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MODELLING POLLUTION OF RADIATION VIA TOPOLOGICAL MINIMAL STRUCTURES

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Abstract. The model of a generalized variable precision rough set is one of the variable precision rough sets used to solve some problems and measurements confront us that was difficult from the view point of science. The behavior of the radio contaminants in the environment is one of these measurements. Throughout this paper, we introduce and study a generalization variable precision rough set via a topological minimal structure. Some characteristics related to generalized upper and lower approximation with a variable precision by minimal structures will be discussed. A dispersion model which is the necessity to predict atmospheric path and danger from an atmospheric plume of hazardous materials will be applied with different types of examples.

1. Introduction and Preliminaries

Pawlak in [17] and [18] introduced rough sets as a formal tool to deal with uncertainty in the data analysis. It was based on the equivalence relation and crisp sets. Dudois et al. [4] introduced the notions of fuzzy roughness of information system in decision making. The connection between rough sets and topological spaces was investigated in [16] and [12]. Ziarko [27] extended rough sets through variable precision rough sets which are not only solve the problems with uncertain data, but also relax the strict definition of a rough set. He also studied the relative error limit of the partition blocks with the inclusion order $A \subseteq_\beta B$ if and only if $C(A, B) \leq \beta$, where β is called the majority inclusion relation, $0 \leq \beta \leq 0.5$. Both of the concept analyses [2] and rough sets are two significant mathematical creatures for the data analysis and knowledge processing. In 2000, Popa et al. [20] introduced the notion of minimal structure. Also, they introduced the notion of m_x-open sets and m_x- closed sets and characterized those sets using m_x-closure and m_x- interior operators, respectively. They defined in [15] separation axioms using the concept of minimal structure spaces and studied m_x^1 m_x^2-open in bimanual structure spaces. Recently, the neighborhood systems and rough sets on information system are used to represent structures such as self-similar fractals [7] and Human Heart [6] which are useful in physics and medicine, respectively. Also, the reduction of information system can be calculated by similarity as in [5].

Many researchers were interested in the atmospheric modeling as Santos et al. in [21] that is shown in Figure 1 for GPU-based implementation of a real-time model for atmospheric dispersion of radionuclides. Also, Cherradi et al. in [3] introduced the model of an atmospheric dispersion modeling microservice for HazMat transportation shown in Figure 2. These models were used by mathematicians to rephrase these models for each aspect of their study.

The aim of this paper is to use the model of Multilevel Meteorological Tower in National Center for Nuclear Safety and Radiation control, AEA, Egypt. We introduce the notion of generalized variable precision rough sets using a minimal structure. Some properties of generalized lower approximation (resp., generalized upper approximation) $\underline{R}_m^\beta$ (resp., $\overline{R}_m^\beta$) operator with e_x (resp., d_x) will be investigated. Atmospheric dispersion models which are important to predict path and danger from an atmospheric plume of hazardous materials with numerous kinds of examples will be applied.

Definition 1.1 ([19])**.** If U is a finite set of objects called universe, R a finite set of equivalence relations on U, called attributes, then the pair $K(U, R)$ is called an information system.

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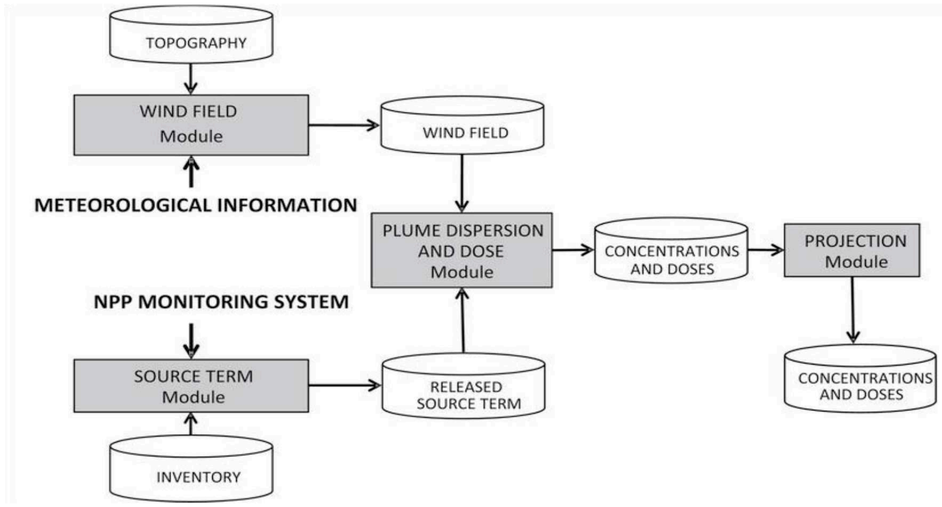

FIGURE 1. CNAAA site and th5e region covered by the ADR system

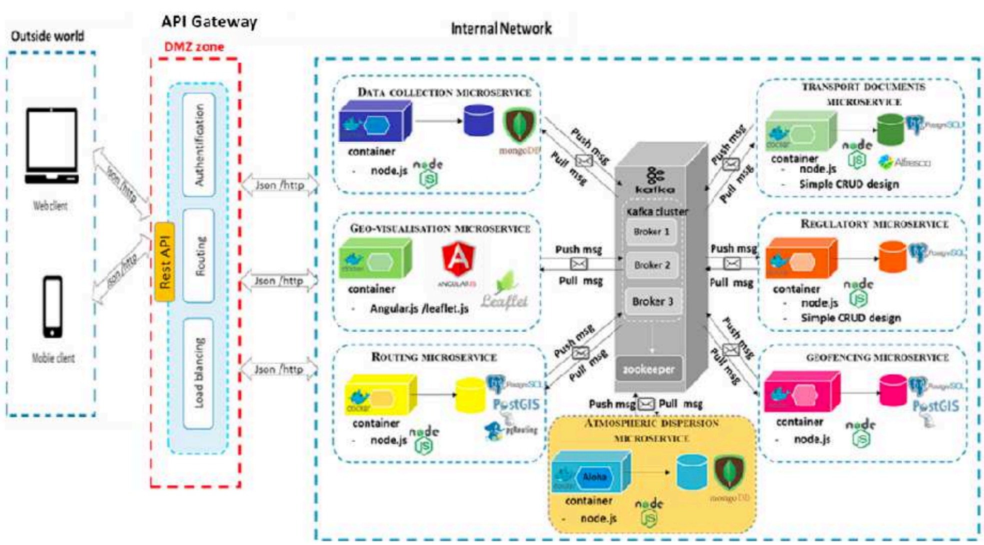

FIGURE 2. Overall system architecture

Definition 1.2 ([19])**.** Let X be a subset of U, the lower and upper approximation of X in U is defined by $\underline{R}(X) = \bigcup\{Y \in U/R : Y \subseteq X\}$, $\overline{R}(X) = \cup\{Y \in U/R : Y \cap X \neq \phi\}$.

Definition 1.3 ([27])**.** Let X and Y be a nonempty subset of a finite universe U. The measure of the relative degree of misclassification of the set X with respect to the set Y, denoted by $C(X,Y)$, is defined by

$$C(X,Y) = \begin{cases} \frac{1-\mathrm{Card}(X\cap Y)}{\mathrm{Card}(X)}, & \text{if} \quad \mathrm{Card}(X) > 0; \\ 0, & \text{if} \quad \mathrm{Card}(X) = 0, \end{cases}$$

where $\mathrm{Card}(X)$ denotes to cardinality of X.

Definition 1.4 ([27])**.** Let X and Y be nonempty subsets of a finite universe set U. $X \subseteq_\beta Y$ if $C(X,Y) \leq \beta$.

Definition 1.5 ([27]). Let $X \subseteq Y$ be a subset of U. The β-lower $\underline{R}_\beta$ and β-upper $\overline{R}^\beta$ approximation of X under a relation R are defined by
$\underline{R}_\beta = \bigcup\{E \in U/R : C(E,X) \leq \beta\}$;
$\overline{R}^\beta = \bigcup\{E \in U/R : C(E,X) \leq 1-\beta\}$.

Definition 1.6 ([14]). Let X be a nonempty set and $P(X)$ the power set of X. A subfamily of m_x is called a minimal structure (m-structure) on X if $\phi \in m_x$ and $X \in m_x$. By (X, m_x), we denote a nonempty set X with an m-structure m_x on X and call it an m-space. Each member of m_x is said to be m_x-open and the complement of an m_x-open set is said to be m_x-closed.

Definition 1.7 ([14]). Let X be a nonempty set and m_x be an m-structure on X. For a subset A of X, the m_x-closure (resp., m_x-interior) of A which is denoted by m_x-$Cl(A)$ (resp., m_x-Int(A)) is defined by m_x-$Cl(A) = \cap\{F : A \subseteq F, X - F \in m_x\}$ (resp., m_x-Int$(A) = \cap\{U : U \subseteq A, U \in m_x\}$).

Lemma 1.8 ([13]). *Let X be a nonempty set and m_x be an m-structure on X. For a subsets $A, B \subseteq X$, the following properties hold:*
(1) $m_x - Cl(X-A) = X - (m_x - \mathrm{Int}(A))$, $m_x - \mathrm{Int}(X-A) = X - (m_x - Cl(A))$.
(2) *If* $X - A \in m_x$ *then,* $m_x - Cl(A) = A$, *if* $A \in m_x$, *then* $m_x - \mathrm{Int}(A) = A$.
(3) $m_x - Cl(\phi) = \phi$, $m_x - Cl(X) = X$, $m_x - \mathrm{Int}(\phi) = \phi$, $m_x - \mathrm{Int}(X) = X$.
(4) *If* $A \subseteq B$, *then* $m_x - Cl(A) \subseteq m_x - Cl(B)$ *and* $m_x - \mathrm{Int}(A) \subseteq m_x - \mathrm{Int}(B)$.
(5) $m_x - Cl(m_x - Cl(A)) = m_x - Cl(A)$, $m_x - \mathrm{Int}(m_x - \mathrm{Int}(A)) = m_x - \mathrm{Int}(A)$.
(6) $A \subseteq m_x - Cl(A)$ *and* $m_x - \mathrm{Int}(A) \subseteq A$.

Lemma 1.9 ([13]). *For the subsets A on m_x an m-structure on X, $x \in m_x - Cl(A)$ if and only if $U \cap A \neq \phi$, for every m_x-open set U containing x.*

2. Generalized Granular Variable Precision Approximation Operators

Definition 2.1. Let $R \subset X \times X$ be a relation and $\beta \in [0,1]$. Two maps $\underline{R}_m^\beta(A)$ and $R_m^\beta(A)$ are defined as follows, $A \subseteq X$:
$\underline{R}_m^\beta(A) = \cup\{E : E \in m_x, e_x(E,A) \geq \beta\}$,
$\overline{R}_m^\beta(A) = \cap\{E^c : E \in m_x, d_x(E^c,A) \leq \beta\}$,
where e_x, d_x are defined as: $e_x(A,B) = \frac{n(A^c \cup B)}{n(X)}$, $d_x(A,B) = \frac{n(A^c \cap B)}{n(X)}$ and $n(X)$ denotes a number of elements of X . Then $\underline{R}_m^\beta(A)$ and $\overline{R}_m^\beta(A)$ are said to be a generalized lower approximation (resp., upper approximation) operator. $(\underline{R}_m^\beta(A), R_m^\beta(A))$ is said to be a generalized variable precision rough set which is determined by e_x and d_x.

Remark 2.2.
(i) If $\beta = 1$, i.e., $e_x(E,A) = 1$, then $E^c \cup A = X$, i.e., $E \subseteq A$ such that $E \in m_x, A \subseteq X$. Hence $\underline{R}_m^1(A) = \bigcup\{E : E \in m_x, E \subset A\}$.
(ii) If $\beta = 0$, i.e., $d_x(E^c,A) = 0$, then $E \cap A = \phi$. So, $A \subseteq E^c$, and then $\overline{R}_m^0(A) = \bigcap\{E^c : E \in m_x, A \subseteq E^c\}$.

Theorem 2.3. *Let R be a relation on X and $\beta \in [0,1]$. The following hold:*
(i) $d_x(E^c, A^c) = 1 - e_x(E,A)$ *for each* $A \subseteq X$.
(ii) $\underline{R}_m^\beta(A) = (\overline{R}_m^{1-\beta}(A^c))^c$ *for each* $A \subseteq X$.
(iii) $\underline{R}_m^1(\phi) = \phi$, $\overline{R}_m^0(X) = X$.
(iv) $\underline{R}_m^1(X) = \cup_{E \subseteq X} E$, $\overline{R}_m^0(\phi) = \cap_{E \subseteq X} E^c$.
(v) *If* $\forall\ x \in X$, $\exists\ E \subseteq X$ *s. t.* $x \in E$, *then* $\underline{R}_m^1(X) = X$, $\overline{R}_m^0(\phi) = \phi$.

Proof. (i) Follows from $d_x(E^c, A^c) = \frac{n(E \cap A^c)}{n(X)} = \frac{n(X) - n(E^c \cup A)}{n(X)} = 1 - e_x(E,A)$.
(ii) Deduced by $(\overline{R}_m^{1-\beta}(A^c))^c = (\bigcap\{E^c : E \in m_x, d_x(E^c, A^c) \leq 1-\beta\})^c = \bigcup\{E : d_x(E^c, A^c) \leq 1-\beta\} = \bigcup\{E : 1 - e_x(E,A) \leq 1-\beta\} = \underline{R}_m^\beta(A)$.
(iii) Follows from Remark 2.2.

(iv) Since $\underline{R}^1_m(X) = \bigcup\{E : E \in m_x, E \subseteq X\} = \bigcup_{E\subseteq X} E$, $\underline{R}^0_m(\phi) = \bigcap\{E^c : E \in m_x, \phi \subset E^c\} = \bigcap_{E\subset X} E^c$.
(v) For each $x \in X$, there exists $E \subseteq X$ such that $x \in\in E$ and $X = \bigcup_{E\subset X} E$. Hence $\underline{R}^1_m(X) = X$, $\overline{R}^0_m(\phi) = \phi$. □

Theorem 2.4. *Let R be a relation on X and $\beta \in [0,1]$, $E \in m_x$, $A \subset X$. The following hold:*
(i) $\underline{R}^1_m(A) \subseteq A$ *and* $A \subseteq \overline{R}^0_m(A)$.
(ii) *If* $\beta_1 \leq \beta_2$, *then* $\underline{R}^{\beta_2}_m(A) \subseteq \underline{R}^{\beta_1}_m(A)$.
(iii) *If* $A_1 \subseteq A_2$, *then* $\underline{R}^{\beta}_m(A_1) \subseteq \underline{R}^{\beta}_m(A_2)$.
(iv) *If* $\beta_1 \leq \beta_2$, *then* $\overline{R}^{\beta_2}_m(A) \subseteq \overline{R}^{\beta_1}_m(A)$.
(v) *If* $A_1 \subseteq A_2$, *then* $\overline{R}^{\beta}_m(A_1) \subseteq \overline{R}^{\beta}_m(A_2)$.
(vi) $\underline{R}^1_m(E) = E$, $\overline{R}^0_m(E^c) = E^c$, $E \in m_x$.
(vii) $\underline{R}^1_m(\underline{R}^{\beta}_m(A)) = \underline{R}^{\beta}_m(A)$ *and* $\overline{R}^0_m(\underline{R}^{\beta}_m(A)) = \underline{R}^{\beta}_m(A)$.

Proof. (i) By Remark 2.2, we have $\underline{R}^1_m(A) = \cup\{E : E \in m_x, E \subset A\} \subset A$. Moreover, $\overline{R}^0_m(A) = \cap\{E^c : E \in m_x, E \subset A^c\} \supset A$.
(ii), (iii), (iv) and (v) are obvious from Definition 2.1.
(vi) Since $e_x(E,E) = \frac{n(E^c\cup E)}{n(X)} = 1$, $\underline{R}^1_m(E) = \bigcup\{E : E \in m_x, E \subseteq E\} \supseteq E$. By (i), $\underline{R}^1_m(E) \subseteq E$, we have $\underline{R}^1_m(E) = E \forall E \in m_x$. Since $d_x(E^c,E^c) = \frac{n(E\cap E^c)}{n(X)} = 0$, $\overline{R}^0_m(E^c) = \bigcap\{E^c : E \in m_x, E^c \subseteq E^c\} \subseteq E^c$. By (i), $\overline{R}^0_m(E^c) \supseteq E^c$. Hence $\overline{R}^0_m(E^c) = E^c \forall E \in m_x$.
(vii) For $E \subseteq \underline{R}^{\beta}_m(A)$, $e_x(E,\underline{R}^{\beta}_m(A)) = \frac{n(E^c\cup \underline{R}^{\beta}_m(A))}{n(X)} = 1$, $\underline{R}^1_m(\underline{R}^{\beta}_m(A)) = \bigcup\{E : E \in m_x, E \subseteq \underline{R}^{\beta}_m(A)\} \supseteq \underline{R}^{\beta}_m(A) \supseteq \underline{R}^{\beta}_m(A)$. By (i), $\underline{R}^1_m(\underline{R}^{\beta}_m(A)) \subseteq \underline{R}^{\beta}_m(A)$. Hence $\underline{R}^1_m(\underline{R}^{\beta}_m(A)) = \underline{R}^{\beta}_m(A)$. For $E^c \supset \underline{R}^{\beta}_m(A)$, $d_x(E^c,\underline{R}^{\beta}_m(A)) = \frac{n(E\cap \underline{R}^{\beta}_m(A))}{n(X)} = 0$, $\overline{R}^0_m(\underline{R}^{\beta}_m(A)) = \bigcap\{E^c : E \in m_x, \underline{R}^{\beta}_m(A) \subseteq E^c\} \subseteq \underline{R}^{\beta}_m(A)$. By (i)$\overline{R}^0_m(\overline{R}^{\beta}_m(A)) \supseteq \overline{R}^0_m(A)$. Hence $\overline{R}^0_m(\overline{R}^{\beta}_m(A)) = \overline{R}^0_m(A)$. □

Theorem 2.5. *Let R be a relation on X and $\beta \in [0,1]$. Define $T_X, F_X \subseteq P(X)$ as $T_X = \{A \subseteq X : A = \underline{R}^{\beta}_m(A)$, $F_X = \{A \subseteq X : A = \overline{R}^0_m(A)\}$*
(i) $\phi \in T_X, E \in T_X, \forall E \in m_x, \underline{R}^{\beta}_m(A) \in T_X$ *for each* $A \subset X$, $\beta \in [0,1]$.
(ii) *If* $A_i \in T_X$ *for each* $i \in I$, *then* $\cup_{i\in I} A_i \in T_X$.
(iii) $X \in F_X, E^c \in F_X \forall E \in m_x$ *and* $\underline{R}^{\beta}_m(A) \in F_X A \subset X$, $\beta \in [0,1]$.
(iv) *If* $A_i \in F_X$ *for each* $i \in I$, *then* $\cap_{i\in I} A_i \in F_X$.
(v) $A \in T_X$ *if and only if* $A^c \in F_X$.
(vi) *If* $\forall\ x \in X$, $\exists\ E \in m_x$ *s.t.* $x \in E$, *then* $X \in T_X$ *and* $\phi \in F_X$.

Proof. Follows directly by Theorems 2.3 and 2.4. □

Definition 2.6. Let R be a relation on X and $\beta \in [0,1]$, $P, Q \subseteq R$ be families of open sets, say, attributes. Then Q is said to depend in degree $\gamma(P,Q,\beta)$ on attributes P. $\gamma(P,Q,\beta) = \frac{|Pos(P,Q,\beta)|}{|X|}$, $\gamma(P,Q,\beta) = \bigcup_{E\in Q} \underline{P}_m(E)$ is P_m-positive region of Q.

Example 2.7. *Define R such that xR_By if $\sum\limits_{i\in B} \frac{|i(x)-i(y)|}{|B|} < \lambda$, where $B \subseteq \{a,b,c,f\}$. Attributes in Table 2 are similar by $\mid c(x_i) - c(y_i) \mid$, where $i,j \in \{1,2,3,4,5,6\}$ for $B = \{c\}$.*

Consider $xR_{\{c\}}y$ if and only if $\mid c(x) - c(y) \mid < 5$, $X = \{x_1,x_2,\ x_3,x_4,\ x_5,x_6,\}$, $m_x = \{\phi, X, \{x_1,x_4,\ x_6\},\{x_2,\ x_3,x_5\},\ \{x_2,x_3,\ x_4,x_5,\ x_6\},\{x_1,\ x_3,x_4,\ x_5,x_6\}\}$, $X - m_x = \{\phi, X, \{x_1,\ x_4,x_6\}, \{x_2,x_3,\ x_5\},\{x_1\}\}$, $\underline{R}^{\beta}_m(A) = \bigcup\{E : E \in m_x,\ e_x(E,A) \geq \beta\} = \bigcup\{E : E \in m_x,\ \frac{n(E^c\cup A)}{n(X)} \geq \beta\}$, $\overline{R}^{\beta}_m(A) = \cap\{E^c : E \in m_x,\ d_x(E^c,A) \leq \beta\} = \bigcap\{E^c : E \in m_x,\ \frac{n(E\cap A)}{n(X)} \leq \beta\}$.

If $\beta = 0.8$, then $\underline{R}^{0.8}_m(A) = \{\phi\} \cup \{x_1,\ x_4,x_6\} = \{x_1,x_4,\ x_6\}$, $\overline{R}^{0.2}_m(A^c) = \overline{R}^{0.2}_m(\{x_2,\ x_4,x_5\} = X \cap \{x_2,\ x_3,x_5\} = \{x_2,x_3,\ x_5\}$, $\underline{R}^{0.8}_m(A) = (\overline{R}^{0.2}_m(A^c)^c = \{x_1,x_4,\ x_6,\}$,$\underline{R}^1_m(\underline{R}^{0.8}_m(A)) = \underline{R}^1_m(\{x_1, x_4,x_6,\} = \{x_1,x_4,\ x_6\} = \overline{R}^{0.2}_m(A) = \overline{R}^{0.2}_m\{x_1,\ x_3,x_6\} = \{x_1,x_4,\ x_6\}$, $\underline{R}^{0.8}_m(A^c) = \underline{R}^{0.8}_m\{x_2,\ x_4,x_5\} =$

$\{x_2, x_3, x_5\}$, $\underline{R}_m^{0.8}(A^c) = (\overline{R}_m^{0.2}(A))^c = \{x_2, x_3, x_5\}$,$\underline{R}_m^{1}(\underline{R}_m^{0.8}(A^c)) = \underline{R}_m^{1}(\{x_2, x_3, x_5, \} = \{x_2, x_3, x_5\} = \underline{R}_m^{0.8}(A^c)$,$\overline{R}_m^{0}(\overline{R}_m^{0.2}(A)) = \overline{R}_m^{0}(\{x_1,\, x_4, x_6\} = \{x_1,\, x_4, x_6\} = \overline{R}_m^{0.2}(A)$.

Table 1. IS with original data

	a	b	c	f
x_1	81	77	84	83
x_2	100	81	93	85
x_3	71	78	89	60
x_4	93	82	88	60
x_5	97	87	91	85
x_6	93	68	87	90

Table 2. IS with similarity

c	x_1	x_2	x_3	x_4	x_5	x_6
x_1	0	9	5	4	7	3
x_2	9	0	4	5	2	6
x_3	5	4	0	1	2	2
x_4	4	5	1	0	3	1
x_5	7	2	2	3	0	4
x_6	3	6	2	1	4	0

3. Application on Pollution of Radiation

As a matter of fact, each industry has its own waste. Most of it are hazardous to a man and his environment. Due to the increasing importance of nuclear power industries and the release of different effluents to air through different pathways such as water and soil, later, they will find their way to a man. So, it becomes important to study the behavior of radio contaminants in environment. Several pathways exist, along which radionuclide can be transported to a man; the atmosphere is a pathway for transport of radioactive release from a nuclear power plant to environment and thereby to a man. Atmospheric dispersion models are the necessity to predict path and danger from an atmospheric plume of hazardous materials.

Atmospheric dispersion models are the computer simulation programs, which combine information about the source of a release and observations of wind and weather considerations with theories of atmospheric behaviour to predict the spread and travel of contaminants. The most widely used model is the Gaussian Plume Model [16]. Most of the countries participating in the NATO plume used this model in Germany. The ground level air concentrations are given from the equation

$$C(x,y) = \frac{2Q}{(2\pi\sigma_y\sigma_x + C_w A)u} \exp\left(\frac{-\lambda x}{u}\right) * \left(\exp \int_0^x \frac{dx}{\exp(\frac{H^2}{2\sigma^2})}\right)^{-(\frac{2}{\pi})^{.5}(vd/u)} * \exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_x^2}\right)^2 - \frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$$

according to the Gaussian Plume model such that $C(Bq.m^{-3})$, $Q(Bq.s^{-1})$, $u(ms^{-1})$, $\sigma_y(m)$, $\sigma_z(m)$, $x(m)$, $y(m)$, $z(m)$, $H(m)$, $\exp(\frac{-\lambda x}{u})$, A and C_w denotes CAP, continuous point source strength, WS at height H, lateral dispersion parameter, VDP, HD in direction of downwind, LD from plume center, height above ground, EH of plume above ground, RD for the specified nuclide, CSA of building normal to wind and shape factor that represents the fraction of A over which the plume is dispersed, respectively. $C_w = 0.5$ is commonly used and V_d denotes the deposition velocity. The dispersion parameters σ_y and σ_z can be estimated from $\sigma_y = cx^d$, where a, b, c, and d are shown in Table 3 [10].

Table 3. Parameters Values

Stability	a	b	c	d
$A-B$	1.46	0.71	0.01	1.54
C	1.52	0.69	0.04	1.17
D	1.36	0.67	0.09	0.95
$E-F$	0.75	0.70	0.40	0.67

Table 4. An Experimental Data

Date(2009)	$Exps$	u	$Stability$	H	$Working$	Obs	Est
14-Mar	x_1	4	A	49	48	0.025	0.114
14-Mar	x_2	4	A	48	49	0.037	0.129
02-May	x_3	4	B	45	1.5	0.091	0.122
03-May	x_4	3	C	46	22	0.197	0.219
03-May	x_5	3	A	45	23	0.272	0.307
03-May	x_6		D	45	24	0.188	0.473
04-May	x_7		E	47	48	0.447	0.484
04-May	x_8		C	46	48.7	0.123	0.456
14-May	x_9		A	47	48.25	0.032	0.109
10-Jan	x_{10}		D	28	5	0.42	0.933
22-Jan	x_{11}		B	28.3	7	0.442	0.756
15-Mar	x_{12}		A	30.8	2	0.67	1.144
15-Mar	x_{13}		A	30.6	4	0.67	1.056

3.1. **Experimental data.** In the following subsection we use some experimental data. Air samples were gathered from 92m to 184m around two industrial locations. The area under consideration is flat and prevailed by a sand soil with poor vegetation cover. Also, it was separated into 16 sectors (with $22.5C^{\circ}$) width for each sector), origin from the north direction. Aerosols were gathered at a height of 0.7 m above the ground of 10.3 cm diameter filter paper with a desired collection efficiency ($\frac{3.4}{100}$) using a high volume air sample with 220V/50Hz bias.

Air sample had air flow rate of approximately $\frac{0.7m^3}{min}$ ($\frac{25ft^3}{min}$). Sample collection time was 30 min with an air volume of $21.2m^3(750ft^3)$. Air volume was adjusted to standard conditions (25 C° and 1013 mb). Filter paper was directly evaluated by energy and efficiency calibrated HPGe detectors relative to 3"x3" NaI (TI) detector were 15.6 and $\frac{30}{100}$ measured at $1.332MeV$ with source to detector distance of 25 cm.

Meteorological data were allowed for Ins has meteorological tower for four months at a smooth flat site (Ins has area, Egypt) for the year (2006). Vertical temperature gradient $\frac{\triangle T}{\triangle Z}$ was decided by measuring temperature at $10-60$m levels from the multilevel meteorological tower of Ins has sitting and Environment Department, National Center for Nuclear Safety and Radiation control, AEA, Egypt. This tower is placed near to our search.

Table 3.1 contains the working data, 13 experiments (Exps), wind speed, stability, the effective height (H) for two locations, observed (Obs) and estimated (Est) concentrations $\frac{Bq}{m^3}$ of Iodine 131(I-131) with the working hours.

3.2. **Analysis of data using topological minimal structure.** In the following, we explain that the observed data, satisfying the properties of our generalization, are used to determine the degree of dependence of the estimated and observed data. Define a relation xRy if and only if $\leftrightarrow \mid x_i - x_j \mid \leq 0.1$, $i, j = 1-13$. Table 3.2 shows the relation R_{Obs} between the observed data. Table 3.2 shows the relation R_{Est} between the observed data.

Table 5. Observation Data by R_{Obs}

Obs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}
x_1	0	0.012	0.066	0.172	0.247	0.163	0.422	0.098	0.007	0.39	0.395	0.64	0.645
x_2	0.012	0	0.054	0.16	0.235	0.151	0.41	0.086	0.005	0.38	0.383	0.63	0.633
x_3	0.066	0.054	0	0.106	0.181	0.097	0.356	0.032	0.059	0.32	0.329	0.57	0.579
x_4	0.172	0.16	0.106	0	0.075	0.009	0.25	0.074	0.165	0.22	0.223	0.47	0.473
x_5	0.247	0.235	0.181	0.075	0	0.084	0.175	0.149	0.24	0.14	0.148	0.39	0.398
x_6	0.163	0.151	0.097	0.009	0.084	0	0.259	0.065	0.156	0.23	0.232	0.48	0.482
x_7	0.422	0.41	0.356	0.25	0.175	0.259	0	0.324	0.415	0.02	0.027	0.22	0.223
x_8	0.098	0.086	0.032	0.074	0.149	0.065	0.324	0	0.091	0.29	0.297	0.54	0.547
x_9	0.007	0.005	0.059	0.156	0.24	0.156	0.415	0.091	0	0.38	0.388	0.63	0.638
x_{10}	0.395	0.383	0.329	0.223	0.148	0.232	0.027	0.297	0.388	0	0	0.25	0.25
x_{11}	0.395	0.383	0.329	0.223	0.148	0.232	0.027	0.297	0.388	0	0	0.25	0.25
x_{12}	0.645	0.633	0.579	0.473	0.398	0.482	0.223	0.547	0.638	0.25	0.25	0	0
x_{13}	0.645	0.633	0.579	0.473	0.398	0.482	0.223	0.547	0.638	0.25	0.25	0	0

Table 6. Observation Data by R_{Est}

Est	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}
x_1	0	0.015	0.008	0.105	0.193	0.359	0.37	0.342	0.005	0.719	0.642	1.03	0.942
x_2	0.015	0	0.007	0.09	0.178	0.344	0.455	0.327	0.02	0.704	0.627	1.015	0.927
x_3	0.008	0.007	0	0.097	0.185	0.351	0.362	0.334	0.013	0.711	0.634	1.022	0.934
x_4	0.105	0.09	0.097	0	0.088	0.254	0.265	0.237	0.11	0.614	0.537	0.925	0.837
x_5	0.193	0.178	0.185	0.088	0	0.166	0.177	0.149	0.198	0.526	0.449	0.837	0.749
x_6	0.359	0.344	0.351	0.254	0.166	0	0.11	0.017	0.364	0.36	0.283	0.671	0.583
x_7	0.37	0.355	0.362	0.256	0.177	0.011	0	0.028	0.375	0.349	0.272	0.66	0.572
x_8	0.342	0.327	0.334	0.237	0.149	0.017	0.028	0	0.347	0.377	0.3	0.688	0.6
x_9	0.005	0.02	0.013	0.11	0.198	0.364	0.375	0.347	0	0.742	0.647	1.035	0.947
x_{10}	0.719	0.704	0.711	0.614	0.526	0.36	0.349	0.377	0.742	0	0.077	0.311	0.223
x_{11}	0.642	0.627	0.634	0.537	0.449	0.283	0.272	0.3	0.647	0.077	0	0.388	0.03
x_{12}	1.03	1.015	1.022	0.925	0.837	0.671	0.66	0.688	1.035	0.311	0.388	0	0.088
x_{13}	0.645	0.633	0.579	0.473	0.398	0.482	0.223	0.547	0.638	0.25	0.25	0	0

If $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}$, $m_{X(Obs)} = \{\phi, X, \{x_1, x_2, x_3, x_8, x_9\}, \{x_1, x_2, x_3, x_6, x_8, x_9\}, \{x_4, x_5, x_6, x_8\}, \{x_4, x_5, x_6\}, \{x_3, x_4, x_5, x_6, x_8\}, \{x_1, x_{10}, x_{11}\}, \{x_1, x_2, x_3, x_4, x_6, x_8, x_9\}, \{x_{12}, x_{13}\}\}$, and then $X(Obs) - m_{X(Obs)} = \{\phi, X, \{x_4, x_5, x_6, x_7, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_4, x_5, x_7, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_1, x_2, x_3, x_7, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_1, x_2, x_3, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_1, x_{10}, x_{11}\}, \{x_1, x_2, x_3, x_4, x_6, x_8, x_9\}, \{x_{12}, x_{13}\}, \{x_1, x_2, x_7, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_8, x_9, x_{12}, x_{13}\}, \{x_5, x_7, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}\}\}$.
Let $A \subset X, \beta \in [0, 1]$, $A = \{x_1, x_2, x_5, x_6\}$. If $\beta = 0.8$, then $\underline{R}_m^{0.8}(A) = \{x_4, x_5, x_6, x_8, x_{12}, x_{13}\}$. If $\beta = 0.2$, then $\overline{R}_m^{0.2}(A^c) = \{x_1, x_2, x_3, x_7, x_9, x_{10}, x_{11}\}$. $\underline{R}_m^{0.8}(A) = (\overline{R}_m^{0.2}(A^c))^c = \{x_4, x_5, x_6, x_8, x_{12}, x_{13}\}$. $\underline{R}_m^{1}(\underline{R}_m^{0.8}(A)) = \underline{R}_m^{1}(\{x_4, x_5, x_6, x_8, x_{12}, x_{13}\}) = \{x_4, x_5, x_6, x_8, x_{12}, x_{13}\} = \underline{R}_m^{0.8}(A)$. $\overline{R}_m^{0}(\overline{R}_m^{0.2}(A^c)) = \overline{R}_m^{0}(\{x_1, x_2, x_3, x_7, x_9, x_{10}, x_{11}\} = \{x_1, x_2, x_3, x_7, x_9, x_{10}, x_{11}\} = \overline{R}_m^{0.2}(A^c)$. $\overline{R}_m^{0.2}(A) = \overline{R}_m^{0.2}(\{x_1, x_2, x_5, x_6\}) = \phi$. $\underline{R}_m^{0.8}(A^c) = \underline{R}_m^{0.8}(\{x_3, x_4, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}) = X$. $\underline{R}_m^{0.8}(A^c) = (\overline{R}_m^{0.2}(A))^c = X$. $\underline{R}_m^{1}(\underline{R}_m^{0.8}(A^c)) = X = \underline{R}_m^{0.8}(A^c)$. $\overline{R}_m^{0}(\overline{R}_m^{0.2}(A)) = \phi = \overline{R}_m^{0.2}(A)$.

Now we use the approach to determine the degree of dependence of the estimated and observed data $m_{X(Est)} = \{\phi, X, \{x_1, x_2, x_3, x_9\}, \{x_1, x_2, x_3, x_4, x_9\}, \{x_4, x_5\}, \{x_6, x_7, x_8\}, \{x_{10}, x_{11}\}, \{x_{12}, x_{13}\}\}$, $X(Est) - m_{X(Est)} = \{\phi, X, \{x_4, x_5, x_6, x_7, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_5, x_6, x_7, x_8, x_{10}, x_{11}, x_{12}, x_{13}\}, \{x_1,$

$x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}$, $\{x_1, x_2, x_3, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}$, $\{x_1, x_2, x_3, x_4, x_5, x_9, x_{10}, x_{11}$, $\{x_{12}, x_{13}\}$, $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{12}, x_{13}\}$, $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}\}\}$. Therefore, $\gamma(P, Q, 0.3) = \frac{|POS(P,Q,0.3)|}{|X|} = 1$.

4. Conclusion and Discussion

The field of mathematical science which goes under the name of minimal structure is concerned with all questions, related directly or indirectly to topology. Therefore, the theory of rough sets is one of the subjects, most important in topology. Also, we give an atmospheric dispersion modeling which is essential to predict the path and danger from an atmospheric plume of hazardous materials. The approach used here can be applied in any IS with quantitative or qualitative data. Moreover, the concepts proposed in this paper can be extended to fuzzy topological structures [1] and thus one can get a more affirmative solution in decision making problems [9, 22–24] in real life solutions.

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