Transactions of A. Razmadze
Mathematical Institute
Vol. **173** (2019), issue 3, 137–145

TOPSIS APPROACH TO MULTI-OBJECTIVE EMERGENCY SERVICE FACILITY LOCATION SELECTION PROBLEM UNDER Q-RUNG ORTHOPAIR FUZZY INFORMATION

GIA SIRBILADZE, ANNA SIKHARULIDZE, BIDZINA MATSABERIDZE, IRINA KHUTSISHVILI, BEZHAN GHVABERIDZE

Abstract. A new model based on q-rung orthopair fuzzy sets (q-ROFS) has been presented to manage the uncertainty in real-world multi-criteria decision-making problems. q-ROFS has much stronger ability than Pythagorean fuzzy set (PFS) or intuitionistic fuzzy set (IFS) to model such uncertainty. A q-rung orthopair fuzzy TOPSIS approach for formation and representing experts knowledge on the parameters of emergency service facility location planning is developed. In this approach, we propose a score function based on the comparison method to identify the q-rung orthopair fuzzy positive ideal solution and the q-rung orthopair fuzzy negative ideal solution. Based on the constructed fuzzy TOPSIS aggregation, a new objective function is formulated. The constructed criterion maximizes service centers' selection index. This criterion together with the second criterion - minimization of a number of selected centers creates the multi-objective facility location set covering problem. The approach is illustrated by the simulation example of emergency service facility location planning for a city in Georgia. More exactly, the example looks into the problem of planning fire stations locations to serve emergency situations in specific demand points critical infrastructure objects.

1. Introduction

Multi-criteria decision making (MCDM) is to find an optimal alternative that has the highest degree of satisfaction from a set of feasible alternatives characterized with multiple criteria, and these kinds of MCDM problems arise in many real-world situations. Considering the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, Bellman and Zadeh [2] introduced the theory of fuzzy sets in the MCDM problems. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) developed by Hwang and Yoon [7] (1981) is one of the most useful distance measure based on the classical approaches to multi-criteria/multi-attribute decision making (MCDM/MADM) problems. It is a practical and useful technique for ranking and selection of a number of externally determined alternatives through distance measures. The basic principle used in the TOPSIS is that the chosen alternative should have the shortest distance from positive-ideal solution (PIS) and farthest from the negative-ideal solution (NIS). There exists a large amount of literature involving TOPSIS theory and applications. In the TOPSIS, the performance ratings and the weights of the criteria are given as crisp values. In classical TOPSIS methods, crisp numerical values are used to express the performance rating and criteria weights. But human judgment, preference values and criteria weights are often ambiguous and cannot be represented by using crisp numerical value in real-life situation. To resolve the ambiguity frequently arising in information from human judgment and preference, the fuzzy set theory has been successfully used to handle imprecision and uncertainty in decision making problems. In this work, a novel decision-making TOPSIS approach is developed to deal effectively with the interactive MCDM problems with q-rung orthopair fuzzy information.

Intuitionistic fuzzy sets (IFS) were introduced by Atanassov [1], as a generalization of a Zadeh's fuzzy sets (FS). Since to each element of IFS, as Intuitionistic fuzzy number (IFN) (μ, ν), is assigned a membership degree (μ), a non-membership degree (ν) and a hesitancy degree $(1 - \mu - \nu)$, IFS is more powerful in dealing with uncertainty and imprecision than FS. The IFS theory has been widely

2010 *Mathematics Subject Classification.* 28E10, 47S40, 62C86, 90B50, 90C70.
Key words and phrases. Emergency Service Facility Location planning; q-Rung orthopair fizzy sets; Fuzzy TOPSIS; Critical infrastructure.

studied and applied to a variety of areas. But an IFN (μ, ν) has a significant restriction - the sum of the degrees of membership and the non-membership is equal to or less than 1. In some cases, a decision maker (DM) may provide data for some attribute that the sum of two degrees is greater than $1(\mu + \nu > 1)$. Yager in [13, 14] presented the concept of the Pythagorean fuzzy set (PFS) as extension of an IFS, where the pair of a Pythagorean fuzzy number (PFN) (μ, ν) has a less significant restrict – a square sum of the degrees of membership and the non-membership is equal to or less than $1(\mu^2 + \nu^2 \leq 1)$. In general, for practical problems, the PFSs can decide significant ones that IFSs cannot do. Therefore, PFSs are more able to process uncertain information and solve complex decision making problems. PFNs have much less, but significant restriction. When the evaluation psychology of a DM is too complicated and contradictory for complex decision making, the attribute's corresponding information is still difficult to express with PFNs. Recently, again Yager decided this problem in [15,16]. He proposed a concept of a q-rung orthopair fuzzy set (q-ROFS), where $q \geq 1$ and the sum of the qth power of the degrees of membership and the non-membership cannot be greater than 1. For a q-rung orthopair fuzzy number (q-ROFN) we have $(\mu^q + \nu^q \leq 1)$. It is obvious that the q-ROFSs are more general than IFSs and PFSs. The IFSs and PFSs are the special cases of the q-ROFSs when $q = 1$ and $q = 2$, respectively. Therefore, q-ROFNs are more convenient and able to describe DM's evaluation information than IFNs and PFNs.

Defnition 1 ([15])**.** Let S be a fixed ordinary set. A q-rung orthopair fuzzy set A on S is defined as membership grades:

$$A = \{\langle s, \mu_A(s), \nu_A(s)\rangle/(s \in S)\},$$

where the functions $\mu_A(s)$ indicate support for membership of $s \in A$ and $\nu_A(s)$ indicates support against membership of $s \in A$, where

$$q \geq 1, \;\; 0 \leq \mu_A(s) \leq 1, \;\; 0 \leq \nu_A(s) \leq 1, \;\; 0 \leq (\mu_A(s))^q + (\nu_A(s))^q \leq 1.$$

$\mathrm{Hes}_q(s) = (1 - (\mu_A(s))^q + (\nu_A(s))^q)^{1/q}$ is called a hesitancy associated with a q-rung orthopair membership grades and $\mathrm{Str}_q(s) = ((\mu_A(s))^q + (\nu_A(s))^q)^{1/q}$ is called a strength of commitment viewed at rung q.

In [15], Yager showed that Attanassov's intuitionistic fuzzy sets [1] are $q = 1$-rung orthopair and Yager's Pythagorean fuzzy sets [14] are $q = 2$ rung orthopair fuzzy sets. For convenience, the authors for every $s \in S$ called $\alpha = \langle s, \mu_\alpha(s), \nu_\alpha(s)\rangle$ a q-rung orthopair fuzzy number (q-ROFN) denoted by $\alpha = (\mu_\alpha, \nu_\alpha)$.

Let us denote by L the lattice of non-empty intervals $L = \{[a; b])/(a, b) \in [0, 1]^2, \; a \leq b\}$. The partial order relation $\leq_L$ is defined as $[a; b] \leq_L [c; d] \Leftrightarrow a \leq c$ and $b \leq d$. The top and bottom elements are $1_L = [1; 1]$ and $0_L = [0; 0]$, respectively. For the lattice of all q-ROFNs the corresponding partial order relation $\leq_{L_{q\text{-ROFNs}}}$ is defined as

$$(\mu_1, \nu_1) \leq_{L_{q\text{-ROFNs}}} (\mu_2, \nu_2) \Leftrightarrow \mu_1 \leq \mu_2 \;\text{ and }\; \nu_1 \geq \nu_2.$$

The top and bottom elements are $1_{L_{q\text{-ROFNs}}} = (1; 0)$ and $0_{L_{q\text{-ROFNs}}} = (0; 1)$, respectively.

Defnition 2 ([15])**.** Suppose $\alpha = (\mu_\alpha, \nu_\alpha)$ is a q-ROFN. a) A score function Sc of α is defined as

$$Sc(\alpha) = \mu_\alpha^q - \nu_\alpha^q; \tag{1}$$

b) An accuracy function Ac of α is defined as follows:

$$Ac(\alpha) = \mu_\alpha^q + \nu_\alpha^q. \tag{2}$$

Based on these definitions, a comparison method of q-ROFNs (total order relation $\leq_t$ on the lattice $L_{q\text{-ROFNs}}$) is defined.

Defnition 3 ([15])**.** Suppose $\alpha = (\mu_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \nu_\beta)$ are any two q-ROFNs and $Sc(\alpha)$, $Sc(\beta)$ are the score functions and $Ac(\alpha)$, $Ac(\beta)$ are the accuracy functions of α and β, respectively, then:

a) if $Sc(\alpha) > Sc(\beta)$, then $\beta <_t \alpha$;

b) if $Sc(\alpha) = Sc(\beta)$, then:
   if $Ac(\alpha) > Ac(\beta)$, then $\beta <_t \alpha$;
   if $Ac(\alpha) = Ac(\beta)$, then $\beta =_t \alpha$.

On the lattice $L_{q\text{-ROFNs}}$, the following basic operations can be defined.

Defnition 4 ([15])**.** Suppose for $\alpha = (\mu_\alpha, \nu_\alpha)$, $\alpha_1, \alpha_2 \in L_{q\text{-ROFNs}}$ we have:

1. $\alpha^c = (\nu_\alpha, \mu_\alpha)$;
2. $\alpha_1 \oplus_q \alpha_2 = \big((\mu^q_{\alpha_1} + \mu^q_{\alpha_2} - \mu^q_{\alpha_1} \cdot \mu^q_{\alpha_2})^{1/q}, \nu_{\alpha_1}\dot{\nu}_{\alpha_2}\big)$;
3. $\alpha_1 \otimes_q \alpha_2 = \big(\mu_{\alpha_1} \cdot \mu_{\alpha_2}, (\nu^q_{\alpha_1} + \nu^q_{\alpha_2} - \nu^q_{\alpha_1} \cdot \nu^q_{\alpha_2})^{1/q}\big)$;
4. $\mathrm{Min}(\alpha_1, \alpha_2) = \big(\min(\mu_{\alpha_1}, \mu_{\alpha_2}), \max(\nu_{\alpha_1}, \nu_{\alpha_2})\big)$;
5. $\mathrm{Max}(\alpha_1, \alpha_2) = \big(\max(\mu_{\alpha_1}, \mu_{\alpha_2}), \min(\nu_{\alpha_1}, \nu_{\alpha_2})\big)$;
6. $\lambda \cdot \alpha = ((1 - (1 - \mu^q_\alpha)^\lambda)^{1/q}, \nu^\lambda_\alpha), \quad \lambda > 0$;
7. $\alpha^\lambda = (\mu^\lambda_\alpha, (1 - (1 - \nu^q_\alpha)^\lambda)^{1/q}), \quad \lambda > 0$.

We define the distance between the q-rung orthopiar fuzzy numbers $\alpha_1, \alpha_2 \in L_{q\text{-ROFNs}}$:

$$d_q(\alpha_1, \alpha_2) = 1/2 \cdot (|(\mu_{\alpha_1})^q - (\mu_{\alpha_2})^q + |(\nu_{\alpha_1})^q - (\nu_{\alpha_2})^q|). \tag{3}$$

It is not difficult to prove that this measure satisfies all properties of a distance function.

2. Description of TOPSIS Approach to Facility Location Selection Problem with Q-Rung Orthopair Fuzzy Information

Location planning for candidate centers is vital in minimizing traffic congestion arising from facility movement in extreme environment. In recent years, transport activity has grown tremendously and this has undoubtedly affected the travel and living conditions in difficult and extreme urban areas. Considering the growth in the number of freight movements and their negative impacts on residents and the environment, municipal administrations are implementing sustainable freight regulations like restricted delivery timing, dedicated delivery zones, congestion charging etc. With the implementation of these regulations, the logistics operators are facing new challenges in location planning for service centers. For example, if service centers are located close to customer locations, then they increase traffic congestion in the urban areas. If they are located far from customer locations, then the service costs for the operators result to be very high. Under these circumstances, it is clear that the location planning for service centers in extreme environment is a complex decision that involves consideration of multiple attributes like maximum customer coverage, minimum service costs, least impacts on geographical points' residents and the environment, and conformance to freight regulations of these points.

Timely servicing from emergency service centers to the affected geographical areas (demand points as customers, for example, critical infrastructure objects) is a key task of the emergency management system. Scientific research in this area focuses on distribution networks decision-making problems, which are known as a Facility Location Problem (FLP) [4]. FLP's models have to support the generation of optimal locations of service centers in complex and uncertain situations. There are several publications about application of fuzzy methods in the FLP. However, all of them have a common approach. They represent parameters as fuzzy values (triangular fuzzy numbers [5] and others) and develop methods for facility location problems called in this case Fuzzy Facility Location Problem (FFLP). Fuzzy TOPSIS approaches for facility location selection problem for different fuzzy environments are developed in [3,8,10,12,17,18]. In this work we consider a new model of FFLP based on the q-rung orthopair fuzzy TOPSIS approach for the optimal selection of facility location centers.

This section first introduces the MCDM problem under q-rung orthopair fuzzy environment. Then, an effective decision-making approach is proposed to deal with such MCDM problems. At length, an algorithm of the proposed method is also presented

First, we are focusing on a multi-attribute decision making approach for location planning for service centers under uncertain and extreme environment. We develop a fuzzy multi-attribute decision making approach for the service center location selection problem for which a fuzzy TOPSIS approach is used.

The formation of expert's input data for construction of attributes is an important task of the centers' selection problem. To decide on the location of service centers, it is assumed that a set of *candidate sites* (CSs) already exists. This set is denoted by $CS = \{cs_1, cs_2, \ldots, cs_m\}$, where we can locate service centers, and $S = \{s_1, s_2, \ldots, s_n\}$ is the set of all attributes (transformed in benefit attributes) which define CS's selection. For example: "access by public and special transport modes to the candidate site", "security of the candidate site from accidents, theft and vandalism", "connectivity of the location with other modes of transport (highways, railways, seaports, airports, etc.)", "costs in vehicle resources, required products and etc. for the location of a candidate site", "impact of the candidate site location on the environment, such as important objects of Critical Infrastructure, air pollution and others", "proximity of the candidate site location from the central locations", "proximity of the candidate site location from customers", "availability of raw material and labor resources in the candidate site", "ability to conform to sustainable freight regulations imposed by managers for e.g. restricted delivery hours, special delivery zones", "ability to increase size to accommodate growing customers" and others. Let $W = \{w_1, w_2, \ldots, w_n\}$ be the weights of attributes. For each expert e_k from invited group of experts (service dispatchers and so on) $E = \{e_1, e_2, \ldots, e_t\}$, let α_{ij}^kbe the fuzzy rating of his/her evaluation in q-ROFNs for each candidate site cs_i $(i = 1, \ldots, m)$, with respect to each attribute s_j $(j = 1, \ldots, n)$. For the expert e_k we construct binary fuzzy relation $A_k = \{\alpha_{ij}^k,\ i = 1, \ldots, m;\ j = 1, \ldots, n\}$ decision making matrix, elements of which are represented in q-ROFNs. If some attribute s_j is cost type, then we transform experts' evaluations and α_{ij}^k is changed by $(\alpha_{ij}^k)^c$. Experts' data must be aggregated in etalon decision making matrix $-A = \{\alpha_{ij},$ $i = 1, \ldots, m;\ j = 1, \ldots, n\}$. Our task is to build fuzzy TOPSIS approach, which for each candidate site cs_i $(i = 1, \ldots, m)$ aggregates presented objective and subjective data into scalar values – site's selection index. This aggregation can be formally represented as a TOPSIS "relative closeness of the alternative" defined on α_{ij}, $j = 1, \ldots, n$:

$$\begin{aligned} \delta_i &= \text{relative closeness of the alternative } (cs_i) \\ &\equiv \text{TOPSIS aggregation } (\alpha_{i1}, \ldots, \alpha_{in}),\ \ i = 1, \ldots, m. \end{aligned} \tag{4}$$

The proposed framework of location planning for candidate sites comprises the following steps:

Step 1: Selection of location attributes. Involves the selection of location attributes for evaluating potential locations for candidate sites. These attributes are obtained from discussion with experts and members of the city transportation group. We use five attributes (n=5) defined above by short names: s_1 ="Accessibility", s_2 ="Security", s_3 ="Connectivity to multimodal transport", s_4 ="Costs", s_5 ="Proximity to customers". The fourth attribute is cost type and the others are benefit types. As mentioned above, cost type evaluation data must be transformed in the benefit forms.

Step 2: Selection of candidate location sites. Involves selection of potential locations for implementing service centers. The decision makers use their knowledge, prior experience in transportation or other aspects of the geographical area of extreme events and the presence of sustainable freight regulations to identify candidate locations for implementing service centers. For example, if certain areas are restricted for delivery by municipal administration, then these areas are barred from being considered as potential locations for implementing urban service centers. Ideally, the potential locations are those that cater for the interest of all city stakeholders, which are city residents, logistics operators, municipal administrations, etc.

Step 3: Assignment of ratings to the attributes with respect to the candidate sites. Let $A_k = \{\alpha_{ij}^k \in$ q-ROFNs, $i = 1, \ldots, m;\ j = 1, \ldots, n\}$ be the performance ratings of each expert e_k $(k = 1, 2, \ldots, t)$ for each candidate site cs_i $(i = 1, 2, \ldots, m)$ with respect to attributes s_j $(j = 1, 2, \ldots, n)$.

Step 4: Computation of the q-ROF decision matrix for the attributes and the candidate sites. Let the ratings of all experts be described by positive numbers ω_k, $\omega_k > 0$, $k = 1, \dots, t$. If ratings of the attributes evaluated by the k-th expert are α_{ij}^k, then the aggregated fuzzy ratings (α_{ij}) of candidate sites with respect to each attribute are given by q-ROF weighted sum

$$\alpha_{ij} = \sum_{k=1}^{t}{}^{\oplus_q} \alpha_{ij}^k \left(\omega_k \Big/ \sum_{l=1}^{t} \omega_l\right). \tag{5}$$

The fuzzy decision matrix $\{\alpha_{ij}\}$ for the candidate sites CS and the attributes S is constructed as follows:

$$\begin{array}{c} \\ cs_1 \\ cs_2 \\ \dots \\ cs_m \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & \dots & s_n \end{array} \\ \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha_{m1} & \alpha_{m2} & \dots & \alpha_{mn} \end{bmatrix} \end{array}$$

Construct the q-rung fuzzy decision matrix $\{\alpha_{ij}\}$ and calculate Sc and Ac functions values (Definition 2) of elements α_{ij}.

Step 5: Identification of q-rung orthopair fuzzy PIS and NIS. TOPSIS approach starts with the definition of the q-rung orthopair fuzzy PIS and the q-rung orthopair fuzzy NIS. Using formulas (1), (2) the PIS is defined as a q-rung orthopair fuzzy set on attributes S: $cs^+ = \{s_j,\ \alpha_j^+ \equiv \text{Max}_i[(\alpha_{ij})] \mid j = 1, 2, \dots, n\}$ and the NIS is defined as a q-rung orthopair fuzzy set on attributes S: $cs^- = \{s_j,\ \alpha_j^- \equiv \text{Min}_i[(\alpha_{ij})] \mid j = 1, 2, \dots, n\}$. In the real MCDM models PIS and NIS are usually not be feasible alternatives. They are extreme alternatives.

Step 6. Calculate the distances between the alternative candidate location site and the q-rung orthopair fuzzy PIS, as well as q-rung orthopair fuzzy NIS, respectively. Then, we proceed to calculate the distances between each alternative and q-rung orthopair fuzzy PIS and NIS. Using equation (3), we define distances between the alternative cs_i and the q-rung orthopair fuzzy PIS and NIS, as a weighted sums of distances between extreme and evaluated q-ROFNs:

$$D(cs_i, sc^+) = \sum_{j=1}^{n} w_j d_q(\alpha_{ij}, \alpha_j^+) = 1/2 \cdot \sum_{j=1}^{n} w_j (|(\mu_{\alpha_{ij}})^q - (\mu_{\alpha_j^+})^q| + |(\nu_{\alpha_{ij}})^q - (\nu_{\alpha_j^+})^q|),$$
$$D(cs_i, sc^-) = \sum_{j=1}^{n} w_j d_q(\alpha_{ij}, \alpha_j^-) = 1/2 \cdot \sum_{j=1}^{n} w_j (|(\mu_{\alpha_{ij}})^q - (\mu_{\alpha_j^-})^q| + |(\nu_{\alpha_{ij}})^q - (\nu_{\alpha_j^-})^q|),$$

Step 7. Calculate the revised closeness or TOPSIS aggregation as a site's selection index for every alternative. In general, the bigger $D(cs_i, sc^-)$ and the smaller $D(cs_i, sc^+)$ the better is the alternative cs_i. In the classical TOPSIS method, the authors usually need to calculate the relative closeness (RC) of the alternative cs_i. We define candidate site's selection index as RC with respect to the q-rung orthopair PIS sc^+ as follows:

$$\delta_i \equiv RC(cs_i) = \frac{D(cs_i, cs^+)}{D(cs_i, cs^+) + D(cs_i, cs^-)}, \quad i = 1, \dots, m. \tag{6}$$

3. Multi-Objective Optimization Model of Facility Location Set Covering Problem

The location set covering problem (LSCP) proposed by C. Toregas and C. Revell in 1972, seeks for a solution for locating the least number of facilities to cover all demand points within the service distance. In some of our works we are focusing on the multi-objective fuzzy set covering problems [9, 11] for extreme conditions. In this work, we construct new fuzzy LSCP model for emergency service facility location planning.

As we discussed in the previous section, the constructed Fuzzy TOPSIS technology forms center's selection rational index. The center's index reflects expert evaluations with respect to the center,

TABLE 1. Fuzzy travel times $\widetilde{t}_{ij}$ from fire stations to critical infrastructure objects (in minutes)

	a_1	a_2	a_3	a_4	a_5	a_6
cs_1	$(3,5,7)$	$(2,4,6)$	$(4,6,7)$	$(4,7,9)$	$(1,3,5)$	$(1,3,4)$
cs_2	$(6,10,14)$	$(4,9,14)$	$(2,4,6)$	$(5,7,10)$	$(1,4,8)$	$(1,4,5)$
cs_3	$(4,8,12)$	$(4,7,11)$	$(4,6,9)$	$(2,4,7)$	$(4,7,10)$	$(4,6,8)$
cs_4	$(4,7,10)$	$(7,11,15)$	$(6,9,13)$	$(4,6,8)$	$(2,4,6)$	$(1,3,5)$
cs_5	$(1,3,5)$	$(2,4,6)$	$(1,3,6)$	$(2,4,7)$	$(4,6,8)$	$(5,9,12)$

considering all actual attributes. If $x = \{x_1, x_2, \dots, x_m\}$ is the Boolean decision vector that defines some selection from candidate centers $CS = \{cs_1, cs_2, \dots, cs_m\}$ for facility location, we can build centers' selection index as a linear sum of $\delta_j x_j$ values: as a result, new objective function – *centers' selection index* $\sum_{j=1}^{m} \delta_j x_j$ is constructed. Maximizing it, we will be able to select a group of centers with the best total ranking from admissible covering selections. Classical facility location set covering problem tries to *minimize the number of centers*, where service facilities can be located – $\sum_{j=1}^{m} x_j$. The problem aims to locate service facilities in minimal travel time from candidate centers. Let customers covered by service centers in distribution networks be denoted by $A = \{a_1, \dots, a_k\}$. The problem aims to locate service facilities in minimal travel time from candidate sites. Let experts evaluated movement fuzzy times (evaluated in triangular fuzzy numbers (TFNs) [5]) between customer and candidate sites be $\widetilde{t}_{ij}$, $a_i \in A$; $cs_j \in CS$. In extreme environment for emergency planning a radius of service center is defined based not on distance but on maximum allowed time T for movement, since the rapid help and servicing is crucial for customers in such situations. Respectively, a set of candidate sites N_i, covering customer $a_i \in A$, is defined as $N_i = \{cs_j, cs_j \in CS/E(\widetilde{t}_{ij}) \leq T\}$, $i = 1, \dots, m$, where

$$E(\widetilde{t}_{ij}) = \widetilde{t}^2_{ij} + (\widetilde{t}^3_{ij} - 2\widetilde{t}^2_{ij} + \widetilde{t}^1_{ij})/4,$$

is an expected value of a TFN $\widetilde{t}_{ij} \equiv (\widetilde{t}^1_{ij}, \widetilde{t}^2_{ij}, \widetilde{t}^3_{ij})$. Then we can state bi-objective facility location set covering problem:

$$\min z_1 = \sum_{j=1}^{m} x_j, \quad \max z_2 = \sum_{j=1}^{m} \delta_j x_j, \tag{7}$$

$$\sum_{s_j \in N_i} x_j \geq 1 \ \ (i = 1, 2, \dots, k); \quad x_j \in \{0, 1\}, \quad j = 1, 2, \dots, m.$$

Based on the epsilon-constraint approach, an algorithm of finding all Pareto solutions [6] is constructed (omitted here).

4. Numerical Simulation of Emergency Service Facility Location Model

We illustrate the effectiveness of the constructed optimization model by the numerical example. Let us consider an emergency management administration of a city in Georgia that wishes to locate some fire stations with respect to timely servicing of critical infrastructure objects. Assume that there are 6 demand points as customers (critical infrastructure objects) and 5 candidate facility centers (fire stations) in the urban area. Let there be 4 experts from Emergency Management Agency (EMA) of Georgia for the evaluation of travel times and the ranking of candidate facility centers. The travel times between demand points and candidate centers are evaluated in triangular fuzzy numbers (see Table 1). According to the standards of EMA (Georgia), the principle of location fire stations is that the fire station can reach the area edge within 5 minutes after receiving the dispatched instruction. Therefore, we set covering radius $T = 5$ minutes.

TABLE 2. Appraisal matrix A_1 by expert-1

	s_1	s_2	s_3	s_4	s_5
cs_1	$(0.7, 0.5)$	$(0.8, 0.3)$	$(0.7, 0.4)$	$(0.7, 0.4)$	$(0.8, 0.4)$
cs_2	$(0.6, 0.5)$	$(0.7, 0.4)$	$(0.4, 0.6)$	$(0.8, 0.4)$	$(0.7, 0.4)$
cs_3	$(0.7, 0.5)$	$(0.9, 0.5)$	$(0.9, 0.7)$	$(0.7, 0.4)$	$(0.8, 0.5)$
cs_4	$(0.6, 0.5)$	$(0.8, 0.4)$	$(0.8, 0.5)$	$(0.9, 0.5)$	$(0.8, 0.5)$
cs_5	$(0.8, 0.6)$	$(0.7, 0.4)$	$(0.9, 0.5)$	$(0.7, 0.4)$	$(0.8, 0.6)$

TABLE 3. Appraisal matrix A_2 by expert-2

	s_1	s_2	s_3	s_4	s_5
cs_1	$(0.7, 0.5)$	$(0.8, 0.4)$	$(0.6, 0.3)$	$(0.6, 0.3)$	$(0.7, 0.4)$
cs_2	$(0.6, 0.5)$	$(0.7, 0.3)$	$(0.7, 0.4)$	$(0.9, 0.4)$	$(0.8, 0.4)$
cs_3	$(0.8, 0.5)$	$(0.9, 0.5)$	$(0.6, 0.4)$	$(0.8, 0.4)$	$(0.6, 0.2)$
cs_4	$(0.6, 0.4)$	$(0.8, 0.3)$	$(0.9, 0.6)$	$(0.7, 0.3)$	$(0.6, 0.2)$
cs_5	$(0.9, 0.7)$	$(0.7, 0.4)$	$(0.9, 0.4)$	$(0.7, 0.3)$	$(0.9, 0.6)$

TABLE 4. Appraisal matrix A_3 by expert-3

	s_1	s_2	s_3	s_4	s_5
cs_1	$(0.7, 0.4)$	$(0.8, 0.3)$	$(0.7, 0.5)$	$(0.7, 0.4)$	$(0.9, 0.5)$
cs_2	$(0.6, 0.5)$	$(0.7, 0.4)$	$(0.5, 0.3)$	$(0.7, 0.2)$	$(0.6, 0.3)$
cs_3	$(0.6, 0.2)$	$(0.9, 0.6)$	$(0.7, 0.5)$	$(0.7, 0.3)$	$(0.6, 0.3)$
cs_4	$(0.8, 0.4)$	$(0.9, 0.4)$	$(0.8, 0.5)$	$(0.8, 0.5)$	$(0.8, 0.3)$
cs_5	$(0.9, 0.7)$	$(0.6, 0.3)$	$(0.9, 0.5)$	$(0.9, 0.6)$	$(0.7, 0.4)$

Covering sets of candidate sites N_i are defined (omitted here). Let experts generated the attributes weights as values of overall importance be based on the consensus:

$$w_1 = 0.25; \quad w_2 = 0.15; \quad w_3 = 0.25; \quad w_4 = 0.20; \quad w_5 = 0.15.$$

Each expert e_k $(k = 1, 2, 3)$ presented the ratings r_{ij}^k for each candidate center s_i $(i = 1, \dots, 5)$ with respect to each attribute s_j $(j = 1, \dots, 5)$.

Let experts have equal ratings $\{\omega_j = 1/3\}$. Using formula (5), experts' evaluations are aggregated in decision making matrix $\{\alpha_{ij}\}$ (Table 5).

Using the algorithm from Section 2 of new fuzzy TOPSIS, we calculated values of candidate centers' selection indices: $\delta_1 = 0.472$, $\delta_2 = 0.803$, $\delta_3 = 0.441$, $\delta_4 = 0.455$, $\delta_5 = 0.377$. After these calculations

Table 5. Accumulated q-rung orthopair fuzzy decision matrix $\{\alpha_{ij}\}$

	s_1	s_2	s_3	s_4	s_5
cs_1	$(0.70, 0.46)$	$(0.80, 0.33)$	$(0.67, 0.39)$	$(0.67, 0.36)$	$(0.83, 0.43)$
cs_2	$(0.60, 0.50)$	$(0.70, 0.36)$	$(0.58, 0.42)$	$(0.83, 0.32)$	$(0.72, 0.36)$
cs_3	$(0.72, 0.37)$	$(0.90, 0.53)$	$(0.79, 0.52)$	$(0.74, 0.36)$	$(0.70, 0.31)$
cs_4	$(0.70, 0.43)$	$(0.84, 0.36)$	$(0.84, 0.53)$	$(0.83, 0.42)$	$(0.76, 0.31)$
cs_5	$(0.88, 0.66)$	$(0.67, 0.36)$	$(0.90, 0.46)$	$(0.80, 0.42)$	$(0.83, 0.52)$

the following Combinatorial Programming Problem (7) has been constructed:

$$\begin{cases} f_1 = x_1 + x_2 + x_3 + x_4 + x_5 \Rightarrow \min, \\ f_2 = 0.472x_1 + 0.803x_2 + 0.441x_3 + 0.455x_4 + 0.377x_5 \Rightarrow \max, \\ x_1 + x_5 \geq 1, \\ x_2 + x_5 \geq 1, \\ x_3 + x_5 \geq 1, \\ x_1 + x_2 + x_4 \geq 1, \\ x_i \in 0, 1, \quad i = 1, 2, 3, 4, 5. \end{cases} \tag{8}$$

Based on the developed software for problem (8), the Pareto solutions [6] are founded. They are:

a) $cs_1, cs_5, \quad f_1 = 2; \quad f_2 = 1.18,$

b) $cs_1, cs_2, cs_3, \quad f_1 = 3; \quad f_2 = 1.716,$

c) $cs_1, cs_2, cs_3, cs_4, \quad f_1 = 4; \quad f_2 = 2.171,$

d) $cs_1, cs_2, cs_3, cs_4, cs_5, \quad f_1 = 5, \quad f_2 = 2.548.$

It is clear that increasing of fire stations number in Pareto solutions results in a more better level of the second objective function – *fire stations' selection index*. But the decision on the choice of the fire stations as service centers depends on the decision making person's preferences with respect to risks of administrative actions.

5. Conclusions

The paper presented new approach for the facility location problem for selection of the locations of service centers in extreme and uncertain situations. The approach utilizes experts knowledge represented by q-rung orthopair fuzzy numbers and considers the suitability of central location (i.e., affordability, security, etc.) using constructed new fuzzy TOPSIS approach. On the other hand, the model also considers the necessity to reach all critical infrastructure points and time that is required to reach them, presented by triangular fuzzy numbers. As a result, the bi-objective set covering problem is obtained. The constructed approach is illustrated by a numerical example for locating fire stations servicing critical infrastructure points in a city in Georgia. For the constructed problem, the Pareto solutions are obtained. For the large-dimension cases of the problem, the epsilon-constraint approach for the Pareto front obtaining is constructed.

Acknowledgments

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [FR-18-466].

References

1. K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **20** (1986), no. 1, 87–96.
2. R. E. Bellman, L. A. Zadeh, Decision-making in a fuzzy environment. *Management Sci.* **17** (1970/71), B141–B164.

3. T. C. Chu, Facility location selection using fuzzy TOPSIS under group decisions. *Internat. J. Uncertain. Fuzziness Knowledge-Based Systems* **10** (2002), no. 6, 687–701.
4. M. S. Daskin, *Network and Discrete Location. Models, Algorithms, and Applications*. Second edition. John Wiley & Sons, Inc., Hoboken, NJ, 2013.
5. D. Dubois, H. Prade, *Possibility Theory. An Approach to Computerized Processing of Uncertainty*. Plenum Press, New York, 1988.
6. M. Ehrgott, *Multicriteria Optimization*. Second edition. Springer-Verlag, Berlin, 2005.
7. C. L. Hwang, K. Yoon, *Multiple Attribute Decision Making. Methods and Applications*. A state-of-the-art survey. Lecture Notes in Economics and Mathematical Systems, 186. Springer-Verlag, Berlin-New York, 1981.
8. G. R. Jahanshahloo, F. Hosseinzadeh Lotfi, M. Izadikhah, Extension of the TOPSIS method for decision-making problems with fuzzy data. *Applied Mathematics and Computation* **181** (2006), no. 2, 1544–1551.
9. G. Sirbiladze, B. Ghvaberidze, B. Matsaberidze, Bicriteria fuzzy vehicle routing problem for extreme environment. *Bull. Georgian Natl. Acad. Sci. (N.S.)* **8** (2014), no. 2, 41–48.
10. G. Sirbiladze, B. Ghvaberidze, B. Matsaberidze, A. Sikharulidze, Multi-objective emergency service facility location problem based on fuzzy TOPSIS. *Bull. Georgian Natl. Acad. Sci. (N.S.)* **11** (2017), no. 1, 23–30.
11. G. Sirbiladze, A. Sikharulidze, B. Ghvaberidze, B. Matsaberidze, Fuzzy-probabilistic aggregations in the discrete covering problem. *Int. J. Gen. Syst.* **40** (2011), no. 2, 169–196.
12. Y. J. Wang, H. S. Lee, Generalizing TOPSIS for fuzzy multiple-criteria group decision-making. *Comput. Math. Appl.* **53** (2007), no. 11, 1762–1772.
13. R. R. Yager, Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems* **22** (2013), no. 4, 958–965.
14. R. R. Yager, Pythagorean fuzzy subsets. *Joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)*, pp. 57-61. IEEE, 2013.
15. R. R. Yager, Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems* **25** (2016), no. 5, 1222–1230.
16. R. R. Yager, N. Alajlan, Y. Bazi, Aspects of generalized orthopair fuzzy sets. *International Journal of Intelligent Systems* **33** (2018), no. 11, 2154–2174.
17. D. Yong, Plant location selection based on fuzzy TOPSIS. *The International Journal of Advanced Manufacturing Technology* **28** (2006), no. 7-8, 839–844.
18. X. Zhang, Z. Xu, Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems* **29** (2014), no. 12, 1061–1078.

(Received 01.08.2019)

Department of Computer Sciences, Faculty of Exact and Natural Sciences, I. Javakhishvili Tbilisi State University, Georgia

E-mail address: gia.sirbiladze@tsu.ge
E-mail address: annasikharulidze@gmail.com
E-mail address: b.matsaberidze@gmail.com
E-mail address: irina.khutsishvili@tsu.ge
E-mail address: bezhan.ghvaberidze@tsu.ge