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MATHEMATICAL STRUCTURES VIA b-OPEN SETS

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Abstract. In view of a kernel, s-kernel and b-kernel in a topological space, we study a new type of generalized closed set through this write-up. This type of generalized closed set splits various types of collections of generalized open sets, as well as different types of collections of generalized closed sets. Since the collection of b-open sets in a topological space is a generalization of both collections of semi-open sets and pre-open sets, the study of generalized closed sets via b-open set is a remarkable part.

1. Introduction

H. Maki [13] in 1986 initiated the concept of Λ-sets in topological spaces. A Λ-set is a set A which coincides with its kernel (= saturated set) i.e., with the intersection of all open sets containing A. In 1997, Arenas *et al.* [2] introduced and studied the notion of λ-closed and λ-open sets by using Λ-sets and closed sets. In 1996, D. Andrijević [1] gave a new type of generalized open sets, called b-open sets, whereas generalized locally closed sets have been studied by Modak and Noiri [15] in 2019. Ekici and Caldas [8] studied b-open sets under the name of γ-open sets.

The aim of this paper is to introduce a new class of sets called $gb\Lambda$-closed sets and $gb\Lambda$-open sets in a topological space and to study their properties and characterizations. Throughout this paper, we denote by η a topological space, where X is a set and τ is a topology on X on which no separation axioms are accepted, unless explicitly mentioned. The collection of all closed sets in a topological space η is denoted by $C(\tau)$. For a subset A of a topological space η, its closure (resp., interior) is denoted by $Cl(A)$ (resp., $\operatorname{Int}(A)$) and they obey $\operatorname{Int}(A) = X \setminus Cl(X \setminus A)$.

2. Known Facts

Let us recall the followings representing mathematical tools for our paper.

For a topological space η, a subset A of X is said to be b-open [1] (resp., semi-open [11], b-closed [1], semi-closed [11]) if $A \subseteq Cl\big(\operatorname{Int}(A)\big) \cup \operatorname{Int}\big(Cl(A)\big)$ (resp., $A \subseteq Cl\big(\operatorname{Int}(A)\big)$, $Cl\big(\operatorname{Int}(A)\big) \cap \operatorname{Int}\big(Cl(A)\big) \subseteq A$, $\operatorname{Int}\big(Cl(A)\big) \subseteq A$).

The family of all b-open (resp., semi-open, b-closed, semi-closed) sets in a topological space η is denoted by $O^b(X)$ (resp., $O^s(X)$, $C^b(\tau)$, $C^s(\tau)$). The intersection of all b-closed (resp., semi-closed) subsets of X containing A is called b-closure (resp., semi-closure) of A and is denoted by $Cl_b(A)$ (resp., $Cl_s(A)$).

The kernels are defined as follows:

Kernel [13] (resp., b-kernel [5], s-kernel [14]) of A is denoted by $\operatorname{Ker}(A)$ (resp., $\operatorname{Ker}_b(A)$, $\operatorname{Ker}_s(A)$) and is defined as $\operatorname{Ker}(A) = \bigcap\big\{U \subseteq X : U \supseteq A,\ U \in \tau\big\}$ (resp., $\operatorname{Ker}_b(A) = \bigcap\big\{U \subseteq X : U \supseteq A,\ U \in O^b(X)\big\}$, $\operatorname{Ker}_s(A) = \bigcap\big\{U \subseteq X : U \supseteq A,\ U \in O^s(X)\big\}$).

In this respect, a subset A of X is said to be a Λ-set [13] if $A = \operatorname{Ker}(A)$.

The collection of all Λ-sets in a topological space η is denoted by $O^\Lambda(X)$. In general, $\operatorname{Ker}(A)$ is neither an open set, nor a closed set.

A subset A of X is called:

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- λ-closed [2] $\big($resp., generalized closed, or briefly, g-closed [12]$\big)$ if $A = B \cap F$, where $B \in O^{\Lambda}(X)$ and $F \in C(\tau)$ $\big($resp., $Cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in \tau\big)$. The complement of a λ-closed $\big($resp., g-closed$\big)$ set is called λ-open $\big($resp., g-open$\big)$. The collection of λ-closed $\big($resp., λ-open, g-closed, g-open$\big)$ sets in a topological space η is denoted by $C^{\lambda}(\tau)$ $\big($resp., $O^{\lambda}(X)$, $C^{g}(\tau)$, $O^{g}(X)\big)$.
- g^*-closed [10] $\big($resp., generalized semi-closed (briefly, gs-closed) [3], semi-generalized closed (briefly, sg-closed) [3], Λg-closed [6], $g\Lambda$-closed [6], $gs\Lambda$-closed [14], weakly closed (briefly, w-closed) [16]$\big)$ set if $Cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in O^{g}(X)$ $\big($resp., $Cl_s(A) \subseteq U$, whenever $A \subseteq U$ and $U \in \tau$, $Cl_s(A) \subseteq U$, whenever $A \subseteq U$ and $U \in O^{s}(X)$, $Cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in O^{\lambda}(X)$, $Cl_{\lambda}(A) \subseteq U$, whenever $A \subseteq U$ and $U \in \tau$, $Cl_{\lambda}(A) \subseteq U$, whenever $A \subseteq U$ and $U \in O^{s}(X)$, $Cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in O^{s}(X)\big)$. The family of all j-closed sets in a topological space η is denoted by $C^{j}(\tau)$, where $j \in \big\{g^*,\ gs,\ sg,\ \Lambda g,\ g\Lambda,\ gs\Lambda,\ w\big\}$.

In view of the above, in [2], it has been shown that A is closed if and only if $A = F \cap Cl(A)$ $\big($where F is a Λ-set$\big)$ if and only if $A = \operatorname{Ker}(A) \cap Cl(A)$; $\tau \subseteq O^{\Lambda}(X) \subseteq C^{\lambda}(\tau)$ and $C(\tau) \subseteq C^{\lambda}(\tau)$.

Recall that a point $x \in X$ is said to be a λ-cluster [4] $\big($resp., λ-interior [4]$\big)$ point of A if for every $\big($resp., there exists a$\big)$ λ-open set U of X containing x, $A \cap U \neq \varnothing$ $\big($resp., such that $U \subseteq A\big)$. The collection of all λ-cluster $\big($resp., λ-interior$\big)$ points of A is called the λ-closure $\big($resp., λ-interior$\big)$ of A and is denoted by $Cl_{\lambda}(A)$ $\big($resp., $\operatorname{Int}_{\lambda}(A)\big)$.

In view of the above, the authors Caldas *et al.* [4] have shown that A is λ-closed if and only if $Cl_{\lambda}(A) = A$; $Cl_{\lambda}(A) = \bigcap\big\{F \in C^{\lambda}(\tau) : A \subseteq F\big\}$; $A \subseteq Cl_{\lambda}(A) \subseteq Cl(A)$, $Cl_{\lambda}(A)$ is λ-closed; $X \setminus \operatorname{Int}_{\lambda}(A) = Cl_{\lambda}(X \setminus A)$ and for $A \subseteq B$, $Cl_{\lambda}(A) \subseteq Cl_{\lambda}(B)$.

Recall that a point $x \in X$ is said to be a λ-limit point [4] of A if for each λ-open set U containing x, $U \cap \big(A \setminus \{x\}\big) \neq \varnothing$. The set of all λ-limit points of A is called λ-derived set of A and is denoted by $D_{\lambda}(A)$.

In this context, the author Caldas *et al.* [4] showed that $D_{\lambda}(A) \subseteq D(A)$ and $Cl_{\lambda}(A) = A \cup D_{\lambda}(A)$ for a subset A of X, where $D(A)$ is the derived set of A.

3. The Role of b-open Sets as a Kernel

In this section, we split the collections τ, $O^{\Lambda}(X)$, $C^{\lambda}(\tau)$, $C^{gb\Lambda}(\tau)$, $C^{g\Lambda}(\tau)$, $C(\tau)$ and $C^{gs\lambda}(\tau)$. We study the collections in a topological space which are not related to the collection $C^{gb\Lambda}(\tau)$.

Definition 1. Let η be a topological space and $A \subseteq X$. A is said to be $gb\Lambda$-closed in X if $Cl_{\lambda}(A) \subseteq U$, whenever $A \subseteq U$ and $U \in O^{b}(X)$.

The collection of all $gb\Lambda$-closed sets in a topological space η is denoted by $C^{gb\Lambda}(\tau)$.

The following example shows the existence of a $gb\Lambda$-closed set in $\mathbb{R}$.

Example 2. Consider the set $\mathbb{R}$ of real numbers with usual topology and $A = (0,1) \cap \mathbb{Q}$, where $\mathbb{Q}$ stands for the set of all rational numbers. Then $\operatorname{Ker}(A) = \operatorname{Ker}\big(\bigcup_{x \in A}\{x\}\big) = \bigcup_{x \in A}\operatorname{Ker}\big(\{x\}\big) = \bigcup_{x \in A}\{x\} = A$ and hence $A = \operatorname{Ker}(A) \cap Cl(A)$. Therefore A is λ-closed implies $Cl_{\lambda}(A) = A$. Thus for any b-open set $U \supseteq A$, $Cl_{\lambda}(A) \subseteq A$. Hence A is $gb\Lambda$-closed in $\mathbb{R}$.

Theorem 3. *Let η be a topological space. Then $C^{\lambda}(\tau) \subseteq C^{gb\Lambda}(\tau)$.*

Proof. Follows from the fact that for a λ-closed set A, $Cl_{\lambda}(A) = A$. □

The following example shows that the reverse inclusion of Theorem 3 does not hold, in general.

Example 4. Let $X = \{e_1, e_2, e_3, e_4, e_5\}$ and $\tau = \Big\{\varnothing, \{e_1\}, \{e_1, e_2, e_3\}, \{e_1, e_4, e_5\}, X\Big\}$. Here, $\{e_2, e_3, e_4, e_5\}$ is $gb\Lambda$-closed, but not λ-closed.

However, we can give the converse of Theorem 3 as follows:

Theorem 5. *Let η be a topological space and $A \subseteq X$. If $A \in O^{b}(X) \cap C^{gb\Lambda}(\tau)$, then $A \in C^{\lambda}(\tau)$.*

Proof. Follows from the definition of a $gb\Lambda$-closed set. □

Theorem 6. *Let η be a topological space and $A \subseteq X$. Then for $A \in C(\tau)$, $A \in C^{gb\Lambda}(\tau)$.*

Proof. Follows from the fact that $C(\tau) \subseteq C^{\lambda}(\tau) \subseteq C^{gb\Lambda}(\tau)$. □

For the converse of Theorem 6 we consider the following example.

Example 7. Let $X = \{e_1, e_2, e_3, e_4, e_5\}$ and $\tau = \Big\{\varnothing, \{e_1\}, \{e_1, e_2, e_3\}, \{e_1, e_4, e_5\}, X\Big\}$. Let $A = \{e_1, e_2, e_3\}$. Since A is open, $\mathrm{Ker}(A) = A$. So, $A = \mathrm{Ker}(A) \cap Cl(A)$ implies that A is λ-closed and hence $Cl_\lambda(A) = A$. So, for any b-open set $U \supseteq A$, $Cl_\lambda(A) \subseteq U$. Hence A is $gb\Lambda$-closed, but not closed in X.

Theorem 8. *Let η be a topological space. Then for $U \in \tau$, $U \in C^{gb\Lambda}(\tau)$.*

Proof. Follows from the fact $\tau \subseteq C^{\lambda}(\tau) \subseteq C^{gb\Lambda}(\tau)$. □

For the converse of this Theorem we intimate the following

Example 9. Let $X = \{e_1, e_2, e_3, e_4, e_5\}$ and $\tau = \Big\{\varnothing, \{e_1\}, \{e_1, e_2, e_3\}, \{e_1, e_4, e_5\}, X\Big\}$. Let $A = \{e_2, e_3\}$. Since A is closed in X, A is $gb\Lambda$-closed but not open in X.

Thus we conclude that every closed subset in a topological space is a $gb\Lambda$-closed set.

Theorem 10. *Let η be a topological space and $A \subseteq X$. Then for $A \in C^{gb\Lambda}(\tau)$, $A \in C^{g\Lambda}(\tau)$.*

Proof. Let A be a $gb\Lambda$-closed set in X and $A \subseteq U$, where U is open in X. Since every open set is b-open [1] and A is $gb\Lambda$-closed, $Cl_\lambda(A) \subseteq U$. Hence A is $g\Lambda$-closed. □

For the converse of Theorem 10 we consider the following

Example 11. Let $X = \{e_1, e_2, e_3, e_4, e_5\}$ and $\tau = \Big\{\varnothing, \{e_1\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}, X\Big\}$. Let $A = \{e_3\}$. Then A is b-open in X. Now $\mathrm{Ker}(A) \cap Cl(A) = \{e_2, e_3\} \cap \{e_2, e_3, e_4, e_5\} = \{e_2, e_3\} \neq A$ implies that A is not λ-closed and hence $A \subsetneqq Cl_\lambda(A)$, where A is b-open. Hence A is not $gb\Lambda$-closed in X. Since $\{e_2, e_3\}$ is λ-closed containing A, $Cl_\lambda(A) \subseteq \{e_2, e_3\} = \mathrm{Ker}(A)$. Thus for any open set $U \supseteq A$, $Cl_\lambda(A) \subseteq U$. Hence A is $g\Lambda$-closed in X.

From the above discussed results, we have the following chains:

- $\tau \subseteq O^{\Lambda}(X) \subseteq C^{\lambda}(\tau) \subseteq C^{gb\Lambda}(\tau) \subseteq C^{g\Lambda}(\tau)$;
- $C(\tau) \subseteq C^{\lambda}(\tau) \subseteq C^{gb\Lambda}(\tau) \subseteq C^{g\Lambda}(\tau)$.

Thus we see that $C^{gb\Lambda}(\tau)$ splits the collections $C^{\lambda}(\tau)$ and $C^{g\Lambda}(\tau)$.
For the next results, we recall the following definition from [9].

Definition 12. A partition topology is a topology which can be induced on any set X by partitioning X into disjoint subsets P; these subsets form the basis for the topology.

Proposition 13. Let η be a topological space. Then:

(1) η is a partition space if and only if $\tau \subseteq C(\tau)$ [9].
(2) For a partition space η, $Cl(A) = Cl_\lambda(A)$, where $A \subseteq X$ [14].

Theorem 14. *In a partition space η, $C^{gb\Lambda}(\tau) \subseteq C^{w}(\tau)$.*

Proof. Let A be a $gb\Lambda$-closed set in a partition space X and $A \subseteq U$, where U is semi-open in X. Since every semi-open set is b-open and A is $gb\Lambda$-closed, $Cl_\lambda(A) \subseteq U$. Since in a partition space $Cl(A) = Cl_\lambda(A)$, $Cl(A) \subseteq U$. Hence A is w-closed. □

Theorem 15. *Let η be a topological space and $A \subseteq X$. Then for $A \in C^{gb\Lambda}(\tau)$, $A \in C^{gs\Lambda}(\tau)$.*

Proof. Follows from the fact $O^{b}(X) \supseteq O^{s}(X)$. □

The converse of Theorem 15 is not true, in general, which is followed by the following

Example 16. Let $X = \{e_1, e_2, e_3, e_4, e_5\}$ and $\tau = \Big\{\varnothing, \{e_1\}, \{e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_4\}, X\Big\}$. Let $A = \{e_2\}$. Then A is b-open in X. Also, $\mathrm{Ker}(A) \cap Cl(A) = \{e_2, e_3, e_4\} \neq A$. Therefore A is not λ-closed and hence by Theorem 5, A is not $gb\Lambda$-closed. Now, the λ-closed sets containing A are $\{e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_4\}, \{e_2, e_3, e_4, e_5\}$ and X. These are also the only semi-open sets containing A. Therefore $Cl_\lambda(A) = \{e_2, e_3, e_4\} = \mathrm{Ker}_s(A)$. Hence A is $gs\Lambda$-closed.

From the above reasoning, we have the following chains:

- $\tau \subseteq O^\Lambda(X) \subseteq C^\lambda(\tau) \subseteq C^{gb\Lambda}(\tau) \subseteq C^{gs\Lambda}(\tau)$;
- $C(\tau) \subseteq C^\lambda(\tau) \subseteq C^{gb\Lambda}(\tau) \subseteq C^{gs\Lambda}(\tau)$.

Thus we see that $C^{gb\Lambda}(\tau)$ splits the collections $C^\lambda(\tau)$ and $C^{gs\Lambda}(\tau)$.

Remark 17. We now mention that the following collections are not related to each other for a topological space η:

(1) $C^g(\tau)$ and $C^{gb\Lambda}(\tau)$;
(2) $C^{\Lambda g}(\tau)$ and $C^{gb\Lambda}(\tau)$;
(3) $C^{gs}(\tau)$ and $C^{gb\Lambda}(\tau)$;
(4) $C^{sg}(\tau)$ and $C^{gb\Lambda}(\tau)$;
(5) $C^{g*}(\tau)$ and $C^{gb\Lambda}(\tau)$;
(6) $C^s(\tau)$ and $C^{gb\Lambda}(\tau)$;
(7) $C^b(\tau)$ and $C^{gb\Lambda}(\tau)$.

Let $X = \{p_1, p_2, p_3, p_4, p_5\}$ and $\tau = \Big\{\varnothing, \{p_1\}, \{p_2, p_3\}, \{p_1, p_2, p_3\}, X\Big\}$.

For (1), $\{p_4\}$ is g-closed, but not $gb\Lambda$-closed and $\{p_1, p_2, p_3\}$ is $gb\Lambda$-closed, but not g-closed.

For (2), $\{p_2, p_3\}$ is $gb\Lambda$-closed, but not Λg-closed and $\{p_4\}$ is Λg-closed, but not $gb\Lambda$-closed.

For (3), $\{p_4\}$ is gs-closed, but not $gb\Lambda$-closed and $\{p_2, p_3\}$ is $gb\Lambda$-closed, but not gs-closed.

For (4), $\{p_4, p_5\}$ is $gb\Lambda$-closed, but not sg-closed and $\{p_1, p_2, p_3, p_4\}$ is sg-closed, but not $gb\Lambda$-closed.

For (5), $\{p_2, p_3\}$ is $gb\Lambda$-closed, but not g^*-closed and $\{p_4\}$ is g^*-closed, but not $gb\Lambda$-closed.

For (6), $\{p_1, p_2, p_3\}$ is $gb\Lambda$-closed, but not semi-closed and $\{p_1, p_4\}$ is semi-closed, but not $gb\Lambda$-closed.

For (7), $\{p_2, p_4\}$ is b-closed, but not $gb\Lambda$-closed and $\{p_1, p_2, p_3\}$ is $gb\Lambda$-closed, but not b-closed.

The following example shows that union of two $gb\Lambda$-closed sets in a topological space is not necessarily a $gb\Lambda$-closed set.

Example 18. Let $X = \{p_1, p_2, p_3, p_4, p_5\}$ and $\tau = \Big\{\varnothing, \{p_1\}, \{p_2\}, \{p_1, p_2\}, \{p_2, p_3\}, \{p_1, p_2, p_3\}, \{p_2, p_3, p_4\}, \{p_1, p_2, p_3, p_4\}, \{p_2, p_3, p_4, p_5\}, X\Big\}$. Let $A = \{p_1, p_2\}$ and $B = \{p_1, p_4, p_5\}$. Then A, being an open set, is $gb\Lambda$-closed in X and B, being a closed set, is $gb\Lambda$-closed in X. Now, $A \cup B = \{p_1, p_2, p_4, p_5\}$ and $\mathrm{Ker}(A \cup B) \cap Cl(A \cup B) = X \neq A \cup B$. Therefore $A \cup B$ is not λ-closed in X. Moreover, $A \cup B$ is b-open in X. Hence by Theorem 5, it follows that $A \cup B$ is not $gb\Lambda$-closed in X.

In a topological space, the intersection of two $gb\Lambda$-closed sets is not necessarily a $gb\Lambda$-closed set which is followed by the following

Example 19. Let $X = \{p_1, p_2, p_3, p_4, p_5\}$ and $\tau = \Big\{\varnothing, \{p_1\}, \{p_2, p_3\}, \{p_1, p_2, p_3\}, \{p_2, p_3, p_4, p_5\}, X\Big\}$. Let $A = \{p_1, p_2, p_4, p_5\}$ and $B = \{p_2, p_3, p_4, p_5\}$. Then A is not λ-closed in X and the only λ-closed set containing A is X. Therefore $Cl_\lambda(A) = X$, where X is the only b-open set containing X. So, A is $gb\Lambda$-closed in X and B, being open, is $gb\Lambda$-closed in X. Now, $A \cap B = \{p_2, p_4, p_5\}$ is b-open, but not λ-closed. Hence by Theorem 5, it follows that $A \cap B$ is not $gb\Lambda$-closed in X.

Thus the collection $C^{gb\Lambda}(\tau)$ for a topological space η does not form a topology on X, in general.

4. Applications of b-open Sets as a Kernel

Theorem 20. *Let η be a topological space and $A \subseteq X$. If $A \in C^{gb\Lambda}(\tau)$, then $F \nsubseteq Cl_\lambda(A) \setminus A$, where $\varnothing \neq F \in C(\tau)$.*

Proof. If possible, suppose that F is a non-empty closed set such that $F \subseteq Cl_\lambda(A) \setminus A$. Then $A \subseteq X \setminus F$. Since A is $gb\Lambda$-closed and $X \setminus F$ is b-open, $Cl_\lambda(A) \subseteq X \setminus F$ which implies $F \subseteq X \setminus Cl_\lambda(A)$. Moreover, $F \subseteq Cl_\lambda(A)$. Hence $F \subseteq \big(X \setminus Cl_\lambda(A)\big) \cap Cl_\lambda(A)$, proving that $F = \varnothing$, a contradiction. Hence $Cl_\lambda(A) \setminus A$ does not contain any non-empty closed set. $\square$

If $Cl_\lambda(A) \setminus A$ does not contain any non-empty closed set, then it is not necessary that $A \in C^{gb\Lambda}(\tau)$.

Example 21. Let $X = \{k_1, k_2, k_3, k_4, k_5\}$ and $\tau = \Big\{\varnothing, \{k_1\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}, X\Big\}$. Let $A = \{k_2\}$. Then $Cl_\lambda(A) = \{k_2, k_3\}$ and hence $Cl_\lambda(A) \setminus A = \{k_3\}$ which does not contain any non-empty closed set. Now, $\mathrm{Ker}(A) \cap Cl(A) = \{k_2, k_3\} \neq A$ implies that A is not λ-closed, where A is b-open. Therefore by Theorem 5, it follows that A is not $gb\Lambda$-closed.

Theorem 22. *Let η be a topological space and $A \subseteq X$. If $A \in C^{gb\Lambda}(\tau)$, then $T \nsubseteq Cl_\lambda(A) \setminus A$, where $\varnothing \neq T \in C^s(\tau)$.*

Proof. If possible, suppose that $\varnothing \neq T \in C^s(\tau)$ such that $T \subseteq Cl_\lambda(A) \setminus A$. Then $A \subseteq X \setminus T$. Since A is $gb\Lambda$-closed and $X \setminus T$ is b-open, $Cl_\lambda(A) \subseteq X \setminus T$ which implies $T \subseteq X \setminus Cl_\lambda(A)$. Moreover, $T \subseteq Cl_\lambda(A)$. Hence $T \subseteq \big(X \setminus Cl_\lambda(A)\big) \cap Cl_\lambda(A)$, proving that $T = \varnothing$, a contradiction. Hence $Cl_\lambda(A) \setminus A$ does not contain any non-empty semi-closed set. $\square$

The converse of Theorem 22 need not hold, in general.

Example 23. Let $X = \{k_1, k_2, k_3, k_4, k_5\}$ and $\tau = \Big\{\varnothing, \{k_1\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}, X\Big\}$. Let $A = \{k_2\}$. Then $Cl_\lambda(A) = \{k_2, k_3\}$ and hence $Cl_\lambda(A) \setminus A = \{k_3\}$ which does not contain any non-empty semi-closed set, but A is not $gb\Lambda$-closed.

Theorem 24. *Let η be a topological space and $A \subseteq X$. If $A \in C^{gb\Lambda}(\tau)$, then $T \nsubseteq Cl_\lambda(A) \setminus A$, where $\varnothing \neq T \in C^b(\tau)$.*

Proof. The proof is straightforward. $\square$

The following example shows that the converse of Theorem 24 is not true, in general.

Example 25. Let $X = \{k_1, k_2, k_3, k_4, k_5\}$ and $\tau = \Big\{\varnothing, \{k_1\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}, \{k_2, k_3, k_4, k_5\}, X\Big\}$. Let $A = \{k_4\}$. Then $Cl_\lambda(A) = \{k_4, k_5\}$ and hence $Cl_\lambda(A) \setminus A = \{k_5\}$ which does not contain any non-empty b-closed set, but A is not $gb\Lambda$-closed.

Theorem 26. *Let η be a topological space. Then for each $x \in X$, either $\{x\} \in C^b(\tau)$ or $X \setminus \{x\} \in C^{gb\Lambda}(\tau)$.*

Proof. Suppose that $\{x\} \notin C^b(\tau)$. Then $X \setminus \{x\} \notin O^b(X)$. Since X is the only b-open set containing $X \setminus \{x\}$, $Cl_\lambda\big(X \setminus \{x\}\big) \subseteq X$. Hence $X \setminus \{x\} \in C^{gb\Lambda}(\tau)$. Thus either $\{x\} \in C^b(\tau)$ or $X \setminus \{x\} \in C^{gb\Lambda}(\tau)$. $\square$

For the next result, we recall that a topological space η is Hausdorff (or T_2) if and only if for each pair of distinct points x and y of X, there exist $U, V \in \tau$ such that $x \in U$, $y \in V$ and $U \cap V = \varnothing$. In this context, a topological space η is called a T_1-space if every singleton set is closed in η. It is obvious that every Hausdorff space is a T_1-space.

Theorem 27. *Let η be a topological space in which each one-point set is closed. Then $C^\lambda(\tau) = C^{gb\Lambda}(\tau)$.*

Proof. Let A be a $gb\Lambda$-closed subset of X. If possible, let A be not λ-closed. Then $Cl_\lambda(A) \setminus A$ is non-empty. Let $x \in Cl_\lambda(A) \setminus A$. Since $\{x\}$ is closed, $Cl_\lambda(A) \setminus A$ contains a non-empty closed set $\{x\}$ which leads towards a contradiction, by Theorem 20. Hence A is λ-closed. Therefore $C^\lambda(\tau) \supseteq C^{gb\Lambda}(\tau)$. Moreover, $C^\lambda(\tau) \subseteq C^{gb\Lambda}(\tau)$. Hence the result follows. $\square$

Corollary 28. *In a Hausdorff space $\big($and hence T_1-space$\big)$ η, $C^\lambda(\tau) = C^{gb\Lambda}(\tau)$.*

Definition 29 ([12])**.** A topological space η is called a $T_{\frac{1}{2}}$-space if every generalised closed subset of X is closed.

Proposition 30 ([2])**.** For a topological space η, the followings are equivalent:
(1) X is a $T_{\frac{1}{2}}$-space;
(2) every subset of X is λ-closed.

Theorem 31. *Let η be a $T_{\frac{1}{2}}$-space. Then for each $A \subseteq X$, $A \in C^{gb\Lambda}(\tau)$.*

Proof. The proof immediately follows from Proposition 30 and Theorem 3. □

Definition 32 ([2])**.** A topological space η is said to be a $T_{\frac{1}{4}}$-space if for every finite subset F of X and every $y \notin F$, there exists a set A_y containing F and disjoint from $\{y\}$ such that A_y is either open or closed.

Proposition 33 ([2])**.** For a topological space η, the followings are equivalent:
(1) X is a $T_{\frac{1}{4}}$-space;
(2) every finite subset of X is λ-closed.

Theorem 34. *Let η be a $T_{\frac{1}{4}}$-space. Then for any finite subset A of X, $A \in C^{gb\Lambda}(\tau)$.*

Proof. Follows immediately from Proposition 33 and Theorem 3. □

Theorem 35. *Let η be a T_1-space. Then $C^{\Lambda g}(\tau) \subseteq C^{gb\Lambda}(\tau)$.*

Proof. Follows from Theorem 6 and the following □

Theorem 36 ([6])**.** *Let η be a T_1-space. Then $C^{\Lambda g}(\tau) \subseteq C(\tau)$.*

Definition 37 ([7])**.** A topological space η is said to be a door space if every subset of X is either open or closed.

Theorem 38. *Let η be a door space. Then $C^{gb\Lambda}(\tau) = \mathcal{P}(X)$, a power set of X.*

Proof. Let A be a subset of a topological space η. Then A is either open or closed in η. Hence A is $gb\Lambda$-closed, by Theorem 6 and Theorem 8. □

For a reason of the converse of Theorem 38, the following example is interesting.

Example 39. Let $X = \{k_1, k_2, k_3, k_4\}$ and $\tau = \Big\{\varnothing, \{k_1\}, \{k_2\}, \{k_1, k_2\}, \{k_1, k_2, k_3\}, \{k_1, k_2, k_4\}, X\Big\}$. By Theorems 6 and 8, it follows that every open and closed subset of X is $gb\lambda$-closed. Now, the only subsets of X which are neither open, nor closed, are $\{k_1, k_3\}$, $\{k_1, k_4\}$, $\{k_2, k_3\}$ and $\{k_2, k_4\}$ which are λ-closed and hence are $gb\Lambda$-closed (from Theorem 3). Therefore every subset of X is $gb\Lambda$-closed, but X is not a door space.

However, in a partition space, the following theorem holds.

Theorem 40. *Let η be a partition space. Then $C^{gb\Lambda}(\tau) \subseteq C^{g}(\tau)$.*

Proof. Let A be a $gb\Lambda$-closed subset of a partition space X and $A \subseteq U$, where U is open. Then $Cl_\lambda(A) \subseteq U$, since U is b-open and A is $gb\Lambda$-closed. Since in a partition space, $Cl(A) = Cl_\lambda(A)$ and hence $Cl(A) \subseteq U$. Consequently, A is g-closed. □

Theorem 41. *Let η be a topological space and A be a $gb\Lambda$-closed subset of X. Then $A \in C^{\lambda}(\tau)$ if and only if $Cl_\lambda(A) \setminus A \in C(\tau)$.*

Proof. Let A be a λ-closed subset of X. Since A is λ-closed, $Cl_\lambda(A) = A$ which implies that $Cl_\lambda(A) \setminus A = \varnothing$, a closed set. Conversely, let A be a $gb\Lambda$-closed subset of X such that $Cl_\lambda(A) \setminus A$ is closed. Since A is $gb\Lambda$-closed, $Cl_\lambda(A) \setminus A$ contains no non-empty closed subset of X, by Theorem 20. Since $Cl_\lambda(A) \setminus A$ is closed, we must have $Cl_\lambda(A) \setminus A = \varnothing$. Therefore $Cl_\lambda(A) = A$ and, consequently, A is λ-closed. □

Theorem 42. *Let η be a topological space. If $C^{gb\Lambda}(\tau) \subseteq C^{\lambda}(\tau)$, then for each $x \in X$, either $\{x\} \in C^b(\tau)$ or $\{x\} \in O^{\lambda}(X)$.*

Proof. If $\{x\} \notin C^b(\tau)$, then $X \setminus \{x\} \notin O^b(X)$. Now, the only b-open set containing $X \setminus \{x\}$ is X. Moreover, $Cl_{\lambda}(X \setminus \{x\}) \subseteq X$. So, $X \setminus \{x\}$ is $gb\Lambda$-closed and by the hypothesis, $X \setminus \{x\}$ is λ-closed. Therefore $\{x\}$ is λ-open. Hence for each $x \in X$, $\{x\}$ is either b-closed or λ-open. □

For the next result, we need the following theorem from [4].

Theorem 43. *Let η be a topological space and $\{A_i : i \in \Lambda\}$ be an arbitrary collection of λ-closed sets. Then $\bigcap_i A_i \in C^{\lambda}(\tau)$.*

Theorem 44. *Let η be a topological space and A, $F \subseteq X$. Then for $A \in O^b(X) \cap C^{gb\Lambda}(\tau)$ and $F \in C^{\lambda}(\tau)$, $A \cap F \in C^{gb\Lambda}(\tau)$.*

Proof. Since A is b-open and $gb\Lambda$-closed, by Theorem 5, we have A is λ-closed. Then by Theorem 43, we get $A \cap F$ is λ-closed. Hence by Theorem 3, it follows that $A \cap F$ is $gb\Lambda$-closed. □

Theorem 45. *Let η be a topological space and $A \subseteq X$. Then for $A \in C^{gb\Lambda}(\tau)$, $Cl_b(\{x\}) \cap A \neq \varnothing$, for every $x \in Cl_{\lambda}(A)$.*

Proof. If possible, suppose that $Cl_b(\{x\}) \cap A = \varnothing$ for some $x \in Cl_{\lambda}(A)$. Then $A \subseteq X \setminus Cl_b(\{x\})$, where $X \setminus Cl_b(\{x\})$ is b-open in X. Therefore $Cl_{\lambda}(A) \subseteq X \setminus Cl_b(\{x\})$. Thus $x \in Cl_{\lambda}(A)$ implies $x \notin Cl_b(\{x\})$, which is a contradiction. Hence $Cl_b(\{x\}) \cap A \neq \varnothing$ for every $x \in Cl_{\lambda}(A)$. □

Theorem 46. *For a topological space η, the following statements are equivalent:*
(1) *$O^b(X) \subseteq C^{\lambda}(\tau)$;*
(2) *$\mathcal{P}(X) \subseteq C^{gb\Lambda}(\tau)$.*

Proof. (1) implies (2): Let A be a subset of X and $A \subseteq U$, where U is b-open in X. Then $Cl_{\lambda}(A) \subseteq Cl_{\lambda}(U)$. Since by the hypothesis, U is λ-closed, $Cl_{\lambda}(U) = U$. Therefore $Cl_{\lambda}(A) \subseteq U$ and hence A is $gb\Lambda$-closed.

(2) implies (1): Let A be b-open in X. By the assumption, A is $gb\Lambda$-closed. Then by Theorem 5, A is λ-closed. □

Theorem 47. *Let η be a topological space. Let A, $B \in C^{gb\Lambda}(\tau)$ with $D(A) \subseteq D_{\lambda}(A)$ and $D(B) \subseteq D_{\lambda}(B)$. Then $A \cup B \in C^{gb\Lambda}(\tau)$.*

Proof. We know that $D_{\lambda}(A) \subseteq D(A)$ and $D_{\lambda}(B) \subseteq D(B)$. Therefore $D(A) = D_{\lambda}(A)$ and $D(B) = D_{\lambda}(B)$. Now, $Cl(A) = D(A) \cup A = D_{\lambda}(A) \cup A = Cl_{\lambda}(A)$. Similarly, $Cl(B) = Cl_{\lambda}(B)$. Now, let $A \cup B \subseteq U$, where U is b-open. Then $A \subseteq U$ and $B \subseteq U$. This implies $Cl_{\lambda}(A) \subseteq U$ and $Cl_{\lambda}(B) \subseteq U$ as A and B are $gb\Lambda$-closed. Now, $Cl_{\lambda}(A \cup B) \subseteq Cl(A \cup B) = Cl(A) \cup Cl(B) = Cl_{\lambda}(A) \cup Cl_{\lambda}(B) \subseteq U$. Hence $A \cup B$ is $gb\Lambda$-closed. □

Following theorem is a characterization of $gb\Lambda$-closed sets.

Theorem 48. *Let η be a topological space and $A \subseteq X$. Then $A \in C^{gb\Lambda}(\tau)$ if and only if $Cl_{\lambda}(A) \subseteq \mathrm{Ker}_b(A)$.*

Proof. Suppose that A is $gb\Lambda$-closed in X. If possible, let $x \in Cl_{\lambda}(A)$ but $x \notin \mathrm{Ker}_b(A)$. Then $x \notin G$ for some b-open set $G \supseteq A$. Since A is $gb\Lambda$-closed, $Cl_{\lambda}(A) \subseteq G$ implies $x \in G$, we have a contradiction. Hence $Cl_{\lambda}(A) \subseteq \mathrm{Ker}_b(A)$.

Conversely, let A be a subset of X such that $Cl_{\lambda}(A) \subseteq \mathrm{Ker}_b(A)$ and $A \subseteq U$, where U is b-open. Then $\mathrm{Ker}_b(A) \subseteq U$. So, $Cl_{\lambda}(A) \subseteq U$. Hence A is $gb\Lambda$-closed in X. □

Theorem 49. *Let η be a topological space. Let A and B be two subsets of X such that $A \in C^{gb\Lambda}(\tau)$ and $A \subseteq B \subseteq Cl_{\lambda}(A)$. Then $B \in C^{gb\Lambda}(\tau)$.*

Proof. Let $B \subseteq U$, where U is b-open. Now, $A \subseteq B \subseteq U$ implies $A \subseteq U$, where U is b-open. Since A $gb\Lambda$-closed, $Cl_{\lambda}(A) \subseteq U$. By the hypothesis, $B \subseteq Cl_{\lambda}(A)$ implies $Cl_{\lambda}(B) \subseteq Cl_{\lambda}(Cl_{\lambda}(A)) = Cl_{\lambda}(A) \subseteq U$. Hence B is a $gb\Lambda$-closed set. □

5. Complement of $gb\Lambda$-closed Set

Throughout this section, we study the complement of $gb\Lambda$-closed sets.

Definition 50. Let η be a topological space and $A \subseteq X$. A is said to be $gb\Lambda$-open if $X \setminus A \in C^{gb\Lambda}(\tau)$.

Equivalently, a subset A of a topological space η is said to be $gb\Lambda$-open if $\mathrm{Int}_\lambda(A) \supseteq F$, whenever $A \supseteq F$ and $F \in C^b(\tau)$.

The collection of all $gb\Lambda$-open sets in a topological space η is denoted as $O^{gb\Lambda}(X)$.

Theorem 51. *Let η be a topological space and $A \subseteq X$. Then $A \in O^{gb\Lambda}(X)$ if and only if $\mathrm{Int}_\lambda(A) \supseteq F$ whenever $A \supseteq F$ and $F \in C^b(\tau)$.*

Proof. Suppose that A is $gb\Lambda$-open in X. Suppose $A \supseteq F$, where F is b-closed. Then $X \setminus A \subseteq X \setminus F$, where $X \setminus A$ is $gb\Lambda$-closed and $X \setminus F$ is b-open. Therefore $Cl_\lambda(X \setminus A) \subseteq X \setminus F$ and hence $F \subseteq X \setminus Cl_\lambda(X \setminus A) = \mathrm{Int}_\lambda(A)$. Thus $\mathrm{Int}_\lambda(A) \supseteq F$.

Conversely, let $\mathrm{Int}_\lambda(A) \supseteq F$, where $A \supseteq F$ and F is b-closed. Then $X \setminus A \subseteq X \setminus F$ and $X \setminus \mathrm{Int}_\lambda(A) \subseteq X \setminus F$, whence $Cl_\lambda(X \setminus A) \subseteq X \setminus F$. Therefore $X \setminus A$ is $gb\Lambda$-closed and A is $gb\Lambda$-open. □

Theorem 52. *Let η be a topological space. Then $O^\lambda(X) \subseteq O^{gb\Lambda}(X)$.*

Proof. Let A be λ-open. Then $X \setminus A$ is λ-closed. Since every λ-closed set is $gb\Lambda$-closed, $X \setminus A$ is $gb\Lambda$-closed and hence A is $gb\Lambda$-open in X. □

Theorem 53. *Let η be a topological space. Then $\tau \subseteq O^{gb\Lambda}(X)$.*

Proof. Let A be open. Then $X \setminus A$ is closed. Therefore $X \setminus A$ is λ-closed. So, $X \setminus A$ is $gb\Lambda$-closed, by Theorem 3. Hence $A \in O^{gb\Lambda}(X)$. □

The converse of Theorem 53 is not necessarily true.

Example 54. Let $X = \{k_1, k_2, k_3, k_4, k_5\}$ and $\tau = \Big\{\varnothing, \{k_1\}, \{k_2\}, \{k_1, k_2\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}, \{k_2, k_3, k_4\}, \{k_1, k_2, k_3, k_4\}, \{k_2, k_3, k_4, k_5\}, X\Big\}$. Then $\{k_1, k_5\}$ and $\{k_1, k_2, k_4, k_5\}$ are $gb\Lambda$-open but none of them is open.

Theorem 55. *Let η be a topological space. Then $C(\tau) \subseteq O^{gb\Lambda}(X)$.*

Proof. Let $A \in C(\tau)$. Then $X \setminus A \in \tau \subseteq C^\lambda(\tau) \subseteq C^{gb\Lambda}(\tau)$. Therefore $A \in O^{gb\Lambda}(X)$. □

For the converse of Theorem 55, we consider the following

Example 56. Let $X = \{k_1, k_2, k_3, k_4, k_5\}$ and $\tau = \Big\{\varnothing, \{k_1\}, \{k_2\}, \{k_1, k_2\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}, \{k_2, k_3, k_4\}, \{k_1, k_2, k_3, k_4\}, \{k_2, k_3, k_4, k_5\}, X\Big\}$. Then $\{k_1, k_2\}$, $\{k_2, k_3, k_4\}$ and $\{k_1, k_2, k_3, k_5\}$ are $gb\Lambda$-open, but none of them is closed.

Theorem 57. *Let η be a topological space and $A \subseteq X$. Then for $A \in C^b(\tau) \cap O^{gb\Lambda}(X)$, $A \in O^\lambda(X)$.*

Proof. Since A is $gb\Lambda$-open and $A \subseteq A$, where A is b-closed, so $A \subseteq \mathrm{Int}_\lambda(A)$. Therefore $X \setminus \mathrm{Int}_\lambda(A) \subseteq X \setminus A$. This implies $Cl_\lambda(X \setminus A) \subseteq X \setminus A$. Moreover $X \setminus A \subseteq Cl_\lambda(X \setminus A)$. Therefore $Cl_\lambda(X \setminus A) = X \setminus A$ and consequently, $X \setminus A$ is λ-closed. Hence $A \in O^\lambda(X)$. □

Theorem 58. *Let η be a topological space and $A \in O^{gb\Lambda}(X)$. Then for $\mathrm{Int}_\lambda(A) \subseteq B \subseteq A$, $B \in O^{gb\Lambda}(X)$.*

Proof. $\mathrm{Int}_\lambda(A) \subseteq B \subseteq A$ implies $X \setminus A \subseteq X \setminus B \subseteq X \setminus \mathrm{Int}_\lambda(A) = Cl_\lambda(X \setminus A)$, where $X \setminus A$ is $gb\Lambda$-closed. Therefore $X \setminus B$ is $gb\Lambda$-closed in X, by Theorem 49. Hence B is $gb\Lambda$-open. □

Theorem 59. *Let η be a topological space. Let $A \in O^{gb\Lambda}(X)$ and $G \in O^b(X)$ with $\mathrm{Int}_\lambda(A) \cup (X \setminus A) \subseteq G$. Then $G = X$.*

Proof. $\text{Int}_\lambda(A) \cup (X \setminus A) \subseteq G$ implies $X \setminus G \subseteq \big((X \setminus \text{Int}_\lambda(A)\big) \cap A = Cl_\lambda(X \setminus A) \setminus (X \setminus A)$. Since $X \setminus A$ is $gb\Lambda$-closed, $Cl_\lambda(X \setminus A) \setminus (X \setminus A)$ does not contain any non-empty b-closed set, by Theorem 24. Since $X \setminus G \subseteq Cl_\lambda(X \setminus A) \setminus (X \setminus A)$, we must have $X \setminus G = \varnothing$. Hence $G = X$. $\square$

Theorem 60. *Let η be a topological space and $A \subseteq X$. Then for $A \in C^{gb\Lambda}(\tau)$, $Cl_\lambda(A) \setminus A \in O^{gb\Lambda}(X)$.*

Proof. Let $F \subseteq Cl_\lambda(A) \setminus A$, where F is b-closed. Since A is $gb\Lambda$-closed, $Cl_\lambda(A) \setminus A$ does not contain any non-empty b-closed subset, by Theorem 24. Therefore $F = \varnothing$. So, $\varnothing = F \subseteq \text{Int}_\lambda(Cl_\lambda(A) \setminus A)$. Therefore $Cl_\lambda(A) \setminus A$ is $gb\Lambda$-open. $\square$

Theorem 61. *Let η be a door space. Then $\mathcal{P}(X) \subseteq O^{gb\Lambda}(X)$.*

Proof. Let A be a subset of a door space η. Since every subset in a door space is either open or closed, A is either open or closed. Therefore by Theorem 53 or 55, we have A is $gb\Lambda$-open in X. $\square$

We consider the following example to show that the converse of Theorem 61 is not true, in general.

Example 62. Consider the topological space in Example 39. In this space, every subset is $gb\Lambda$-closed and hence $gb\Lambda$-open, but the space is not a door space.

Theorem 63. *Let η be a topological space. Then for each $x \in X$, either $\{x\} \in C^b(\tau)$ or $\{x\} \in O^{gb\Lambda}(\tau)$.*

Proof. Suppose that $\{x\} \notin C^b(\tau)$. Then $X \setminus \{x\} \notin O^b(X)$. Since X is the only b-open set containing $X \setminus \{x\}$, $Cl_\lambda(X \setminus \{x\}) \subseteq X$. Hence $X \setminus \{x\} \in C^{gb\Lambda}(\tau)$ implies $\{x\} \in O^{gb\Lambda}(\tau)$. Hence either $\{x\} \in C^b(\tau)$ or $\{x\} \in O^{gb\Lambda}(\tau)$. $\square$

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