions.* (Russian) Naukova Dumka, Kiev, 1992.
150. K. Tschirpke, On the dissection of simplices into orthoschemes. *Geom. Dedicata* **46** (1993), no. 3, 313–329.
151. J. B. Wilker, Tiling $\mathbf{R}^3$ with circles and disks. *Geom. Dedicata* **32** (1989), no. 2, 203–209.
152. V. A. Zalgaller, G. A. Los', Solution of the Malfatti problem. (Russian) *translated from Ukrain. Geom. Sb.* **35** (1992), 14–33, 161 *J. Math. Sci.* **72** (1994), no. 4, 3163–3177.

(Received 28.05.2024)

[1]A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, 2 Merab Aleksidze II Lane, Tbilisi 0193, Georgia

[2]Department of Mathematics, Georgian Technical University, 77 Kostava Str., Tbilisi, 0160, Georgia

[3]Department of Mathematics, Georgian Technical University, 77 Kostava Str., Tbilisi 0171, Georgia

[4]I. Vekua Institute of Applied Mathematics, 2 University Str., Tbilisi 0186, Georgia
Email address: kirtadze2@yahoo.com
Email address: tengiztetunashvili@gmail.com; t.tetunashvili@gtu.ge