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THE INFLUENCE OF CHANGES ON THE RHEOLOGICAL PROPERTIES OF A GAS-LIQUID MIXTURE ON THE DYNAMICS OF ITS MOTION, TAKING INTO ACCOUNT THE HEAT EXCHANGE PROCESS

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Abstract. A model of non-stationary motion of a gas-liquid mixture in the reservoir-well system is constructed, taking into account the heat exchange process between the flow of the gas-liquid mixture in the riser pipe and its environment, and the solutions of boundary value problems are given. In the first approximation, the influence of changes of the rheological properties of the gas-liquid mixture depending on temperature on the dynamics of its motion is determined. Analytical formulas have been obtained that make it possible to determine the dynamics of pressure at the bottom of the well and the productivity of the reservoir depending on the parameters of the system. Numerical calculations are carried out for practical values of the system parameters.

1. Introduction

A change in rheological properties may have a significant impact on the dynamics of the motion of a gas-liquid mixture in the reservoir-well system. The rheological properties of a gas-liquid mixture are sensitive to changes of the temperature of its environment. When the gas-liquid mixture moves through the pipeline, heat exchange occurs between the flow of the gas-liquid mixture and its environment. This leads to a change of the viscosity and density of the gas-liquid mixture and, as a result, to a change of the dynamics of its movement. In addition, the density of the gas-liquid mixture still strongly depends on pressure, which also affects the dynamics of its movement. The issue of the influence of changes in rheological properties on the dynamics of the movement of a gas-liquid mixture is the subject of works [2, 3, 7, 10–13, 15, 16], but so far it remains poorly understood. The problem of heat transfer during the motion of a dropping viscous liquid, taking into account the dependence of the viscosity coefficient on temperature, was posed in 1922 by Acad. L. S. Leibenson [11]. The paper solves the problem of determining the change in the viscosity of a dropping viscous liquid depending on the heat exchange process during its stationary movement through the pipeline. And in this work, the problem of determining the effect of changes in the viscosity and density of the gas-liquid mixture depending on temperature on the dynamics of its movement in the reservoir-well system is solved.

2. Problem Statement and Methods for Solving it

Consider the motion of the gas-liquid mixture in the riser pipes. Due to the smallness of the liquid fraction in the mass of the gas-liquid mixture, in the first approximation, we will assume it to be homogeneous with a reduced density ρ_m. Then the equation of motion in the gas-liquid mixture of the pipe and the continuity condition, due to the assumptions made, are described by the I. A. Charny equations [1–3, 6]:

$$-\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial t} + 2aQ + \rho_m g, \quad -\frac{1}{c^2}\frac{\partial P}{\partial t} = \frac{\partial Q}{\partial x}, \quad Q = \rho_m u, \tag{2.1}$$

ρ_m is determined by the formula (2.2) [3, 11]:

$$\rho_{\text{m}} = \frac{(1+\varepsilon)\rho_{oil}\rho_g}{\rho_{\text{oil}} + \rho_g \varepsilon}, \tag{2.2}$$

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ρ_g and ρ_{oil} are, respectively, the density of the gas and the oil, c is a speed of sound propagation in mixture, t is time, x is coordinate, a is a resistance coefficient, which is determined by the formula [5]:

$$a = \frac{16\nu}{d_T^2}, \tag{2.3}$$

where ν is kinematic viscosity of the mixture, d_T is diameter of the inner wall of the pipe, ε is the mass fraction of oil in gas.

The change in the viscosity of the mixture is associated with the dynamics of its motion. However, it is known that as the temperature of the mixture increases, its viscosity decreases. To simplify the solution of the problem, in the first approximation, we assume that the change in the viscosity of the gas-liquid mixture with temperature occurs linearly,

$$\nu = \nu_{Tc} - \frac{\nu_{T0} - \nu_{Tc}}{T_c - T_0}(T - T_c), \tag{2.4}$$

where ν_{Tc} and ν_{T0} are, respectively, the kinematic viscosities of the mixture at the bottom and at the wellhead; T is temperature of the mixture in any cross-section of the column of lifting pipes; T_c and T_0-respectively, the temperatures of the mixture at the bottom and at the wellhead.

The temperature T is determined from the heat exchange process between the flow of the gas-liquid mixture and its environment during its movement through the pipeline. The process itself is non-stationary. However, in the first approximation, neglecting the inertial component of the flow, taking into account only convective heat transfer [14], placing the origin of the X-coordinate axis in the lower section of the pipe, we obtain [7, 12, 15]:

$$\frac{\partial T}{\partial x} + \beta T = \beta T_1. \tag{2.5}$$

The boundary condition

$$T|_{x=0} = T_c, \tag{2.6}$$

where

$$\beta = \frac{2\alpha}{c_m \rho_m \nu_x r_T},$$

x is coordinate, T_1 is temperature of the medium surrounding the pipe,
α is a heat transfer coefficient, r_T is radius of the inner wall of the pipe,
ν_x is averaged over the cross-section of the pipe axial velocity of the gas flow,
c_m- is a specific heat capacity of the mixture under the normal conditions.

The temperature distribution of the medium surrounding the pipeline in the vertical direction occurs linearly and has the form [7, 12]

$$T_1 = T_2 + \frac{l - x}{l}(T_3 - T_2), \tag{2.7}$$

where T_2 and T_3 are, respectively, the temperature of the medium surrounding the pipeline at the wellhead and at the bottom of the well; l is a depth of the descent of the pipe string.

Substituting expression (2.7) into equation (2.5) and integrating the resulting equation, taking into account the boundary condition (2.6), we obtain

$$T = T_c \exp(-\beta x) + T_3(1 - \exp(-\beta x)) - \frac{T_3 - T_2}{l}x + \frac{T_3 - T_2}{\beta l}(1 - \exp(-\beta x)). \tag{2.8}$$

For the values of the practical parameters of the system $\beta l << 1$, therefore $\exp(-\beta x)$ can be represented as

$$\exp(-\beta x) = 1 - \beta x.$$

Then expression (2.8) looks like

$$T = T_c \left(1 - \left(1 - \frac{T_3}{T_c}\right)\beta x\right). \tag{2.9}$$

We find the gas density in the pipe ρ_3 from the expression [3]

$$\rho_g = \frac{\rho_{atm} P T_{atm}}{P_{atm} T}, \tag{2.10}$$

where ρ_{atm} is the gas density at atmospheric pressure; P is the gas pressure;

T_{atm} is gas temperature at atmospheric pressure; T is temperature of the gas in any cross-section of the pipe.

Substituting expression (2.9) into formula (2.10), we obtain

$$\rho_g = \frac{\rho_{atm} P T_{atm}}{P_{atm} T_c \left(1 - \left(1 - \frac{T_3}{T_c}\right)\beta\, x\right)}. \tag{2.11}$$

Moreover, always, for practical values of the system parameters, $\left(1 - \frac{T_3}{T_c}\right)\beta\, l << 1$.
Therefore $\frac{1}{\left(1-\left(1-\frac{T_3}{T_c}\right)\beta\, x\right)}$ can be represented as a series

$$\frac{1}{\left(1 - \left(1 - \frac{T_3}{T_c}\right)\beta\, x\right)} = 1 + \left(1 - \frac{T_3}{T_c}\right)\beta\, x + \left(1 - \frac{T_3}{T_c}\right)^2 \beta^2\, x^2 + \cdots.$$

Then, in the first approximation, taking into account only one member of this series, from expression (2.11) we obtain

$$\rho_g = \frac{\rho_{atm} P T_{atm}}{P_{atm} T_c} \left(1 + \left(1 - \frac{T_3}{T_c}\right)\beta\, x\right). \tag{2.12}$$

Substituting expression (2.9) into formula (2.4), and then the resulting expression into formula (2.3), we obtain

$$a = \alpha_0 + \beta_0 x, \tag{2.13}$$

where

$$\alpha_0 = \frac{16 v_{T0}}{d_T^2}, \quad \beta_0 = \frac{16}{d_T^2} \frac{\nu_{T0} - \nu_{Tc}}{T_c - T_0} T_c \left(1 - \frac{T_3}{T_c}\right)\beta.$$

Substituting expression (2.12) into formula (2.2), after some transformations, we obtain

$$\rho_m = d_2 P(x)\,(1 + d_1 x)\left[1 - d_3 P(x)\,(1 + d_1 x)\right], \tag{2.14}$$

where $P(x)$ is pressure distribution in the pipeline at stationary mode,

$$d_1 = \left(1 - \frac{T_3}{T_c}\right)\beta, \qquad d_2 = (1 + \varepsilon)\frac{\rho_{atm} T_{atm}}{P_{atm} T_c}, \qquad d_3 = \frac{\varepsilon \rho_{atm} T_{atm}}{\rho_{oil} P_{atm} T_c}.$$

Substituting expressions (2.13) and (2.14) into equation (2.1) and then differentiating the first equation with respect to the $\tilde{o}$-coordinate, and the second with respect to time t, and subtracting one from the other, we obtain

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x^2} + 2c^2 \beta_0 Q_0 - 2ac^2 \frac{\partial P}{\partial t} + \frac{\partial \rho_m}{\partial x} g. \tag{2.15}$$

The initial and boundary conditions

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = 0, \quad 0 < x \le l, \tag{2.16}$$

$$P(x,0)|_{t=0} = f(x), \quad 0 < x \le l, \tag{2.17}$$

$$P|_{x=0} = P_c(t), \quad t > 0, \tag{2.18}$$

$$P|_{x=l} = P_y(t), \quad t > 0. \tag{2.19}$$

The solution of equation (2.15), satisfying the boundary conditions (2.18) and (2.19), will be sought in the form

$$P = P_c(t) - \frac{P_c(t) - P_y(t)}{l} x + \sum_{i=1}^{\infty} \sin\frac{i\pi x}{l} \varphi_i\,(t). \tag{2.20}$$

Substituting expression (2.20) into equation (2.15), multiplying both parts of the resulting equation by $\sin\left(\frac{i\pi x}{l}\right)$ and integrating it from 0 to l, for $i = 1$ [4,8,9], we get:

$$\ddot{\varphi} + c_2\dot{\varphi} + c_3\varphi = a_5\ddot{P}_c + a_6\ddot{P}_y + a_3\dot{P}_c + a_4\dot{P}_y + a_1 P_c + a_2 P_y + a_7, \tag{2.21}$$

where

$$c_2 = 2\alpha_0 + \beta_0 l, c_3 = -\frac{c^2(gl^2d_1d_2 - 2\pi^2)}{2l^2},$$

$$a_1 = \frac{4gc^2d_2\big[((d_1{}^2l^2 + d_1l + 2)\pi^2 - 8d_1{}^2l^2)d_3P_k - \pi^2\big]}{l\pi^3},$$

$$a_2 = -\frac{8gc^2d_2\big[((d_1{}^2l^2 + \frac{3}{2}d_1l + 1)\pi^2 - 4d_1{}^2l^2)d_3P_k - \frac{\pi^2}{2}(d_1l + 1)\big]}{l\pi^3},$$

$$a_3 = -\frac{4\alpha_0}{\pi} - \frac{8\beta_0 l}{\pi^3}, \quad a_4 = \frac{16\beta_0 l}{\pi^3} - \frac{4\alpha_0}{\pi} - \frac{4\beta_0 l}{\pi}, \quad a_5 = -\frac{2}{\pi}, \quad a_6 = -\frac{2}{\pi},$$

$$a_7 = \frac{8c^2\beta_0 Q_0}{\pi}.$$

Applying the Laplace transform from equation (2.21), we obtain

$$\bar{\varphi} = \frac{\varphi_0(s + c_2) + \dot{\varphi}_0}{s^2 + c_2 s + c_3} - \frac{(a_5 s^2 + a_3 s + a_1)\bar{P}_c - (a_5 s + a_3)P_{c0} - a_5\dot{P}_{c0}}{s^2 + c_2 s + c_3}$$
$$-\frac{(a_6 s^2 + a_4 s + a_2)\bar{P}_y - (a_4 s + a_6)P_{y0} - a_6\dot{P}_{y0}}{s^2 + c_2 s + c_3} - \frac{a_7}{s(s^2 + c_2 s + c_3)}. \tag{2.22}$$

The initial values of the function φ_0 and its derivative $\dot{\varphi}_0$ will be determined from the initial conditions (2.16) and (2.17).

At the initial moment, the well operates in a stationary mode. Therefore from equation (2.1), taking into account formulas (2.13) and (2.14), we have

$$-\frac{\partial P(x)}{\partial x} = 2aQ_0 + gd_2P(x)\ (1 + d_1\,x)\left[1 - d_3P(x)\ (1 + d_1\,x)\right]. \tag{2.23}$$

The boundary condition

$$P|_{x=0} = P_{c0}. \tag{2.24}$$

In formula (2.23), following L.S. Leibenson [11], for the linearization, it is assumed that $P^2(x) = BP(x)$. As a first approximation, $B = P_k$. Integrating expression (2.23), we obtain

$$P_{x0} = -2Q_0\left(\int a\exp\Big(-\frac{1}{6}gd_2x(2Bd_1{}^2d_3x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6)\Big)dx + P_{c0}\right)$$
$$\times\exp\Big(\frac{1}{6}gd_2x(2Bd_1{}^2d_3x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6)\Big). \tag{2.25}$$

For practical values of the system parameters (even when $x = l$), the expression

$$-\frac{1}{6}gd_2x(2Bd_1{}^2d_3x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6) << 1.$$

Therefore, expanding

$$\exp\Big(-\frac{1}{6}gd_2x(2Bd_1{}^2d_3x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6)\Big)$$

in a series and taking into account only the first term, we obtain

$$\exp\Big(-\frac{1}{6}gd_2x(2Bd_1{}^2d_3x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6)\Big)$$
$$\approx 1 - \frac{1}{6}gd_2x(2Bd_1{}^2d_3x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6). \tag{2.26}$$

Then from expression (2.25), taking into account the boundary condition (2.24) and formula (2.26), we obtain

$$P_x|_{t=0} = \big(-2Q_0(b_1x^5 + b_2x^4 + b_3x^3 + b_4x^2 + \alpha_0x) + P_{c0}\big)b_5, \tag{2.27}$$

where

$$b_1 = -\frac{1}{15} gB{d_1}^2 d_2 d_3 \beta_0,$$

$$b_2 = -\frac{1}{24} gd_2\big(2B{d_1}^2 d_3\alpha_0 + (6Bd_1d_3 - 3d_1)\beta_0\big),$$

$$b_3 = -\frac{1}{6} gd_2\big((2Bd_3 - 1)d_1\alpha_0 + 2(Bd_3 - 1)\beta_0\big),$$

$$b_4 = -\frac{1}{2}\big(gd_2(Bd_3 - 1)\alpha_0 - \beta_0\big),$$

$$b_5 = 1 + \frac{1}{6} gd_2 x(2B{d_1}^2 d_3 x^2 + 6Bd_1d_3x + 6Bd_3 - 3d_1x - 6).$$

From expression (2.20) for $t = 0$, we obtain

$$P_x\,|_{t=0} = P_{c0} - \frac{P_{c0} - P_{y0}}{l} x + \sum_{i=1}^{\infty} \varphi_{i0} \Big(\sin\frac{i\pi x}{l}\Big). \tag{2.28}$$

Equating expressions (2.27) and (2.28) we find that

$$\sum_{i=1}^{\infty} \varphi_{i0} \Big(\sin\frac{i\pi x}{l}\Big) = \big(-2Q_0(b_1x^5 + b_2x^4 + b_3x^3 + b_4x^2 + \alpha_0 x) + P_{c0}\big)b_5 + \frac{P_{c0} - P_{y0}}{l} x - P_{c0}. \tag{2.29}$$

Multiplying both parts of expression (2.29) by $\big(\sin\frac{i\pi x}{l}\big)$ and integrating from 0 to l taking into account, we get only one member of the series

$$\varphi_0 = w_1 l^8 + w_2 l^7 + w_3 l^6 + w_4 l^5 + w_5 l^4 + w_6 l^3 + w_7 l^2 + w_8 l + w_9, \tag{2.30}$$

where

$$w_1 = -\frac{4}{3\pi^9} BQ_0 b_1 {d_1}^2 d_2 d_3 g(\pi^8 - 56\pi^6 + 1680\pi^4 - 20160\pi^2 + 80640),$$

$$w_2 = -\frac{4}{3\pi^7} Q_0 d_1 d_2 g(\pi^6 - 42\pi^4 + 840\pi^2 - 5040)\Big(Bb_2d_1d_3 + 3Bb_1d_3 - \frac{3}{2}b_1\Big),$$

$$w_3 = -\frac{4}{\pi^7} Q_0 d_2 g(\pi^6 - 30\pi^4 + 360\pi^2 - 1440)\Big(\frac{1}{3}Bb_3{d_1}^2d_3 + b_2d_1\Big(Bd_3 - \frac{1}{2}\Big) + b_1(Bd_3 - 1)\Big),$$

$$w_4 = -\frac{4}{\pi^5} Q_0(\pi^4 - 20\pi^2 + 120)\Big(b_1 + g\Big(\frac{1}{3}Bb_4{d_1}^2d_3 + b_3d_1\Big(Bd_3 - \frac{1}{2}\Big) + b_2d_2(Bd_3 - 1)\Big)\Big),$$

$$w_5 = -\frac{4}{\pi^5} Q_0(\pi^4 - 12\pi^2 + 48)\Big(b_2 + g\Big(\frac{1}{6}B{d_1}^2d_3\alpha_0 + b_4d_1\Big(Bd_3 - \frac{1}{2}\Big) + b_3d_2(Bd_3 - 1)\Big)\Big),$$

$$w_6 = -\frac{4}{\pi^3}(\pi^2 - 6)\Big(Q_0\Big(b_3 + gd_2\Big(\Big(\frac{1}{2}Bd_3\alpha_0 - \frac{1}{4}\alpha_0\Big)d_1 + b_4d_2(Bd_3 - 1)\Big)\Big) - \frac{1}{6}BP_{c0}{d_1}^2d_2d_3\Big),$$

$$w_7 = -\frac{2}{\pi^3}(\pi^2 - 4)\Big(2Q_0b_4 - d_2g\Big(P_{c0}d_1\Big(Bd_3 - \frac{1}{2}\Big) - Q_0\alpha_0(Bd_3 - 1)\Big)\Big),$$

$$w_8 = -\frac{2}{\pi}(Q_0\alpha_0 - P_{c0}d_2g(Bd_3 - 1)),$$

$$w_9 = -\frac{2}{\pi}(P_{y0} - P_{c0}).$$

To determine $\dot{\varphi}_0$, we differentiate expression (2.20) with respect to time t,

$$\dot{P} = \dot{P}_c(t) - \frac{\dot{P}_c(t) - \dot{P}_y(t)}{l} x + \sum_{i=1}^{\infty} \dot{\varphi}_i\,(t)\,\Big(\sin\frac{i\pi x}{l}\Big).$$

From expression (2.30), taking into account the initial condition (2.16), we obtain

$$\dot{\varphi}_0 = 0.$$

Let us determine the mass flow rate of the mixture coming per unit area of the flow section of the pipe. Towards this end, substituting expressions (2.13) and (2.14) into the first equation of expression

(2.1), we obtain

$$-\frac{\partial P}{\partial x}=\frac{\partial Q}{\partial t}+2aQ+gd_2P(x)\ (1+d_1x)\left[1-d_3P(x)\ (1+d_1\,x)\right]. \tag{2.31}$$

Differentiating equation (2.20) with respect to the X-coordinate and substituting it into equation (2.31), applying the Laplace transform, we have

$$\bar{Q}=\frac{Q_0}{s+2a}+\frac{\bar{P}_c}{l(s+2a)}-\frac{\bar{P}_y}{l(s+2a)}-\frac{\pi\bar{\varphi}\cos\left(\frac{\pi x}{l}\right)}{l(s+2a)}$$
$$-\frac{gd_2P(x)\ (1+d_1x)\left[1-d_3P(x)\ (1+d_1\,x)\right]}{l(s+2a)}.$$

$$\bar{Q}|_{x=0}=\frac{Q_0}{s+2\alpha_0}+\frac{\bar{P}_c-\bar{P}_y}{l(s+2\alpha_0)}+\frac{gd_2(Bd_3-1)}{(s+2\alpha_0)}\,\bar{P}_c-\frac{\pi}{l(s+2\alpha_0)}\bar{\varphi}. \tag{2.32}$$

The mass flow of the mixture into the well per unit time can be determined from the expression [3]

$$\bar{Q}_m=\frac{G_0}{(s+\alpha)}-B_1\,\frac{s\bar{P}_c-P_{c0}}{s+\alpha}, \tag{2.33}$$

where G_0 is the initial mass flow rate of the mixture, $B_1=kh\pi\frac{P_{c0}+P_k}{D\,\mu_m\,\beta_1}$.

Then, based on the continuity condition, we have

$$\bar{Q}_m=\left.f\,\bar{Q}\right|_{x=0}. \tag{2.34}$$

Substituting expressions (2.32) and (2.33) into formula (2.34), we obtain the following equation:

$$\frac{G_0}{f(s+\alpha)}-B_1\frac{s\bar{P}_c-P_{c0}}{f(s+\alpha)}=\frac{Q_0}{s+2\alpha_0}+\frac{\bar{P}_c-\bar{P}_y}{l(s+2\alpha_0)}-\frac{\pi\ \bar{\varphi}}{l(s+2\alpha_0)}+\frac{d_2g(Bd_3-1)\bar{P}_c}{(s+2\alpha_0)} \tag{2.35}$$

which allows us to determine $P_c(t)$. Substituting expressions (2.22) into formula (2.35), we obtain $\bar{P}_c$:

$$\bar{P}_c=\frac{P_{c1}+P_{c2}+P_{c3}+P_{c4}}{(s-q_1)(s-q_2)(s-q_3)(s-q_4)}, \tag{2.36}$$

where

$$P_{c1}=(s+\alpha)\big((s^2+c_2s+c_3)-\pi(a_6s^2+a_4s+a_2)\big)\bar{P}_y,$$
$$P_{c2}=\big(B_1l(s+2\alpha_0)(s^2+c_2s+c_3)+\pi(s+\alpha)(a_5s+a_3)\big)P_{c0},$$
$$P_{c3}=\pi(s+\alpha)\Big((a_4s+a_6)P_{y0}+a_6\dot{P}_{y0}+a_5\dot{P}_{c0}+\varphi_0(s+c_2)+\dot{\varphi}_0-\frac{a_7}{s}\Big),$$
$$P_{c4}=l(s^2+c_2s+c_3)\big((s+2\alpha_0)G_0-(s+\alpha)Q_0\big),$$

q_1, q_2, q_3, q_4 are the roots of the equation

$$(s^2+c_2s+c_3)((s+\alpha)(ld_2g(Bd_3-1)+1)+B_1l(s+2\alpha_0))+(a_5s^2+a_3s+a_1)(s+\alpha)\pi=0.$$

Applying the Laplace transform and taking into account the convolution and inversion theorems, from expressions (2.33) and (2.36), taking into account the practical values of the system parameters

$$r_c=0.075\ m;\quad \rho=0.668kg/m^3;\quad P_{atm}=10^5Pa;\quad \pi=3.14;\quad h=10m;\quad P_{c0}=2.7\cdot10^7Pa;$$
$$P_k=3\cdot10^7Pa;\quad P_{cT}=10^7Pa;\quad P_{yT}=8\cdot10^6Pa;\quad P_{y0}=2.3\cdot10^7Pa;\quad \mu=1.2\cdot10^{-5}Pa\cdot s;$$
$$f=\pi r_T^2;\quad a=10^{-2}c^{-1};\quad m=0.2;\quad T_n=180\,day;\quad k=10^{-13}m^2;\quad T_2=10^0C;\quad T_3=80^0C;$$
$$T_c=100^0C;\quad m=0.2;\quad c=300\frac{m}{s};\quad T_n=180\,day;\quad T_{atm}=20^0C;$$
$$\nu_0=5.28\cdot10^{-6}m^2/s;\quad \nu_T=1.76\cdot10^{-6}m^2/s;\quad r_T=3\cdot10^{-2}m;\quad R_k=200m;$$
$$\alpha=8.33\cdot10^{-4}\frac{kkal}{m^2{}^0C};\quad \beta=\frac{2\alpha}{c_g\rho_g r_T u_x};\quad l=2300m;\quad G_0=5.052kg/s$$

we get the values for P_c and Q_m.

The calculation results are presented in Figures 1, 2 and 3. Figure 1 shows the dynamics of pressure changes at the bottom of the well for small time values. It can be seen that in the initial period, the pressure at the bottom of the pulsating well drops. Further, as the bottomhole pressure stabilizes, it almost linearly participates in a stationary association (Figure 1 and Figure 2). Figure 3 shows the dynamics of well productivity.

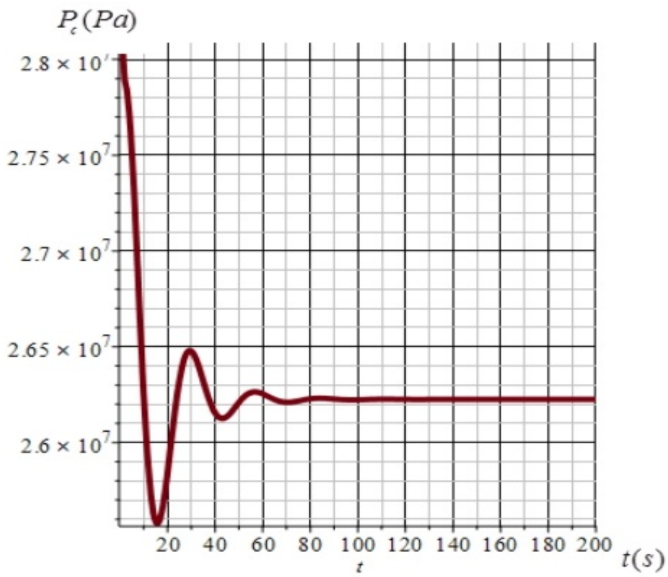

FIGURE 1. Dynamics of the change of pressure at the bottom of the well for small time values.

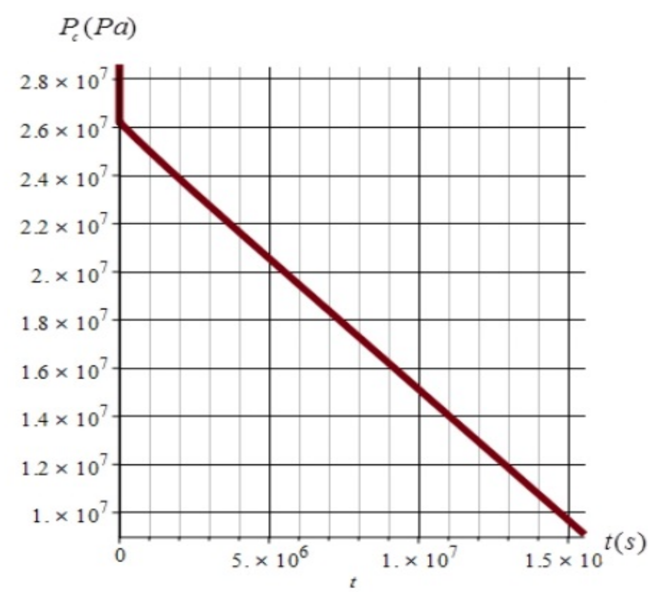

FIGURE 2. Dynamics of pressure changes at the bottom of the well.

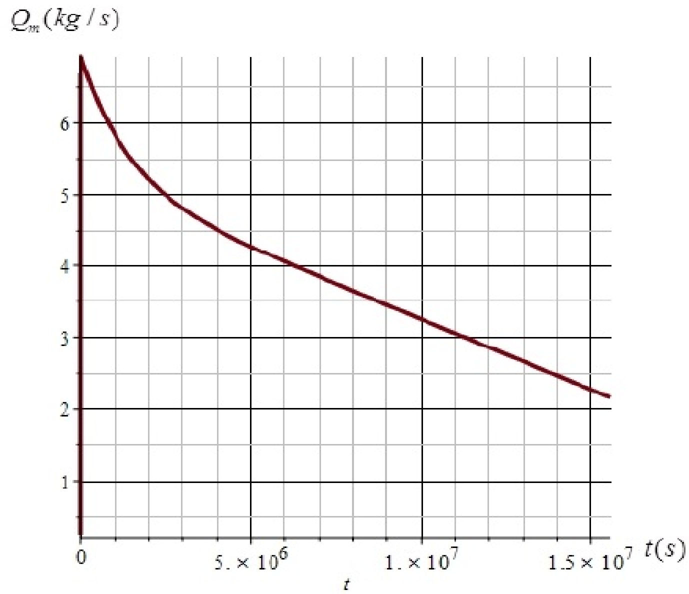

Figure 3. Dynamics of well productivity.

3. Conclusion

A model of the motion of a gas-liquid mixture in the reservoir-well system has been built, taking into account the heat exchange process between the flow of the mixture in the pipe and its environment. The boundary value problem is solved and the influence of the heat exchange process on the performance of the well is determined.

Analytical expressions have been obtained to determine the pressure dynamics at the bottom of the well, which in turn allows to determine the volume of the gas-liquid mixture flowing through any cross-section of the lifting pipes per unit time, and numerical calculations have been carried out for practical system parameters.

Denotation

P is pressure at any point of the formation; $P_c(t)$ is pressure at the bottom of the well;
P_k is pressure on the reservoir contour; ρ_m is density of the mixture of oil and gas;
r is coordinate; h is formation thickness; m is reservoir porosity coefficient;
ρ_{oil} is density of oil; ρ_g is gas density; P_{atm} is atmosphere pressure;
ρ_{atm} is density of the gas at atmospheric pressure; ε is mass fraction of oil in gas;
μ_m is viscosity of the mixture, k is formation permeability coefficient;
P_{c0}, P_{cT} is pressure at the bottom of the well at the beginning and end of operation;
c is speed of sound propagation in gas, t is time, x is coordinate, a is drag coefficient, $P_y(t)$ is pressure at the wellhead, f is area of the pipe flow area, $\varphi_i(t)$ is unknown, time-dependent function, l is depth of descent of the pipe.

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