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TWO-DIMENSIONAL UNSTEADY PULSATION FLOW OF A VISCOUS INCOMPRESSIBLE FLUID BETWEEN THE POROUS WALLS

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Abstract. Two-dimensional unsteady pulsation flow of a viscous incompressible fluid through a porous channel is considered. This motion gets excited from the periodical time change of a pressure drop and a percolation velocity.

1. Introduction

The problems of viscous conducting fluid flows in channels are classical problems of magnetic hydrodynamics. Beginning with the work of Hartmann, who considered the flow in a planar channel, up to present days, a considerable number of studies have been devoted to this issue. Recently, interest in such flows has increased due to applications to MHD generators. There we have to deal with the flow of a conducting fluid in a common rectangular cross-section channel with two non-conducting and two conducting walls, with a transverse magnetic field applied along the latter walls. A similar problem for perfectly conducting electrodes and ideally insulating sidewalls was solved in the articles [1,2,6,8,9,12]. However, its solution was either not obtained in a finite form, or it was impossible to obtain integral characteristics of the flow for large Hartmann numbers from a formal solution.

The approximate method presented below gives the possibility to find a solution in a practically convenient form, as well as take into account the final conductivity of the channel walls.

2. Basic Part

Let us consider the unsteady flow of a viscous fluid in a porous channel with a constant cross-section. If ox is directed in parallel to the walls, and the axis oy is perpendicular to them, then the equations of non-steady two-dimensional motion of a viscous incompressible fluid will be as in [3–5,7,10,11,13–15]:

$$\begin{aligned} &\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),\\ &\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right),\\ &\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \end{aligned} \tag{1}$$

The desired velocities $u(x,y,t)$ and $v(x,y,t)$ must satisfy the following limiting conditions:

$$\begin{aligned} u(x,y,0)=&0, \quad v(x,y,0)=0,\\ u(x,-h,t)=&0, \quad v(x,-h,t)=v_{w_1}(t),\\ u(x,h,t)=&0, \quad v(x,h,t)=v_{w_2}(t). \end{aligned} \tag{2}$$

Let us introduce the following dimensionless quantities:

$$u=u_0u_1, \quad v=v_0v_1, \quad x=lx_1=\frac{u_0}{v_0}hx_1, \quad y=hy_1, \quad t=\frac{A^2}{\nu}t_1,$$
$$\rho=\frac{\nu u_0^2}{v_0h}P_1, \quad v_{w_1}=v_0v_{01}, \quad v_{w_2}=v_0v_{02}.$$

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Then from system (1) we will have equations in a dimensionless form:

$$\begin{aligned}&\left(\frac{v_0}{u_0}\right)^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial t}=R_0\Big(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\Big)+\frac{\partial p}{\partial x},\\&\left(\frac{v_0}{u_0}\right)^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\frac{\partial v}{\partial t}=R_0\Big(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\Big)+\Big(\frac{u_0}{v_0}\Big)\frac{\partial p}{\partial y},\\&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\end{aligned}\tag{3}$$

where u_0,v_0 - are the characteristic average velocity and rate of infiltration, accordingly;

l - is the length of the channel,

h - is half distance between the walls,

$R_0=\frac{v_0 h}{\nu}$ - is the number of Reynolds infiltration. In system (3) indices are down for the sake of simplicity.

We are looking for solutions of system (3) in the following form:

$$u(x,y,t)=(1-x)\frac{\partial f(y,t)}{\partial y},\quad v(x,y,t)=f(y,t).$$

Then it will be as

$$\begin{aligned}\frac{\partial^3 f}{\partial y^3}-\frac{\partial^2 f}{\partial y\partial t}=&R_0\left[f\frac{\partial^2 f}{\partial y^2}-\Big(\frac{\partial f}{\partial y}\Big)^2\right]+\frac{1}{1-x}\frac{\partial p}{\partial x}\\\frac{\partial^2 f}{\partial y^2}-\frac{\partial f}{\partial t}=&R_0 f\frac{\partial f}{\partial y}+\frac{u_0}{v_0}\frac{\partial p}{\partial y}.\end{aligned}\tag{4}$$

Let the values $\frac{1}{1-x}\frac{\partial p}{\partial x}$, $v_{01}(t)$ and $v_{02}(t)$ vary according to the periodic law:

$$\begin{aligned}\frac{1}{1-x}\frac{\partial p}{\partial x}&=a+\varepsilon e^{i\omega t}b,\\v_{01}&=c\left(1+\varepsilon e^{i\omega t}\right),\end{aligned}$$

where a , b are unknown constants determined from the boundary conditions;

c and d are the stated constants.

We will search for the function a$f(y,t)$ in the following form:

$$f(y,t)=\varphi(y)+\varepsilon e^{i\omega t}\phi(y).\tag{5}$$

Substituting (5) into system (4) and neglecting the terms containing and above from the first equation of system (4), we have

$$\varphi^{'''}=R_0\left(\varphi\varphi^{'}-\varphi^{'2}\right)+a,\tag{6}$$

$$\phi^{'''}-i\omega\phi^{'}=R_0\left(\phi\varphi^{''}+\varphi\phi^{''}-2\varphi^{'}\phi^{'}\right)+b,\tag{7}$$

and from (2), we have the following boundary conditions:

$$\begin{aligned}\varphi(-1)=c,&\quad\varphi(1)=d\\\varphi^{'}(-1)=0,&\quad\varphi^{'}(1)=0,\\\phi(-1)=c,&\quad\phi(1)=d,\\\phi^{'}(-1)=0,&\quad\phi^{'}(1)=0.\end{aligned}$$

Assume that the Reynolds number of infiltration $R_0 = \frac{v_0 h}{\nu}$ is a small quantity. Here, we present the functions $\varphi(y)$ and $\phi(y)$, as well as the unknown constants a and b as the series on powers R_0:

$$\begin{gathered}\varphi(y) = \sum_{k=0}^{\infty} R_0^k, \quad \phi(y) = \sum_{k=0}^{\infty} R_0^k \phi_k(y), \\ a = \sum_{k=0}^{\infty} R_0^k a_k, \quad b = \sum_{k=0}^{\infty} R_0^k b_k.\end{gathered} \tag{8}$$

Substituting (8) into equations (6) and (7) and equating the coefficients at the same powers R_0, we get in the first two approximations:

$$\begin{aligned}&\varphi_0^{'''} = a_0, \\ &\phi_0^{'''} - i\omega\phi_0^{'} = b_0, \\ &\ldots\ldots\ldots\ldots\ldots \\ &\varphi_1^{'''} = \varphi_0\varphi_0^{''} - \varphi^{'2} + a_1, \\ &\phi_1^{'''} - i\omega\phi_1^{'} = b_1 + \varphi_0\phi_0^{''} + \varphi_0^{''}\phi_0 - 2\varphi_0^{'}\phi_0^{'}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,\end{aligned} \tag{9}$$

where the functions φ_0, ϕ_0, φ_1 and ϕ_1 functions must satisfy the following boundary conditions:

$$\begin{gathered}\varphi_0(-1) = c, \quad \varphi_0(1) = d, \quad \varphi_0^{'}(-1) = 0, \quad \varphi_0^{'}(1) = 0, \\ \phi_0(-1) = c, \quad \phi_0(1) = d, \quad \phi_0^{'}(-1) = 0, \quad \phi_0^{'}(1) = 0, \\ \varphi_k(-1) = 0, \quad \varphi_k(1) = 0, \quad \phi_k^{'}(-1) = 0, \quad \phi_0^{'}(1) = 0,\end{gathered}$$

where $k \geq 1, 2, \ldots$, and strokes show derivatives on y.

The solution of system (9) is not difficult. We find the functions φ_0, φ_1 and ϕ_0, as well as the values a_0, b_0, a_1. Finding the function φ_1 and value b_1 in the allowed approximation does not make sense, since the terms ϕ_1 and b_1 have coefficients as the product of two infinitely small values εR_0.

Thus, for φ_0, ϕ_0, φ_1, a_0, b_0 and a_1, we obtain the following expressions:

$$\begin{gathered}\varphi_0(y) = A\left(y^3 - 3y\right) + B, \\ \phi_0(y) = \frac{1}{D}\left[4A\left(sh\sqrt{i\omega}y - y\sqrt{i\omega}ch\sqrt{i\omega}\right) + B\right], \\ a_0 = 6A, \quad b_0 = \frac{1}{D}\left[4a(i\omega)^{3/2}ch\sqrt{i\omega}\right], \quad a_1 = \frac{324}{35}A^2 \\ \varphi_1 = \frac{AB}{4}\left(y^2 - 1\right)^2 - \frac{A^2}{70}\left(y^7 - 3y^3 + 2y\right),\end{gathered}$$

with the notation

$$A = \frac{c-d}{4}, \quad B = \frac{c+d}{2}, \quad D = 2\left(sh\sqrt{i\omega} - \sqrt{i\omega}ch\sqrt{i\omega}\right).$$

Finally, for the components of velocity and pressure drops along and across the main flow, in the proposed approximation, we will have

$$\begin{aligned}&u(x,y,t) = (1-x)\left[\varphi_0^{'}(y) + R_0\varphi_1^{'}(y) + \varepsilon e^{i\omega t}\phi_0^{'}(y)\right], \\ &v(x,y,t) = \varphi_0(y) + R_0\varphi_1(y) + \varepsilon e^{i\omega t}\left[\phi_0(y) + R_0\phi_1(y)\right], \\ &\frac{\partial p}{\partial x} = (1-x)[a_0 + R_0 a_1 + \varepsilon e^{i\omega t} b_0], \\ &\frac{\partial p}{\partial y} = \frac{v_o}{u_0}\left[\varphi_0^{''} + R_0\varphi_1^{''} + \varepsilon e^{i\omega t}\left(\phi_0^{''} - i\omega\phi_0\right) - R_0\varphi_0\varphi_0^{'}\right].\end{aligned}$$

3. Conclusion

Thus, the pulsating flow of a viscous incompressible fluid between porous walls was studied. Fluid flow is caused by pulsating pressure drop and pulsating movement of porous walls [3, 10, 11, 14].

References

1. K. V. Deshikachar, R. A. Ramachandra, Magnetohydrodynamic unsteady flow in a tube of variable cross section in an axial magnetic field. *The Physics of fluids* 30 (1987), no. 1, 278–279.
2. L. A. Jikidze, V. N. Tsutskiridze, Approximate Method for Solving an Unsteady Rotation Problem for a Porous Plate in the Conducting Fluid with Regard for the Heat Transfer in the Case of Electroconductivity. In: *Several Problems of Applied Mathematics and Mechanics*, pp. 157-164, Series: Science and Technology Mathematical Physics (ebook), New York, 2013.
3. L. A. Jikidze, V. N. Tsutskiridze, The unsteady flow of conductind liquid squeezed between two parallel infinite rotating porous disks taking into account the strong magnetic field and the heat transfer. *Transactions of GTU (Georgian Technical University)*, **510** (2018), no. 4, 126–135.
4. L. A. Landau, E. M. Lifshits, *Electrodynamics of Continua.* (Russian) GITTL, Moscow, 1982.
5. L. G. Loitsiansky, *Mechanic of a Fluid and Gas.* (Russian) Sciences, Moscow, 1987.
6. J. T. Ramos, N. S. Magnetohydrodynamic channel flow study. *Phys. Fluids* **29** (1986), no. 4, 992–997.
7. J. Sharikadze, V. Tsutskiridze, L. Jikidze, Pulsating flow of weakly conductive liquid with heat transfer. *Appl. Math. Inform. Mech.* **18** (2013), no. 1, 61–73.
8. J. V. Sharikadze, V. Tsutskiridze, L. Jikidze, The unsteady flow of incompressible fluid in a constant cross section pipes in an external uniform magnetic field. *International scientific journal of IFTOMM Problems of Mechanics.* **1** (2013), no. 50, 77–83.
9. V. Tsutskiridze, Hartmann problem for circular pipe. In: *The International scientific Conference on "Mechanics 2016". Proceedings of Mechanics 2016, Tbilisi*, 2016, pp. 89–92.
10. V. Tsutskiridze, Flow of viscous fluid in initial section of plane channel with porous walls. *International Scientific Journal of IFTOMM Problems of Mechanics.* **2** (2018), no. 71, 43–47.
11. V. Tsutskiridze, The stationary flow of laminar liquid in an circular pipe of infinite length. *Appl. Math. Inform. Mech.*, **23** (2018), no. 1, 47–51.
12. V. Tsutskiridze, L. Jikidze, The conducting liquid flow between porous walls with heat transfer. *Proc. A. Razmadze Math. Inst.* **167** (2015), 73–89.
13. V. Tsutskiridze, L. Jikidze, The nonstationary flow of a conducting fluid in a plane pipe in the presence of a transverse magnetic field. *Trans. A. Razmadze Math. Inst.* **170** (2016), no. 2, 280–286.
14. V. Tsutskiridze, L. Jikidze, The flow of weakly electroconductive liquid between porous walls with heat transfer. *AMIM*, Tbilisi **24** (2019), no. 1, 31–44.
15. A. B. Vatazin, G. A. Lubimov, C. A. Regirer, *Magnetohydrodynamic Flow in Channels.* (Russian) Nauka, Moscow, 1970.

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