

DISCRETE INTERACTION OF AN ELASTIC WEDGE-SHAPED PLATE WITH AN ELASTIC STRINGER

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Abstract. An elastic isotropic plate in the form of an angle or a half-plane is considered. One side of the angle is free from external stresses and the other side is discretely reinforced with a straight stringer by rivets which are spaced at a constant intervals. Concentrated force is applied to the end of the stringer along its axis. Using the methods of the theory of analytic functions and integral transformations, the problem reduces to an infinite system of linear algebraic equations. The quasi-regularity of this system in a space of quadratically summable sequences is proved. In the case of a half-plane, we get a system of equations with difference indices (of convolution type) which by using the discrete Fourier transformation reduces to the Riemann problem for a circle in a class of Wiener functions.

INTRODUCTION

The contact problems for various areas reinforced with elastic thin-walled elements of variable stiffness are investigated and asymptotic elements of contact stresses at the ends of the contact line are obtained depending on the law of variation of geometric and physical parameters of these elements. These problems were preceded by the studies of such authors as E. Melan, E. Reisner, V.T. Koiter, E.L. Buell, E.V. Benskoter, R. Myki, E. Stenberg, etc. Continuous and discrete interaction, as well as adhesive contact of thin-walled elements (stringers and inclusions) with massive deformable bodies are allowed.

In this paper, the elastic isotropic plate in the form of an angle or a half-plane is considered, when one side of the angle is free from external stresses and the other side is reinforced with a straight stringer in the condition of a discrete interaction.

I. An elastic isotropic plate on the plane $z = x + iy$ occupies the angle $-\alpha < \arg z < 0$, $0 < \alpha < 2\pi$. One side of the angle $z = -\alpha$ is free from external stresses and the other side, $\arg z = 0$, is reinforced with a straight stringer of constant cross-section F_0 , with the Young modulus E_0 and thickness h_0 . Interaction between the stringer and the wedge is realised discretely, through the rivets which are spaced by the law $x_k = k$, $k = 1, 2, \dots$. The end of the stringer at the point $x = 0$ is under the action of concentrated force of intensity P , directed along the Ox -axis.

We admit the following assumptions: 1) there is no friction force between the plate and stringer; 2) the effect of essentric (with respect to the midplane of the plate) attachment of the stringer is neglected; 3) a plane strassed state is realized in the plate, and the rivets in the plate are simulated by circular rigid inclusions; 4) the stringer works only in a tension-compression regime and its weakening due to the applied rivets is not taken into account, that is, we assume that the stringer bending stiffness is negligibly small [5].

Separate mentally the stringer from the plate and apply unknown interaction forces X_k and $-X_k$ to the centers of rivets x_k of the plate and stringer, respectively. Let N_j be the stress in the bar acting on the segment between the rivets j and $j + 1$, $j = 1, 2, 3, \dots$

From the equilibrium of that part of the stringer which lies in the vicinity of rivets it follows that

$$X_k = N_k - N_{k-1}, \quad k = 1, 2, 3, \dots \quad (1)$$

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This, in particular, implies that

$$N_k = \sum_{j=1}^k X_j - \frac{P}{h_0}, \quad N_0 = -\frac{P}{h_0}$$

The problem consists in finding the unknown concentrated forces X_k ($k = 1, 2, \dots$) and an elastic equilibrium in the plate.

Using Kolosov-Muskhelishvili's formulas, the above stated problem reduces to finding the two functions $\Phi(z)$ and $\Psi(z)$ of complex variables, the so-called complex potentials, holomorphic in the angle $-\alpha < \arg z < 0$, by the following boundary conditions [3]:

$$\Phi(t) + \overline{\Phi(t)} + t\overline{\Phi'(t)} + \overline{\Psi(t)} = -i \sum_{k=1}^{\infty} X_k \delta(t-k), \quad t > 0, \quad (2)$$

$$\Phi(t) + \overline{\Phi(t)} + e^{2i\alpha} \left[t\overline{\Phi'(t)} + \overline{\Psi(t)} \right] = 0, \quad \arg t = -\alpha. \quad (3)$$

where $\delta(x)$ is the Dirichlet function.

The conditions of compatibility of mutual shifts of adjacent rivets in the plate and stringer are written now as follows:

$$\operatorname{Re} [w(x_{j+1}) - w(x_j)] = \frac{N_j}{E_0 F_0}, \quad j = 1, 2, \dots, \quad (4)$$

$$2\mu h w(z) = \varkappa \varphi(z) - z \overline{\varphi'(z)} - \overline{\psi(z)},$$

where $w(z) = u(z) + iv(z)$ is a complex displacement vector, h is the plate thickness, μ is the Lamé constant of the plate material, $\varphi(z)$ and $\psi(z)$ are complex potentials

$$\varphi'(z) = \Phi(z), \quad \psi'(z) = \Psi(z).$$

The formulas

$$2\mu h (u'(t) + iv'(t)) = \varkappa \Phi(t) - \left[\overline{\Phi(t)} + t\overline{\Phi'(t)} + \overline{\Psi(t)} \right],$$

$$Y_y - iX_y = \Phi(t) + \left[\overline{\Phi(t)} + t\overline{\Phi'(t)} + \overline{\Psi(t)} \right]$$

result in

$$2\mu h (u' + iv') + Y_y - iX_y = (\varkappa + 1)\Phi(t).$$

In our statement of the problem, on the boundary of a half-plane we have $Y_y = 0$, therefore we have

$$u'(t) = \frac{\varkappa + 1}{4\mu h} \left[\Phi(t) + \overline{\Phi(t)} \right].$$

Then condition (4) takes the form

$$\frac{\varkappa + 1}{4\mu h} \int_k^{k+1} \left[\Phi(t) + \overline{\Phi(t)} \right] dt = \frac{N_k}{E_0 F_0}, \quad k = 1, 2, \dots \quad (5)$$

Introduce the notation

$$\Psi_1(z) = \Phi(z) + z\Phi'(z) + \Psi(z), \quad -\alpha < \arg z < 0,$$

then formulas (2) and (3) can be rewritten as follows:

$$\Phi(t) + \overline{\Psi_1(t)} = -i \sum_{j=1}^{\infty} X_j \delta(t-j), \quad t > 0, \quad (6)$$

$$\Phi(t) + (1 + e^{2i\alpha}) \left(t\overline{\Phi(t)} \right)' - e^{2i\alpha} \overline{\Psi_1(t)} = 0, \quad \arg t = -\alpha. \quad (7)$$

The functions $\Phi(z)$ and $\Psi_1(z)$ are required to have for large $|z|$ the form

$$\Phi(z) = O\left(\frac{1}{z}\right), \quad \Psi_1(z) = O\left(\frac{1}{z}\right),$$

and in the neighbourhood of angle vertices to satisfy the conditions

$$z\Phi(z) \rightarrow 0, \quad z\Psi_1(z) \rightarrow 0, \quad z \rightarrow 0.$$

Analytic functions $\Phi(z)$ and $\Psi_1(z)$ in the domain $-\alpha < \arg z < 0$ will be sought in the form

$$\Phi(z) = \frac{1}{\sqrt{2\pi z}} \int_{-\infty}^{\infty} \frac{A_1(t)}{t} e^{-it \ln z} dt - \frac{c_1}{z}, \quad -\alpha < \arg z < 0, \quad (8)$$

$$\Psi_1(z) = \frac{1}{\sqrt{2\pi z}} \int_{-\infty}^{\infty} \frac{A_2(t)}{t} e^{-it \ln z} dt - \frac{c_2}{z}, \quad -\alpha < \arg z < 0, \quad (9)$$

where

$$\begin{aligned} c_k &= \lim_{z \rightarrow 0} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{A_k(t)}{t} e^{-it \ln z} dt \\ &= \lim_{z \rightarrow 0} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{A_k(t) - A_k(0)}{t} e^{-it \ln z} dt + \frac{A_k(0)}{\sqrt{2\pi}} \lim_{z \rightarrow 0} \int_{-\infty}^{\infty} \frac{e^{-it \ln z}}{t} dt = i\sqrt{\frac{\pi}{2}} A_k(0). \end{aligned} \quad (10)$$

(By virtue of the Riemann–Lebesgue theorem, the first integral, being the Fourier transform of the integrable function, tends to zero, and the second limit is calculated by means of the following integral:

$$\int_{-\infty}^{\infty} \frac{e^{ixt} dx}{x} = \pi i \operatorname{sgn} t).$$

In formulas (8) and (9), the integrals at the point $t = 0$ are understood in the sense of the Cauchy principal value [3].

Analogously to our previous discussion, it is proved that

$$\lim_{z \rightarrow \infty} z\Phi(z) = -2c_1, \quad \lim_{z \rightarrow \infty} z\Psi_1(z) = -2c_2.$$

We require that $t\Phi(t) + \overline{t\Psi_1(t)} \rightarrow 0$ as $t \rightarrow \infty$, consequently, $c_1 + \overline{c_2} = 0$, i.e., $A_1(0) = \overline{A_2(0)}$.

If we insert the values of (8), (9) into formulas (6) and (7) and perform the Fourier transformation [1], we obtain

$$\begin{aligned} A_1(t) - \overline{A_2(-t)} &= -itT(t), \\ e^{-\alpha t} A_1(t) - i(1 - e^{2i\alpha}) t e^{\alpha t} \overline{A_1(-t)} - e^{\alpha t} \overline{A_2(-t)} &= 0, \end{aligned} \quad (11)$$

where

$$T(t) = \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{\infty} \int_{-\infty}^{\infty} e^s X_j \delta(e^s - j) e^{ist} dt = \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{\infty} X_j e^{it \ln j}. \quad (12)$$

Taking into account the fact that $T(t) = \overline{T(-t)}$, the solution of system (11) takes the form

$$A_1(t) = -\frac{e^{2\alpha t} - 1 - 2te^{i\alpha} \sin \alpha}{4(sh^2 \alpha t - t^2 \sin^2 \alpha)} itT(t), \quad (13)$$

$$A_2(t) = \overline{A_1(-t)} + itT(t). \quad (14)$$

Substituting the (8), (9) formulas into condition (5), we obtain

$$\frac{\varkappa + 1}{4\mu h} \int_k^{k+1} \left[\frac{1}{\sqrt{2\pi x}} \int_{-\infty}^{\infty} \frac{A_1(t) e^{-it \ln x} + \overline{A_1(t)} e^{it \ln x}}{t} dt \right] dx - \frac{(\varkappa + 1) \ln(1 + k^{-1})}{4\mu h} (c_1 + \overline{c_1}) = \frac{N_k}{E_0 F_0}.$$

In view of (1) and (13), we have

$$\begin{aligned} \frac{\varkappa + 1}{4\mu h} \int_{-\infty}^{\infty} \frac{sh2\alpha\sigma - \sigma \sin 2\alpha}{sh^2\alpha\sigma - \sigma^2 \sin^2\alpha} \frac{T(\sigma)}{\sigma} (e^{-i\sigma \ln(k+1)} - e^{-i\sigma \ln k}) d\sigma - \frac{(\varkappa + 1) \ln(1 + k^{-1})}{2\mu h} \operatorname{Re} c_1 \\ = \frac{1}{E_0 F_0} \left(\sum_{j=1}^k X_j - \frac{P}{h_0} \right), \quad (15) \\ c_1 = i\sqrt{\frac{\pi}{2}} A_1(0) = \frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{\alpha - e^{i\alpha} \sin \alpha}{\alpha^2 - \sin^2 \alpha} T(0). \end{aligned}$$

From (12) follows $T(0) = \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{\infty} X_j = \frac{1}{\sqrt{2\pi}} \frac{P}{h_0}$ and, thus $\operatorname{Re} c_1 = \frac{2\alpha - \sin 2\alpha}{2(\alpha^2 - \sin^2 \alpha)} \frac{P}{4h_0}$.

Introducing the notations

$$\begin{aligned} G_k(t) &= \frac{sh2\alpha t - t \sin 2\alpha}{sh^2\alpha t - t^2 \sin^2 \alpha} \frac{e^{-it \ln(k+1)} - e^{-it \ln k}}{t} \equiv G(t) \left[e^{-it \ln(k+1)} - e^{-it \ln k} \right], \quad \omega_0 = \frac{(\varkappa + 1) E_0 F_0}{4\mu h}, \\ \varphi_k &= \frac{(\varkappa + 1) E_0 F_0}{2\mu h} \ln(1 + k^{-1}) \operatorname{Re} c_1, \end{aligned}$$

equation (15) takes the form

$$\omega_0 \int_{-\infty}^{\infty} G_k(t) T(t) dt = \left(\sum_{j=1}^k X_j - \frac{P}{h_0} \right) + \varphi_k, \quad k = 1, 2, \dots \quad (16)$$

Taking now into account the relations (1), (12) and substituting the last expression into (16), after certain calculations

$$\begin{aligned} \int_{-\infty}^{\infty} G_k(t) \left(\sum_{j=1}^{\infty} X_j e^{it \ln j} \right) dt &= \int_{-\infty}^{\infty} G_k(t) \left(\sum_{j=1}^{\infty} (N_j - N_{j-1}) e^{it \ln j} \right) dt \\ &= \int_{-\infty}^{\infty} G_k(t) \sum_{j=1}^{\infty} N_j \left(e^{it \ln j} - e^{it \ln(j+i)} \right) dt \\ -N_0 \int_{-\infty}^{\infty} G_k(t) dt &= \int_{-\infty}^{\infty} G(t) \left(e^{-it \ln(k+1)} - e^{-it \ln k} \right) \sum_{j=1}^{\infty} N_j \left(e^{it \ln j} - e^{it \ln(j+i)} \right) dt - N_0 \int_{-\infty}^{\infty} G_k(t) dt \\ &= \int_{-\infty}^{\infty} G(t) \left(\sum_{j=1}^{\infty} R_{kj}(t) N_j \right) dt - N_0 \int_{-\infty}^{\infty} G_k(t) dt, \end{aligned}$$

we arrive at

$$\int_{-\infty}^{\infty} G(t) \left(\sum_{j=1}^{\infty} R_{kj}(t) N_j \right) dt = \omega_0 N_0 \int_{-\infty}^{\infty} G_k(t) dt + N_k + \varphi_k, \quad k = 1, 2, \dots, \quad (17)$$

where

$$R_{kj}(t) = \frac{\omega_0}{\sqrt{2\pi}} \left(\frac{j}{k} \right)^{it} \left[(1 + k^{-1})^{-it} - 1 \right] \left[1 - (1 + j^{-1})^{it} \right]. \quad (18)$$

In the left-hand side of (18), the order of integration and summation can be changed, since the series $\sum_{j=1}^{\infty} R_{kj} b_j$ converges uniformly and its sum yields a continuous function on $(-\infty, \infty)$, hence

$$\int_{-\infty}^{\infty} G(t) \left(\sum_{j=1}^{\infty} R_{kj}(t) N_j \right) dt = \sum_{j=1}^{\infty} \Gamma_{kj} N_j,$$

where

$$\Gamma_{kj} = \int_{-\infty}^{\infty} G(t)R_{k,j}(t)dt.$$

Then expression (17) takes the form

$$\sum_{j=1}^{\infty} \Gamma_{kj}N_j = N_k + \tilde{\varphi}_k, \quad k = 1, 2, 3, \dots, \tag{19}$$

where

$$\tilde{\varphi}_k = \varphi_k + \omega_0 N_0 \int_{-\infty}^{\infty} G_k(t)dt. \tag{20}$$

In formula (20), the integral at the point $t = 0$ is understood in the sense of the Cauchy principal value. Thus we have obtained the infinite system of linear algebraic equations of type (19), where $\Gamma = \{\Gamma_k\}_1^{\infty}$, $\tilde{\varphi} = \{\tilde{\varphi}_k\}_1^{\infty}$ are the given and $N = \{N_k\}_1^{\infty}$ is an unknown vector from the space of quadratically summable sequences l_2 .

We now investigate system (19) for its regularity. It follows from expressions (18) and (20) that

$$R_{k,j}(t) = \begin{cases} O(k^{-1}), & k \rightarrow \infty, \\ O(j^{-1}), & j \rightarrow \infty, \end{cases} \quad \tilde{\varphi}_k = O(k^{-1}), \quad k \rightarrow \infty$$

and

$$\sum_{k,j=1}^{\infty} \Gamma_{kj}^2 < \infty, \quad \sum_{k=1}^{\infty} \tilde{\varphi}_k^2 < \infty \tag{21}$$

respectively.

The investigation of system (19) under condition (21) in the class l_2 (i.e., $\sum_{k=1}^{\infty} N_k^2 < \infty$) is reduced to that of a finite system of linear algebraic equations.

If a homogeneous system corresponding to (19) has in l_2 a unique solution (zero, obviously), then the given system has a unique solution, as well.

One of the methods of solving system (19) is the method of reduction which consists in replacing the infinite system (19) by a system of n equations with n unknowns. A solution of such an finite system is considered as an approximate solution of the initial system. System (19) under condition (21) is quasi-regular [2].

II. Let an elastic isotropic plate on the plane $z = x + iy$ occupy the lower half-plane $\text{Im } z < 0$. Along the positive semi-axis, the plate is reinforced with a straight stringer through the rivets, but along the negative semi-axis it is free from external stresses. The rivets are located by the law $x_k = k$, $k = 1, 2, \dots$. The stringer end at the point $x = 0$ is under the action of concentrated force of intensity P , directed along the Ox -axis. r is the radius of the rivet.

For the problem under consideration, the unknown complex potentials have the form [3]

$$\begin{aligned} \Phi(z) &= -\frac{1}{2\pi} \sum_{k=1}^{\infty} \frac{X_k}{z - x_k}, \\ \Psi(z) &= \frac{1}{2\pi} \sum_{k=1}^{\infty} \frac{X_k}{z - x_k} - \frac{1}{2\pi} \sum_{k=1}^{\infty} \frac{x_k X_k}{(z - x_k)^2}, \end{aligned} \tag{22}$$

where X_k ($k = 1, 2, \dots$) are unknown interaction forces of the plate and the stringer, applied at the points $x_k = k$.

Relying on formula (5), we have

$$-\frac{\nu + 1}{4\mu h\pi} \sum_{j=1}^{\infty} \Gamma_{k-j} X_j = \frac{N_k}{E_0 F_0}, \quad k = 1, 2, \dots,$$

where

$$\Gamma_k = \begin{cases} \ln(1 + k^{-1}), & k \neq 0, -1, \\ -\ln r, & k = 0, \\ \ln r, & k = -1. \end{cases}$$

It follows from (1) that

$$\sum_{j=1}^{\infty} \Gamma_{k-j} X_j = \sum_{j=1}^{\infty} \Gamma_{k-j} (N_j - N_{j-1}) = \sum_{j=1}^{\infty} (\Gamma_{k-j} - \Gamma_{k-j-1}) N_j - \Gamma_{k-1} N_0, \quad k = 1, 2, \dots$$

Then the system of equations (22) takes the form

$$\omega \sum_{j=1}^{\infty} B_{k-j} N_j = N_k - \omega \Gamma_{k-1} N_0, \quad k = 1, 2, 3, \dots, \tag{23}$$

where

$$B_k = \begin{cases} -2 \ln r, & k = 0, \\ \ln 2r, & |k| = 1, \\ \ln(1 - k^{-2}), & |k| = 2, 3, \dots, \end{cases} \quad \omega = -\frac{(\varkappa + 1)E_0 F_0}{4\mu h \pi}.$$

Let us consider the following system:

$$\omega \sum_{j=1}^{\infty} B_{k-j} N_j = N_k - \omega \Gamma_{k-1} k^{-\varepsilon} N_0, \quad k = 1, 2, 3, \dots, \tag{24}$$

where ε is an arbitrarily small positive number.

As is known, l_1 denotes a class of vectors (of infinite sequences of complex numbers) $a = \{a_k\}_{k=-\infty}^{\infty}$ which satisfy the following restriction:

$$|a_k| < \frac{M}{k^{1+\lambda}}, \quad 0 < \lambda \leq 1;$$

the space l_1 is a commutative normed ring in which the process of multiplication is defined by the convolution

$$\Gamma b = \Gamma * b = \left\{ \sum_{j=-\infty}^{\infty} \Gamma_{k-j} b_j \right\} = \left\{ \sum_{j=-\infty}^{\infty} b_{k-j} \Gamma_j \right\} = b * \Gamma = b \Gamma.$$

To every vector from the space l_1 there corresponds the function which is a sum of an absolutely summable Fourier series, and vice versa, to every function defined on the circular $|t| = 1$, expandable into an absolutely summable Fourier series, there corresponds the vector of a ring l_1 , i.e.,

$$\Psi = \{\Psi_n\}_{n=-\infty}^{\infty} \in l_1, \quad \Psi_n \rightarrow \Psi(t), \quad \Psi(t) = \sum_{n=-\infty}^{\infty} \Psi_n t^n, \quad |t| = 1,$$

$$\Psi(t) \rightarrow \{\Psi_n\}_{n=-\infty}^{\infty}; \quad \Psi_n = \frac{1}{2\pi i} \int_{\gamma} \Psi(t) t^{-(n+1)} dt.$$

A class of functions defined on the circular $\gamma : |t| = 1$, expandable into an absolutely converging Fourier series, is called Wiener's class and denoted by W . The norm is defined as follows: $\|\Psi\|_W = \|\Psi\|_{l_1} = \sum_{n=-\infty}^{\infty} |\Psi_n|$. It is proved that the functions of the class W satisfy Hölder's (H) condition [4].

Let us define each of equations (24) so that they are valid for all integers k :

$$\omega \sum_{j=-\infty}^{\infty} B_{k-j} N^+_j = N^+_k - \omega \Gamma_{k-1} k^{-\varepsilon} N_0 + M^-_k, \quad k = 0, \pm 1, \pm 2, \dots, \tag{25}$$

where

$$N^+_k = \begin{cases} N_k, & k \geq 0, \\ 0, & k < 0, \end{cases} \quad M^-_k = \begin{cases} 0, & k \geq 0, \\ M_k, & k < 0. \end{cases}$$

Performing the discrete Fourier transformation [1] of system (25) and using the property of convolution, we obtain the following boundary value problem of the theory of analytic functions, the Riemann problem for the circle

$$(B(t) - 1) N_{\varepsilon}^{+}(t) = M^{-}(t) + \varphi_{\varepsilon}(t), \quad \gamma : |t| = 1, \quad (26)$$

where

$$B(t) = \omega \sum_{k=-\infty}^{\infty} B_k t^k, \quad N_{\varepsilon}^{+}(t) = \sum_{k=-\infty}^{\infty} N_k^{+} t^k, \quad M^{-}(t) = \sum_{k=-\infty}^{\infty} M_k^{-} t^k, \\ \varphi_{\varepsilon}(t) = -\omega N_0 \sum_{k=1}^{\infty} \Gamma_{k-1} k^{-\varepsilon} t^k \quad B(t), \varphi_{\varepsilon}(t) \in H.$$

It is proved that $B(t)$ is the real function and, consequently, $\text{Ind}_{\gamma}(B(t) - 1) = 0$ and $B(t) - 1 \neq 0$ on γ . A solution of the boundary value problem (26) can be constructed by the well-known method [4], and the solution of the infinite system (25) has the form

$$N_k^{+} = \frac{1}{2\pi i} \int_{|t|=1} \frac{N^{+}(t) dt}{t^{k+1}}, \quad k = 1, 2, \dots, \quad N^{+}(t) = \lim_{\varepsilon \rightarrow 0} N_{\varepsilon}^{+}(t).$$

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