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THE EFFECT OF STIGMATIZATION OF HUMANS WITH ERECTILE DYSFUNCTION DUE TO UNHEALTHY LIFE STYLE AND GENETIC FACTORS: A MATHEMATICAL MODEL WITH STOCHASTIC PERTURBATION

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Abstract. In this study, we present a novel model for understanding the dynamics of Erectile Dysfunction (ED) in various male populations, using ordinary differential equations with stochastic perturbations. Our model takes into account the impact of both unhealthy lifestyle choices and genetic factors, as well as the reluctance of some individuals to disclose their ED status due to fear of stigmatization. We establish the existence, uniqueness, and global positive solution of the stochastic model, demonstrating its well-posed nature. By examining the drift and diffusion components and utilizing the Eulers-Maruyama (E-M) numerical method, we derive closed-form solutions and compare the stochastic model's behavior to deterministic model solutions. Through simulations, we reveal that the number of individuals unwilling to disclose their ED status increases over time without interventions to reduce ED prevalence.

1. Introduction

Erectile Dysfunction (ED), or male impotence, refers to a man's inability to maintain an erection during sexual intercourse. ED complications include unsatisfactory sex life, stress, anxiety, embarrassment, self-doubt, marital problems, and infertility. Globally, ED prevalence ranges from 2% to 86% in men under and over 40 years old, respectively, with age-adjusted incidence rates in Nigeria, Egypt, and Pakistan at 57%, 63.6%, and 80% [19]. Recent findings suggest genetic factors also contribute to ED [13, 17]. Treatment can be achieved by abstaining from unhealthy lifestyles such as smoking, excessive intake of drugs, alcohol and junk foods, getting rid of emotional problems of trauma, depression, and addressing emotional issues [8].

Mathematical modeling has been widely employed in physical, biological, and social research. Several models have been developed to describe ED, including linear and nonlinear models accounting for age and depression, penile and physiological problems, penile vascular dysfunction, and leak area in impotent men. Both deterministic and stochastic models have been formulated to describe biological and epidemiological processes, with stochastic models proving particularly relevant due to their incorporation of random perturbations found in many biological scenarios. Wald et al. [22], studied the linear and nonlinear models of moderate ED with age and depression factor, which gives room for future investigation of age related ED. Ng, Ng and Chia [16] studied penile and physiological problems using mathematical modeling. Their results showed that a difference of $0.10.3^oC$ can be observed and hence used to develop database of smart diagnosis for ED using the machine artificial intelligence. Mulhall and Damaser [15], studied the area of leak using data on ED in normal men and men with impotence, where the found that the area of leak in impotent men is higher than normal men.

Barnea, Hayun and Gillon [6], derived a mathematical model to describe penile vascular dysfunction. They showed through simulations that leak should be avoided by applying controls. Several deterministic and stochastic models have been formulated to describe biological and epidemiological process [1-5]. It is therefore pertinent to consider the stochastic model formulation since random perturbations are involved in evolution of many biological scenarios [20, 21]. The dynamic system can practically undergo random disturbances, otherwise called Brownian motion or white noise. A deterministic model with a random term is a stochastic model [7, 10, 12]. Several iterative schemes have been used by authors to get the closed form solutions of the stochastic model, but the numerical

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method of interest in this work in E-M method. The E-M method is extended from Euler's method for ordinary to stochastic differential equation with strong convergence [11, 24, 25].

In this study, we propose a population-based ED model using linear ordinary differential equations with stochastic perturbations. The model accounts for individuals at risk of ED due to unhealthy lifestyle and genetic factors, considering treatment availability and the fear of stigmatization for those who disclose or conceal their ED status. To our knowledge, this research problem has not been previously explored.

The article is organized into distinct sections. Section 2 presents the mathematical model formulation with stochastic perturbations. Section 3 deals with the qualitative analysis of the stochastic model using the Lipschitz continuity theorem and analysis of the global positive solutions. Section 4 employs the Eulers-Maruyama (E-M) numerical scheme to obtain approximate stochastic model solutions via numerical simulations. Lastly, Section 5 concludes the work.

2. The Mathematical Model Formulation

In order to formulate the model, we made some assumptions as follows:

- Erectile dysfunction is a non-communicable disease and humans with ED who engage in unhealthy practices like eating sugary substance, intake of alcohol, smoking, are much more that humans with ED due to genetic predisposition [13, 17].
- Humans with ED may or may not disclose their ED status due to the fear of stigmatization, but humans with ED who disclose their status are considered for treatment [18].

In the model, $H_e(t)$ denotes the number of humans who are at the risk of having ED, $E_l(t)$ denotes the number of humans with ED due to unhealthy lifestyle, $E_g(t)$ denotes the number humans with ED due to genetic factors, $E_d(t)$ denotes the number of humans who disclosed their ED status without the fear of stigmatization, $E_n(t)$ denotes the number of humans who did not disclose their ED status due to the fear of stigmatization and $T_d(t)$ denotes the number of humans treated for ED, these variables collectively give the deterministic model given by

$$\left.\begin{aligned}
\dot{H}_e &= \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e,\\
\dot{E}_l &= k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l,\\
\dot{E}_g &= \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g,\\
\dot{E}_d &= k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d,\\
\dot{E}_n &= \beta_2 E_l + \beta_4 E_g - \mu_p E_n,\\
\dot{T}_d &= \sigma E_d - \mu_p T_d,
\end{aligned}\right\} \tag{2.1}$$

under the initial conditions $H_e \geq 0, E_l \geq 0, E_g \geq 0, E_d \geq 0, E_n \geq 0$ and $T_d \geq 0$. In (2.1), the recruitment rate of humans who are at the risk of ED is denoted ψ and the incidence rates of the ED in humans with unhealthy practices and genetic factors are denoted δ_1 and δ_2, while k_o represents the modification factor that accounts for the increased infection in humans who engage in unhealthy practices relative to humans with ED due to genetic factors. Moreover, $\beta_i(i = 1-4)$ denotes the progression rates of human who engage in unhealthy lifestyle and genetically pre-disposed to ED, while k_1 and k_2 represents the modification factors that describes the increasing rate of disclosure of ED status relative to humans who did not disclose their ED status. Finally, σ represents the recovery rate of humans treated from ED due to disclosure of their ED status.

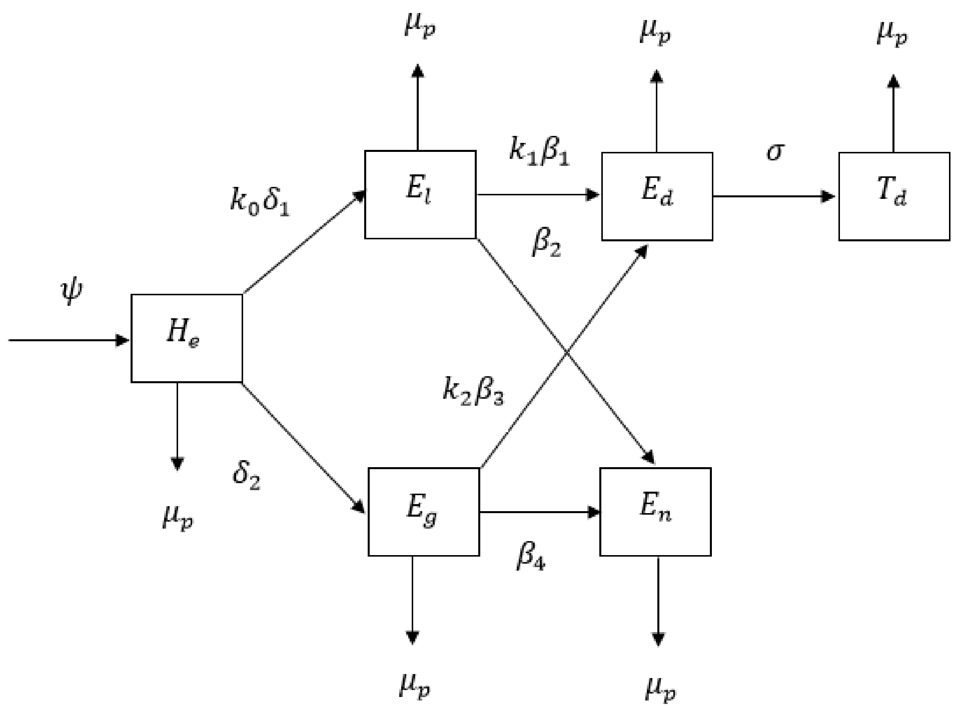

FIGURE 1. Diagrammatic description of the ED incidence progression in different categories of humans

Taking the environmental flunctuations into consideration, we set $\chi_i(t)(i = 1{-}6)$ with $\chi(0) = 0$, $\forall \chi_i$ as the standard Brownian motion and $\rho_i(i = 1 - 6)$ is denoted the white noise so that (2.1) becomes a model based on stochastic differential equation given by

$$\left.\begin{aligned}
\dot{H}_e &= \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e + \rho_1 H_e(t)d\chi_1(t),\\
\dot{E}_l &= k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l + \rho_2 E_l(t)d\chi_2(t),\\
\dot{E}_g &= \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g + \rho_3 E_g(t)d\chi_3(t),\\
\dot{E}_d &= k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d + \rho_4 E_d(t)d\chi_4(t),\\
\dot{E}_n &= \beta_2 E_l + \beta_4 E_g - \mu_p E_n + \rho_5 E_n(t)d\chi_5(t),\\
\dot{T}_d &= \sigma E_d - \mu_p T_d + \rho_6 T_d(t)d\chi_6(t).
\end{aligned}\right\} \tag{2.2}$$

Theorem 2.1. *The ED stochastic model (2.2) is presented under the compact shape given by*

$$\frac{dX(t)}{dt} = f(t, X(t)) + Q(t, X(t))\frac{d\chi}{dt} \tag{2.3}$$

where $\chi = (\chi_j)^T_{j=1\ldots6}$ represents a Brownian motion, where $f(t, X(t))$ represent variation speed vector given by

$$f(t, X(t)) = \begin{pmatrix} \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e, \\ k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l, \\ \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g, \\ k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d, \\ \beta_2 E_l + \beta_4 E_g - \mu_p E_n, \\ \sigma E_d - \mu_p T_d. \end{pmatrix}, \tag{2.4}$$

while the noise matrix $G = G(t, X(t))$ is given by

$$G = \begin{pmatrix} G_1 & -G_2 & -G_3 & -G_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & -G_5 & -G_6 & -G_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4 & 0 & 0 & 0 & -G_8 & -G_9 & -G_{10} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 & 0 & 0 & G_9 & 0 & -G_{11} & -G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_7 & 0 & 0 & G_{10} & 0 & 0 & -G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_{12} & 0 & -G_{14} \end{pmatrix}, \tag{2.5}$$

where the following representations are defined; $G_1 = \sqrt{\psi}, G_2 = \sqrt{\mu_p H_e}, G_3 = \sqrt{k_0\delta_1 H_e}, G_4 = \sqrt{\delta_2 H_e}, G_5 = \sqrt{\mu_p E_l}, G_6 = \sqrt{k_1\beta_1 E_l}, G_7 = \sqrt{\beta_2 E_l}, G_8 = \sqrt{\mu_p E_g}, G_9 = \sqrt{k_2\beta_3 E_g}, G_{10} = \sqrt{\beta_4 E_g}, G_{11} = \sqrt{\mu_p E_d}, G_{12} = \sqrt{\sigma E_d}, G_{13} = \sqrt{\mu_p E_n}, G_{14} = \sqrt{\mu_p T_d}$. Matrix G possess the following properties such that the singular value decomposition of G is $G = PDQ$. Furthermore P and Q represents the orthogonal matrices of dimensions (6×6) and (14×14) respectively, while D denotes (6×14) dimensional matrix with $r(r \leq 6)$ positive diagonal entries. It follows that, $V = GG^T = PDQ(PDQ)^T = PD(QQ^T)D^T P^T = P(DD^T)P^T$, where

$$\sqrt{V} = P(DD^T)^{\frac{1}{2}} P^T. \tag{2.6}$$

Proof. Let the random variables be denoted by $\Delta X = (\Delta H_e, \Delta E_l, \Delta E_g, \Delta E_d, \Delta E_n, \Delta T_d)$ during a variational time Δt, such that P_j denotes the probability of state changes and suppose that there exist m changes of possible states, $j = 1, 2, , m$, we need to take into account two states X^j and X^{j+1} which represents both the initial and intermediate states at time t and $t + \Delta t$ respectively. Therefore, making $\Delta X^j = X^{j+1} - X^j$, we get, $P_j = Prob(\Delta X^j) = Prob\left\{X^{j+1}/X^j\right\}$. To this effect, 15 possible states changes exists for model (2.1) which are summarized below in Table 1.

Table 1. Possible transitions in the stochastic model process

Possibility Change of state (ΔX^j)	Probability $P_j = Prob(\Delta X^j)$
$\Delta X^1 = [1, 0, 0, 0, 0, 0]^T$	$P_1 = \Psi\Delta t$
$\Delta X^2 = [-1, 0, 0, 0, 0, 0]^T$	$P_2 = \mu_p H_e \Delta t$
$\Delta X^3 = [-1, 1, 0, 0, 0, 0]^T$	$P_3 = k_0\delta_1 H_e \Delta t$
$\Delta X^4 = [-1, 0, 1, 0, 0, 0]^T$	$P_4 = \delta_2 H_e \Delta t$
$\Delta X^5 = [0, -1, 0, 0, 0, 0]^T$	$P_5 = \mu_p E_l \Delta t$
$\Delta X^6 = [0, -1, 0, 1, 0, 0]^T$	$P_6 = k_1\beta_1 E_l \Delta t$
$\Delta X^7 = [0, -1, 0, 0, 1, 0]^T$	$P_7 = \beta_2 E_l \Delta t$
$\Delta X^8 = [0, 0, -1, 0, 0, 0]^T$	$P_8 = \mu_p E_g \Delta t$
$\Delta X^9 = [0, 0, -1, 1, 0, 0]^T$	$P_9 = k_2\beta_3 E_g \Delta t$
$\Delta X^{10} = [0, 0, -1, 0, 1, 0]^T$	$P_{10} = \beta_4 E_g \Delta t$
$\Delta X^{11} = [0, 0, 0, -1, 0, 0]^T$	$P_{11} = \mu_p E_d \Delta t$
$\Delta X^{12} = [0, 0, 0, -1, 0, 1]^T$	$P_{12} = \sigma E_d \Delta t$
$\Delta X^{13} = [0, 0, 0, 0, -1, 0]^T$	$P_{13} = \mu_p E_n \Delta t$
$\Delta X^{14} = [0, 0, 0, 0, 0, -1]^T$	$P_{14} = \mu_p T_d \Delta t$
$\Delta X^{15} = [0, 0, 0, 0, 0, 0]^T$	$P_{15} = 1 - \sum_{j=1}^{14} P_j$
Otherwise: $\Delta X^i, i \neq 1, 2, 15$	$P_i = 0$

Therefore the mean and variance of the ΔX are then given by

$$\mathbf{E}(\Delta X) = \sum_{j=1}^{m} P_j \Delta X^j = f(t, X)\Delta t \tag{2.7}$$

and

$$V(\Delta X) = \mathbf{E}((\Delta X)(\Delta X)^T - \mathbf{E}(\Delta X)\cdot\mathbf{E}(\Delta X)^T = \sum_{j=1}^{m} P_j(\Delta X^j)(\Delta X^j)^T = V(t,X)\Delta t, \tag{2.8}$$

where

$$\mathbf{E}(\Delta X) = \begin{pmatrix} \psi - (k_0\delta_1 + \delta_2 + \mu_p)H_e \\ k_0\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l \\ \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g \\ k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d \\ \beta_2 E_l + \beta_4 E_g - \mu_p E_n \\ \sigma E_d - \mu_p T_d \end{pmatrix} \Delta t \tag{2.9}$$

and

$$\mathbf{E}((\Delta X)(\Delta X)^T) = \begin{pmatrix} c_1 & -P_3 - P_4 & 0 & 0 & 0 & 0 \\ -P_3 & c_2 & -P_6 - P_7 & 0 & 0 & 0 \\ -P_4 & 0 & c_3 & -P_9 - P_{10} & 0 & 0 \\ 0 & -P_6 & -P_9 & c_4 & -P_{12} & 0 \\ 0 & -P_7 & -P_{10} & 0 & c_5 & 0 \\ 0 & 0 & 0 & -P_{12} & 0 & c_6 \end{pmatrix} \Delta t \tag{2.10}$$

where

$$\left.\begin{array}{l} P_j = \dfrac{P_6}{\Delta t}, j = 1, 2, ..., 14. \\ c_1 = P_1 + P_2 + P_3 + P_4 = \psi + (k_0\delta_1 + \delta_2 + \mu_p)H_e, \\ c_2 = P_3 + P_5 + P_6 + P_7 = k_0\delta_1 H_e + (k_1\beta_1 + \beta_2 + \mu_p)E_l, \\ c_3 = P_4 + P_8 + P_9 + P_{10} = \delta_2 H_e + (k_2\beta_3 + \beta_4 + \mu_p)E_g, \\ c_4 = P_6 + P_9 + P_{11} + P_{12} = k_1\beta_1 E_l + k_2\beta_3 E_g + (\sigma + \mu_p)E_d, \\ c_5 = P_7 + P_{10} + P_{13} = \beta_2 E_l + \beta_4 E_g + \mu_p E_n, \\ c_6 = P_{12} + P_{14} = \sigma E_d + \mu_p T_d, \end{array}\right\} \tag{2.11}$$

which implies that

$$V(t,X) = \begin{pmatrix} c_1 & -(k_0\delta_1 + \delta_2)H_e & 0 & 0 & 0 & 0 \\ -k_0\delta_1 H_e & c_2 & -(k_1\beta_1 + \beta_2)E_l & 0 & 0 & 0 \\ -\delta_2 H_e & 0 & c_3 & -(k_1\beta_1 + \beta_2)E_l & 0 & 0 \\ 0 & -k_1\beta_1 E_l & -k_2\beta_3 E_g & c_4 & -\sigma E_d & 0 \\ 0 & -\beta_2 E_l & -\beta_4 E_g & 0 & c_5 & 0 \\ 0 & 0 & 0 & -\sigma E_d & 0 & c_6 \end{pmatrix} \tag{2.12}$$

It is clear that $X(t + \Delta t) = X(t) + \Delta X$, is represented in a discrete form given by:

$$X^{j+1} = X^j + \Delta X^j, \ j = 1, , n. \tag{2.13}$$

Note that for $j = 1, , n$, a normal distribution with mean $E(\Delta X^j)$ and variance $V(\Delta X^j)$ exist. Using the Gaussian approximation, we obtain:

$$\Delta X^j = \mathbf{E}(\Delta X^j) + \sqrt{V(\Delta X^j)\Delta\bar{W}^j}, \tag{2.14}$$

where $\Delta\bar{W}^j \sim N(0,1)$, and by the use of the central limit theorem, we get

$$\Delta X = f(t, X(t))\Delta t + \sqrt{(V(t, X(t)))}\Delta t\bar{\omega} \tag{2.15}$$

where the random noise $\bar{\omega} = (\bar{\omega_1}, \bar{\omega_2}, \bar{\omega_3}, \bar{\omega_4}, \bar{\omega_5}, \bar{\omega_6})$ is such that $\bar{\omega_1} \sim N(0,1)(i = 1, 2, 3, 4, 5, 6)$, and it is observed that as $\Delta t \to 0$, $X(t)$ converges strongly towards model (2.2). In order to reduce the model (2.15), the following Lemma is introduced, see [2].

Lemma 1:
The *Itô* stochastic differential equations (EDS_*) and (EDS_{**}) below are shown to be equivalent,

$$\left.\begin{aligned} \frac{dX}{dt} &= f(t,X) + \sqrt{(V(t,X))}\frac{dW}{dt}, \quad (EDS_*),\\ \frac{dX^*}{dt} &= f(t,X^*) + G(t,X^*)\frac{dW^*}{dt}, (EDS_{**}). \end{aligned}\right\} \tag{2.16}$$

where $f : \mathbf{T}\times\mathbf{R}^d \to \mathbf{R}^d,\ G : \mathbf{T}\times\mathbf{R}^d \to R^{d\times m}$ and $V : \mathbf{T}\times\mathbf{R}^d \to R^{d\times d}$ are such that $V = GG^T$. Also the sample solution paths of (EDS_*) and (EDS_{**}) are the same, with the probability axioms satisfying the Lipschitzian condition of (EDS_*) and (EDS_{**}). Moreover, the solutions X and X^* of (EDS_*) and (EDS_{**}) has the same random distribution.
By Lemma 1, we can easily find that model (2.16) is equivalent to (2.3). Given that

$$\frac{dX^*}{dt} = f(t,X^*) + G(t,X^*)\frac{dW^*}{dt}, \tag{2.17}$$

We can observe different events of changes of state of every component of the vector $\Delta X(\Delta X = \Delta H_e, \Delta E_l, \Delta E_g, \Delta E_d, \Delta E_n, \Delta T_d)$ to determine matrix G as $GG^T = V$.
Let Poisson probability be denoted by $\mathbb{P}^*$, so that

$$\left.\begin{aligned} \Delta H_e &= k_1 - k_2 - k_3 - k_4,\\ \Delta E_l &= k_3 - k_5 - k_6 - k_7,\\ \Delta E_g &= k_4 - k_8 - k_9 + k_{10},\\ \Delta E_d &= k_6 + k_9 - k_{11} - k_{12},\\ \Delta E_n &= k_7 + k_{10} - k_{13},\\ \Delta T_d &= k_{12} - k_{14}, \end{aligned}\right\} \tag{2.18}$$

with $k_1 \sim \mathbb{P}(\psi\Delta t), k_2 \sim \mathbb{P}(\mu_p H_e\Delta t), k_3 \sim \mathbb{P}(k_0\delta_1 H_e\Delta t), k_4 \sim \mathbb{P}(\delta_2 H_e\Delta t), k_5 \sim \mathbb{P}(\mu_p E_l\Delta t), k_6 \sim \mathbb{P}(k_1\beta_1 E_l\Delta t), k_7 \sim \mathbb{P}(\beta_2 E_l\Delta t), k_8 \sim \mathbb{P}(\mu_p E_g\Delta t), k_9 \sim \mathbb{P}(k_2\beta_3 E_g\Delta t), k_{10} \sim \mathbb{P}(\beta_4 E_g\Delta t), k_{11} \sim \mathbb{P}(\mu_p E_d\Delta t), k_{12} \sim \mathbb{P}(\sigma E_d\Delta t), k_{13} \sim \mathbb{P}(\mu_p E_n\Delta t), k_{12} \sim \mathbb{P}(\mu_p T_d\Delta t)$.
Then system (2.18) becomes

$$\left.\begin{aligned} &\Delta H_e = \psi\Delta t + \sqrt{\psi\Delta t\omega_1} - \mu_p H_e\Delta t - \sqrt{(\mu_p H_e\Delta t)}\omega_2 - k_0\delta_1 H_e\Delta t - \sqrt{(k_0\delta_1 H_e\Delta t)}\omega_2 - \delta_2 H_e\Delta t\\ &- \sqrt{(\delta_2 H_e\Delta t)\omega_4},\\ &\Delta E_l = k_0\delta_1 H_e\Delta t + \sqrt{(k_0\delta_1 H_e\Delta t)}\omega_2 - \mu_p E_l\Delta t - \sqrt{(\mu_p E_l\Delta t)\omega_5} - k_1\beta_1 E_l\Delta t - \sqrt{(k_1\beta_1 E_l\Delta t)\omega_6}\\ &- \beta_2 E_l\Delta t - \sqrt{(\beta_2 E_l\Delta t)\omega_7},\\ &\Delta E_g = \delta_2 H_e\Delta t + \sqrt{(\delta_2 H_e\Delta t)}\omega_4 - \mu_p E_g\Delta t - \sqrt{(\mu_p E_g\Delta t)\omega_8} - k_2\beta_3 E_g\Delta t - \sqrt{(k_2\beta_3 E_g\Delta t)\omega_9}\\ &- \beta_4 E_g\Delta t - \sqrt{(\beta_4 E_g\Delta t)\omega_{10}},\\ &\Delta E_d = k_1\beta_1 E_l\Delta t + \sqrt{(k_1\beta_1 E_l\Delta t)}\omega_6 + k_2\beta_3 E_g\Delta t + \sqrt{(k_2\beta_3 E_g\Delta t)\omega_9} - \mu_p E_d\Delta t - \sqrt{(\mu_p E_d\Delta t)\omega_{11}}\\ &- \sigma E_d\Delta t - \sqrt{(\sigma E_d\Delta t)\omega_{12}},\\ &\Delta E_n = \beta_2 E_l\Delta t + \sqrt{(\beta_2 E_l\Delta t)\omega_7} + \beta_4 E_g\Delta t + \sqrt{(\beta_4 E_g\Delta t)\omega_{10}} - \mu_p E_n\Delta t - \sqrt{(\mu_p E_n\Delta t)\omega_{13}},\\ &\Delta T_d = \sigma E_d\Delta t + \sqrt{(\sigma E_d\Delta t)\omega_{12}} - \mu_p T_d\Delta t - \sqrt{(\mu_p T_d\Delta t)\omega_{14}}, \end{aligned}\right\} \tag{2.19}$$

that is $u_j \sim N(0,1)$ for $j = 1,2,...,14$. As $\Delta t \to 0$, (2.19) converges to the stochastic differential equation (2.3); $\frac{dX(t)}{dt} = f(t,X(t)) + G(t,X(t))\frac{d\chi(t)}{dt}$, which ends the proof.

3. Qualitative Analysis of the Stochastic Model

3.1. Existence and Uniqueness Results of the Stochastic Model.

Theorem 3.1. *For* $\frac{dX(t)}{dt} = f(t, X(t)) + G(t, X(t))\frac{d\chi(t)}{dt}$, *the following properties hold*

- *Both f(X(t)) and G(X(t)) are continuous on* $t \in [t_o T], T > 0$
- *The coefficient functions* f *and* G *satisfies the Lipschitz condition* $|f(x,t) - f(y,t)| + |G(x,t) - G(x,t)| \leq K|x - y|$ *for some constant* K *and all* $t \in [t_o, T], T > 0$.
- *The coefficient functions* f *and* G *satisfies the growth condition* $|f(y,t)|^2 + |G(x,t)|^2 \leq K^2(1 + |x|^2)$ *for some constant* K^2 *for all* $t \in [t_o, T], T > 0$ *is continuous with probability 1.*

Assume the coefficients in the following system

$$dX_t^i = a_i(t, X_t)dt + \sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij}(t, X_t)dW_t^j \tag{3.1}$$

where

$$\left.\begin{aligned} X_t &= (X_t^1, X_t^2, ..., X_t^n)^T, \\ W_t &= (W_t^1, W_t^2, ..., W_t^m)^T, \end{aligned}\right\} \tag{3.2}$$

where such that $a_i(t, X_t)$ *and* $b_{ij}(t, X_t)$ *is an* $n \times m$ *dimensional matrix with entries* $a_i(t, X_t)$ *and* $b_{ij}(t, x)$ *satisfy the Lipschitz and growth conditions for some constant* $k < \infty$ *and for all* $t \in \Re$ *and* $x, y \in \Re^n$ *such that*

$$\left.\begin{aligned} &||a_i(t,x) - a_i(t,y) \leq k||x - y||, \\ &||b_{ij}(t,x) - b_{ij}(t,y)|| \leq k||x - y||, \\ &||a_i(t,x)|| \leq k||x||, \\ &||b_{ij}(t,x)|| \leq k||x||, \\ &||b|| = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij}(x)^2}, \\ &||a|| = \sqrt{\sum_{i=1}^{n} a_i(x)^2} \end{aligned}\right\} \tag{3.3}$$

where for each $x_o \in \Re^n$, *there exist a unique solution to the system (2.3) such that* $X_o = x_o$

Proof. Let

$$\left.\begin{aligned} f_1 &= \psi - (k_o\delta_1 + \delta_2 + \mu_p)H_e, \\ f_2 &= k_o\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l, \\ f_3 &= \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g, \\ f_4 &= k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d, \\ f_5 &= \beta_2 E_l + \beta_4 E_g - \mu_p E_n, \\ f_6 &= \sigma E_d - \mu_p T_d. \end{aligned}\right\} \tag{3.4}$$

$$\left.\begin{array}{l}
\left|\frac{\partial f_1}{\partial H_e}\right| = |k_o\delta_1 + \delta_2 + \mu_p| \leq M,\ \left|\frac{\partial f_1}{\partial E_l}\right| = \left|\frac{\partial f_1}{\partial E_g}\right| = \left|\frac{\partial f_1}{\partial E_d}\right| = \left|\frac{\partial f_1}{\partial E_n}\right| = \left|\frac{\partial f_1}{\partial T_d}\right| = 0 \\
\left|\frac{\partial f_2}{\partial H_e}\right| = |k_o\delta_1| \leq M, \left|\frac{\partial f_2}{\partial E_l}\right| = |k_1\beta_1 + \beta_2 + \mu_p| \leq M,\ \left|\frac{\partial f_2}{\partial E_g}\right| = \left|\frac{\partial f_2}{\partial E_d}\right| = \left|\frac{\partial f_2}{\partial E_n}\right| = \left|\frac{\partial f_2}{\partial T_d}\right| = 0 \\
\left|\frac{\partial f_3}{\partial H_e}\right| = |\delta_2| \leq M,\ \left|\frac{\partial f_3}{\partial E_l}\right| = |k_2\beta_3 + \beta_4 + \mu_p| \leq M,\ \left|\frac{\partial f_3}{\partial E_g}\right| = \left|\frac{\partial f_3}{\partial E_d}\right| = \left|\frac{\partial f_3}{\partial E_n}\right| = \left|\frac{\partial f_3}{\partial T_d}\right| = 0 \\
\left|\frac{\partial f_4}{\partial H_e}\right| = \left|\frac{\partial f_4}{\partial T_d}\right| = \left|\frac{\partial f_4}{\partial E_n}\right| = 0,\ \left|\frac{\partial f_4}{\partial E_l}\right| = |k_1\beta_1| \leq M,\ \left|\frac{\partial f_4}{\partial E_g}\right| = |k_2\beta_3| \leq M,\ \left|\frac{\partial f_4}{\partial E_d}\right| = |\sigma + \mu_p| \leq M \\
\left|\frac{\partial f_5}{\partial H_e}\right| = \left|\frac{\partial f_5}{\partial E_d}\right| = \left|\frac{\partial f_1}{\partial T_d}\right| = 0,\ \left|\frac{\partial f_5}{\partial E_l}\right| = |\beta_2| \leq M,\ \left|\frac{\partial f_5}{\partial E_g}\right| = |\beta_4| \leq M,\ \left|\frac{\partial f_5}{\partial E_n}\right| = |\mu_p| \leq M \\
\left|\frac{\partial f_6}{\partial H_e}\right| = \left|\frac{\partial f_6}{\partial E_l}\right| = \left|\frac{\partial f_6}{\partial E_g}\right| = \left|\frac{\partial f_6}{\partial E_n}\right| = 0,\ \left|\frac{\partial f_6}{\partial E_d}\right| = |\sigma| \leq M,\ \left|\frac{\partial f_6}{\partial T_d}\right| = |\mu_p| \leq M
\end{array}\right\} \tag{3.5}$$

It is observed in (3.24) that the elements of the diffusion/or drift parts of the model are continuously differentiable. Also for the stochastic/diffusion parts of (2.3), we obtain $||f|| = \sqrt{\sum_{i=1}^{6} f_i(x)^2}$ and $||G|| = \sqrt{\sum_{i=1}^{6}\sum_{j=1}^{14} b_{ij}(x)^2}$, which implies that both f_i and g_{ij} are continuously differentiable and bounded and satisfy the Lipschitz condition. Hence, it implies that the model exist and is unique.

3.2. **Positive Global Bounded Solutions of the Stochastic Model.** We define the following set

$$\Theta = \left\{(H_e, E_l, E_g, E_d, E_n, T_d) \in \Re^{+6} : N_e = H_e + E_l + E_g + E_d + E_n + T_d \leq \frac{\psi}{\mu_p}\right\}$$

, then we have the following analytical result

Theorem 3.2. *Let* $X_o = (H_e(0), E_l(0), E_g(0), E_g(0), E_d(0), E_n(0), T_d(0))$ *and*

$$X(t) = (H_e(t), E_l(t), E_g(t), E_d(t), E_n(0), T_d(0))$$

for $t \geq 0$*, where* $X(t)$ *is the solution curve of the model (2.3) going through* X_o*, then* $X(t)$ *stays in* Θ *for all* $t \geq 0$*, with probability 1.*

Proof. Let the random variable $N_d(t) = H_e(t) + E_l(t) + E_g(t) + E_d(t) + E_n(t) + T_d(t)$ be the total number of human host population, so that

$$\frac{dN_d(t)}{dt} = \psi - \mu_p N_d(t) - \Big(D_1\frac{d\chi_1(t)}{dt} + D_2\frac{d\chi_2(t)}{dt} + D_5\frac{d\chi_5(t)}{dt} + D_8\frac{d\chi_8(t)}{dt} + D_{11}\frac{d\chi_{11}(t)}{dt} + D_{13}\frac{d\chi_{13}(t)}{dt} + D_{14}\frac{d\chi_{14}(t)}{dt}\Big). \tag{3.6}$$

since $\xi\Big(X(t), \frac{d\chi(t)}{dt}\Big) = \Big(D_1\frac{d\chi_1(t)}{dt} + D_2\frac{d\chi_2(t)}{dt} + D_5\frac{d\chi_5(t)}{dt} + D_8\frac{d\chi_8(t)}{dt} + D_{11}\frac{d\chi_{11}(t)}{dt} + D_{13}\frac{d\chi_{13}(t)}{dt} + D_{14}\frac{d\chi_{14}(t)}{dt}\Big)$ If $X(c) \in R^6_+$ for all $0 \leq c \leq t$ almost surely, we obtain the following inequality

$$\frac{dN_d(t)}{dt} \leq \psi - \mu_p N_d(c), \tag{3.7}$$

almost surely. Therefore

$$N_d(c) \leq \frac{\psi}{\mu_p}, \tag{3.8}$$

so that $H_e(c), E_l(c), E_g(c), E_d(c), E_n(c), T_d(c) \in \Big[0, \frac{\psi}{\mu_p}\Big]$ for all $c \in [0, t]$.
It is clear that for f and G which are locally Lipschitz continuous, a unique locally Lipschitz solution $X(c)$ on $[0, t]$ exist for any given initial value X_0, in order to show that the solution $X(t)$ is global for $t \in [0, x_e]$, where x_e denotes the explosion time,
Let $\gamma_o > 0$ such that $(H_e(0), E_l(0), E_g(0), E_d(0), E_n(0), T_d(0)) > \gamma_o$ and for $\gamma \leq 0$, the stopping times is defined given by

$$x_\gamma = \inf\{t \in [0, x_e], H_e(t) < \gamma, E_l(t) < \gamma, E_g(t) < \gamma, E_d(t) < \gamma, E_n(t) < \gamma, T_d(t) < \gamma\} \tag{3.9}$$

we have $\lim_{\gamma\to 0} x_\gamma = x_o$ with

$$x_o = \{t \in [0, x_e], H_e(t) < \gamma, E_l(t) < \gamma, E_g(t) < \gamma, E_d(t) < \gamma, E_n(t) < \gamma, T_d(t) < 0\} \tag{3.10}$$

Let's consider the function C defined for $X = (H_e, E_l, E_g, E_d, E_n, T_d)$ by

$$C = -\ln\Big(\frac{\psi}{\mu_p}H_e\Big) - \ln\Big(\frac{\psi}{\mu_p}E_l\Big) - \ln\Big(\frac{\psi}{\mu_p}E_g\Big) - \ln\Big(\frac{\psi}{\mu_p}E_d\Big) - \ln\Big(\frac{\psi}{\mu_p}E_n\Big) - \ln\Big(\frac{\psi}{\mu_p}T_d\Big) \tag{3.11}$$

By the use of the formula Itô formula multidimensional problems on the interval $[0, \min(t, x_\gamma)]$, we have for all z;

$$\begin{aligned} dC(X(z)) = \Big(&\frac{\partial C(X(z))}{\partial z} + \sum_{i=1}^{6} f_i(z, X(z))\frac{\partial C(X(z))}{\partial X_i} + \\ &\frac{1}{2}\sum_{i,j=1}^{6}(GG^T)_{ij}\frac{\partial^2 C(X(z))}{\partial X_i \partial X_j}\Big)dz + \sum_{i=1}^{6}\sum_{j=1}^{14} G_{ij}dW_j(z)\frac{\partial C(X(z))}{\partial X_i} \end{aligned} \tag{3.12}$$

with

$$G = (G_{ij})_{i=1,\dots,6,\quad j=1,2,\dots,14} \tag{3.13}$$

and

$$(GG^T)_{ij} = \sum_{k=1}^{14} G_{ik}G_{kj}, \tag{3.14}$$

that is,

$$\left.\begin{aligned} &dC(X) = [6\mu_p + k_o\delta_1 + \delta_2 + k_1\beta_1 + \beta_2 + k_2\beta_3 + \beta_4 + \sigma]dz - \\ &\Big[\psi\frac{(2H_e-1)}{2H_e^2} + (k_o\delta_1 H_e)\frac{(2E_l-1)}{2E_l^2} + (\delta_2 H_e)\frac{(2E_g-1)}{2E_g^2} + (k_1\beta_1 E_l + k_2\beta_3 E_g)\frac{(2E_d-1)}{2E_d^2} + \\ &(\beta_2 E_l + \beta_4 E_g)\frac{(2E_n-1)}{2E_n^2} + (\sigma E_d)\frac{(2T_d-1)}{2T_d^2}\Big]dz + \Big[(k_o\delta_1 + \delta_2 + \mu_p)\frac{1}{2H_e} + (k_1\beta_1 + \beta_2 + \mu_p)\frac{1}{2E_l} \\ &+ (k_2\beta_3 + \beta_4 + \mu_p)\frac{1}{2E_g} + (\sigma + \mu_p)\frac{1}{2E_d} + \mu_p\frac{1}{2E_n} + \mu_p\frac{1}{2T_d}\Big]dz - \frac{1}{H_e}(G_1dW_1 + G_2dW_2 + G_3dW_3 + G_4dW_4) - \\ &\frac{1}{E_l}(G_3dW_3 - G_5dW_5 - G_6dW_6 - G_7dW_7) - \frac{1}{E_g}(G_4dW_4 - G_8dW_8 - G_9dW_9 - G_{10}dW_{10}) - \\ &\frac{1}{E_d}(G_7dW_7 + G_9dW_9 - G_{11}dW_{11} - G_{12}dW_{12}) - \frac{1}{E_n}(G_7dW_7 + G_{10}dW_{10} - G_{13}dW_{13}) \\ &+ \frac{1}{T_d}(G_{12}dW_{12} - G_{14}dW_{14}). \end{aligned}\right\} \tag{3.15}$$

Also, for almost surely for $z \in [0, t]$,

$$\left.\begin{aligned} &dC(X(z))t \le [6\mu_p + 2k_o\delta_1 + 2delta_2 + 2k_1\beta_1 + 2\beta_2 + 2k_2\beta_3 + 2\beta_4 + 2\sigma]dz + \frac{1}{H_e}(G_1dW_1) + \frac{1}{E_l}(G_3dW_3) + \\ &\frac{1}{E_g}(G_4dW_4) + \frac{1}{E_d}(G_7dW_7 + G_9dW_9) + \frac{1}{E_n}(G_7dW_7 + G_{10}dW_{10}) + \frac{1}{T_d}(G_{12}dW_{12}). \end{aligned}\right\} \tag{3.16}$$

Integrating the inequality in (3.16), we obtain

$$\left.\begin{aligned} &dC(X(z))t \le Qt + \int_0^t \frac{1}{H_e}(G_1dW_1(z)) + \int_0^t \frac{1}{E_l}(G_3dW_3(z)) + \int_0^t \frac{1}{E_g}(G_4dW_4(z)) + \\ &\int_0^t \frac{1}{E_d}(G_7dW_7(z) + G_9dW_9(z)) + \int_0^t \frac{1}{E_n}(G_7dW_7(z) + G_{10}dW_{10}(z)) + \int_0^t \frac{1}{T_d}(G_{12}dW_{12}(z)) \end{aligned}\right\} \tag{3.17}$$

where $Q = 6\mu_p + 2k_o\delta_1 + 2delta_2 + 2k_1\beta_1 + 2\beta_2 + 2k_2\beta_3 + 2\beta_4 + 2\sigma$. Taking the mathematical expectation $\mathbf{E}(.)$ in (3.16), we obtain

$$E[C(Q(t))] \leq Qt + \Big(E\Big[\int_0^t \frac{1}{H_e}(G_1 dW_1(z))\Big] + E\Big[\int_0^t \frac{1}{E_l}(G_3 dW_3(z))\Big] + E\Big[\int_0^t \frac{1}{E_g}(G_4 dW_4(z))\Big] + E\Big[\int_0^t \frac{1}{E_d}(G_7 dW_7(z) + G_9 dW_9(z))\Big] + E\Big[\int_0^t \frac{1}{E_n}(G_7 dW_7(z) + G_{10} dW_{10}(z))\Big] + E\Big[\int_0^t \frac{1}{T_d}(G_{12} dW_{12}(z))\Big], \tag{3.18}$$

Therefore (3.18) leads to the following Lemma below, see [23].

Lemma 2.

The stochastic integral $\int_0^t B(z)dW(z)$ yields its quadratic variation given by $\int_0^t B^2(z)dz \leq Qt$, while the local Martingales for strong law of implies that

$$\lim_{t\to\infty} \int_0^t B(z)dW(z) = 0, \tag{3.19}$$

almost surely. Thus, we have

$$\left.\begin{aligned}
&\mathbf{E}\Big[\int_0^t \frac{1}{H_e}(G_1 dW_1(z))\Big] = 0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_l}(G_3 dW_3(z))\Big] = 0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_g}(G_4 dW_4(z))\Big] = 0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_d}(G_7 dW_7(z) + G_9 dW_9(z))\Big] = 0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{E_n}(G_7 dW_7(z) + G_{10} dW_{10}(z))\Big] = 0,\\
&\mathbf{E}\Big[\int_0^t \frac{1}{T_d}(G_{12} dW_{12}(z))\Big] = 0.
\end{aligned}\right\} \tag{3.20}$$

Then for all $t \geq 0$, we have

$$\mathbf{E}[C(X(\min(t, x_\gamma)))] \leq Q\min(t, x_\gamma) \leq Qt. \tag{3.21}$$

Since $V(Q(\min(t, x_\gamma))) > 0$, then

$$\mathbf{E}[V(Q(\min(t, x_\gamma)))] = \mathbf{E}\Big[V(Q(\min(t, x_\gamma)))\mathbf{1}_{(x_\gamma \leq t)}\Big] + \mathbf{E}\Big[V(Q(\min(t, x_\gamma)))\mathbf{1}_{(x_\gamma \leq t)}\Big] \geq \mathbf{E}\Big[V(Q(\min(t, x_\gamma)))\mathbf{1}_{(x_\gamma \leq t)}\Big], \tag{3.22}$$

where $\mathbf{1_A}$ is the characteristic function of A, and there exist some component of $Q(x_\gamma)$ equal to γ, then, $C(Q(x_\gamma)) \geq -\ln\left(\frac{\mu_p \gamma}{\psi}\right)$. Hence

$$\mathbf{E}[C(Q(\min(t, x_\gamma)))] \geq -\ln\left(\frac{\mu_p \gamma}{\psi}\right)\mathbf{P}^*(x_\gamma \leq t). \tag{3.23}$$

Taking $\gamma \to 0$ into (3.23), one obtain for all $t \geq 0$;

$$\mathbf{P}^*(x_\gamma \leq t) = 0. \tag{3.24}$$

Hence $\mathbf{P}^*(x = \infty) = 1$. As $x_\gamma \geq x$, then $x_\gamma = x = \infty$ almost surely. This ends the proof to the theorem.

4. Numerical Method of Solution and Simulations

4.1. **Numerical Method of Solution.** Here, we employ Euler-Maruyama numerical scheme [11], given by, $X(t_{n+1}) = X(t_n) + fX(t_n)\Delta t + GX(t_n)\Delta \frac{d\chi}{dt}$ where $n = i = 0, 1, 2, ..., N1$, with $X(0) = X_o$ as the initial condition, and $0 < t < T$, where $\chi(t)$ is the Brownian motion. We apply the numerical scheme to the model (2.3) to obtain

$$\begin{pmatrix} H_{e_{n+1}} \\ E_{l_{n+1}} \\ E_{g_{n+1}} \\ E_{d_{n+1}} \\ E_{n_{n+1}} \\ T_{d_{n+1}} \end{pmatrix} = \begin{pmatrix} H_{e_n} \\ E_{l_n} \\ E_{g_n} \\ E_{d_n} \\ E_{n_n} \\ T_{d_n} \end{pmatrix} + \begin{bmatrix} \psi - (k_0\delta_1 + \delta_2 + \mu_p)H_e \\ k_0\delta_1 H_e - (k_1\beta_1 + \beta_2 + \mu_p)E_l \\ \delta_2 H_e - (k_2\beta_3 + \beta_4 + \mu_p)E_g \\ k_1\beta_1 E_l + k_2\beta_3 E_g - (\sigma + \mu_p)E_d \\ \beta_2 E_l + \beta_4 E_g - \mu_p E_n \\ \sigma E_d - \mu_p T_d \end{bmatrix} \Delta t$$
$$+ \sqrt{\left[\begin{bmatrix} c_1 & -P_3 - P_4 & 0 & 0 & 0 & 0 \\ -P_3 & c_2 & -P_6 - P_7 & 0 & 0 & 0 \\ -P_4 & 0 & c_3 & -P_9 - P_{10} & 0 & 0 \\ 0 & -P_6 & -P_9 & c_4 & -P_{12} & 0 \\ 0 & -P_7 & -P_{10} & 0 & c_5 & 0 \\ 0 & 0 & 0 & -P_12 & 0 & c_6 \end{bmatrix} \Delta t\right]} \Delta\chi_n, \tag{4.1}$$

where Δt is a discretized parameter and $c_i's$ are defined in (2.11). We implement (4.1), using MATLAB computational software to yield the numerical simulations below.

4.2. **Numerical Simulations.** Here, we estimate parameters describing ED incidence in Nigeria [18, 19]. The recruitment rate ψ which determine the population size since the asymptotic carrying capacity of the population is $\frac{\psi}{\mu} \approx 10^6$, but for the purpose of simulation, we put ψ at 500 per week. Also, we assumed μ to be inversely related to life expectancy at birth which is approximately 45 years in Nigeria, that is, $\frac{1}{\mu} = \frac{1}{45} = 0.222$ per year, while we take the recovery rate σ to be the inverse of time between ED activation and recovery by treatment. Also, we consider the treatment period to be between 2 to 10 weeks, and the recovery period to be 10 weeks, that is $\frac{1}{10} = 0.1$ per week. The prevalence of ED in Nigeria is given as 60.5% per week [18], while we assume the prevalence of ED due unhealthy lifestyle and genetic factors and the modification rates of the disease prevalence to be between 0 and 1 We present Figures 2 - 5 as the stochastic and deterministic behavior of the model solutions of (2.2). We observe that the wavy curves (stochastic solutions) is slower that the smooth curve (deterministic solutions) because the latter considers randomness or environmental fluctuations, which implies that the wavy curve is near the real solution of the ED model than the smooth curve.

In Figure 2, individuals at risk of developing Erectile Dysfunction (ED), represented as H_e, show a declining trend over time due to a lack of proactive health measures. Meanwhile, those with ED resulting from an unhealthy lifestyle, denoted as E_l, decrease over time as individuals disclose their ED status E_d and receive appropriate treatments. Cases of ED attributed to genetic factors E_g experience a slight increase and peak within the first two weeks but gradually decline with proper consultation and treatment. Conversely, the population of individuals not disclosing their ED status E_n increases over time, reflecting a lack of healthy interventions due to the fear of stigmatization. Those who do disclose their status and receive treatment T_d gradually increase over a 10-week period. In Figure 3, an increase in parameter values for initial conditions leads to a rapid convergence of the stochastic curve within 4.5 weeks, highlighting the swift manifestation of ED within a short time frame compared to the deterministic curve. Figure 4 illustrates that an escalation in initial conditions and parameters results in a sudden increase in individuals with ED who choose not to disclose their status due to stigmatization fears. Simultaneously, there is an increase in the stochastic behavior of those who disclose their status and undergo treatment over time. Figure 5 demonstrates that, with decreased parameter values and increased initial conditions, there is a rapid biological manifestation similar to Figure 2. Both stochastic and deterministic curves converge quickly within 2.5 weeks.

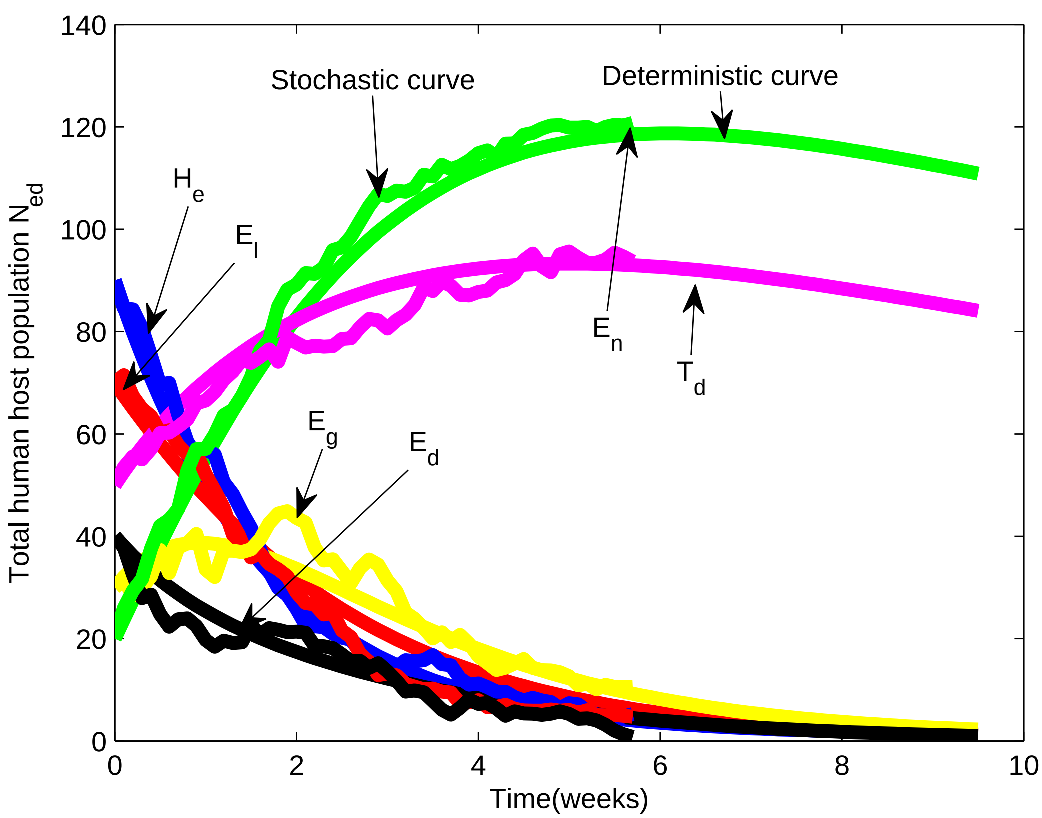

FIGURE 2. Behavior of the stochastic and deterministic model solutions using the fixed parameters in Table 1 together with initial starts $H_e = 90, E_l = 70, E_g = 35, E_d = 40, E_n = 20$ and $T_d = 55$ in ten weeks

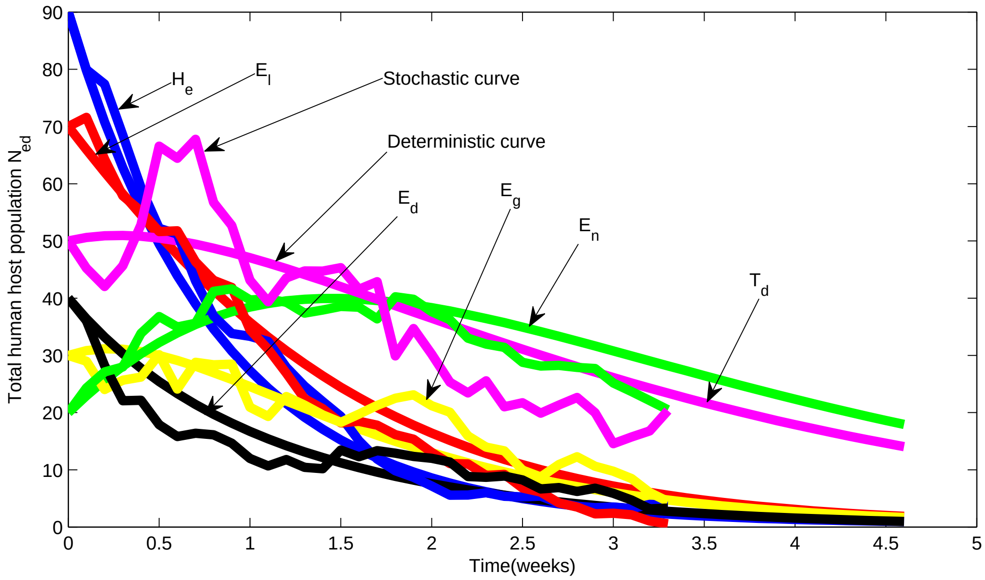

FIGURE 3. Behavior of the stochastic and deterministic model solutions with varied parameters in Table 1 together with initial starts $H_e = 90, E_l = 70, E_g = 35, E_d = 40, E_n = 20$ and $T_d = 55$ in five weeks

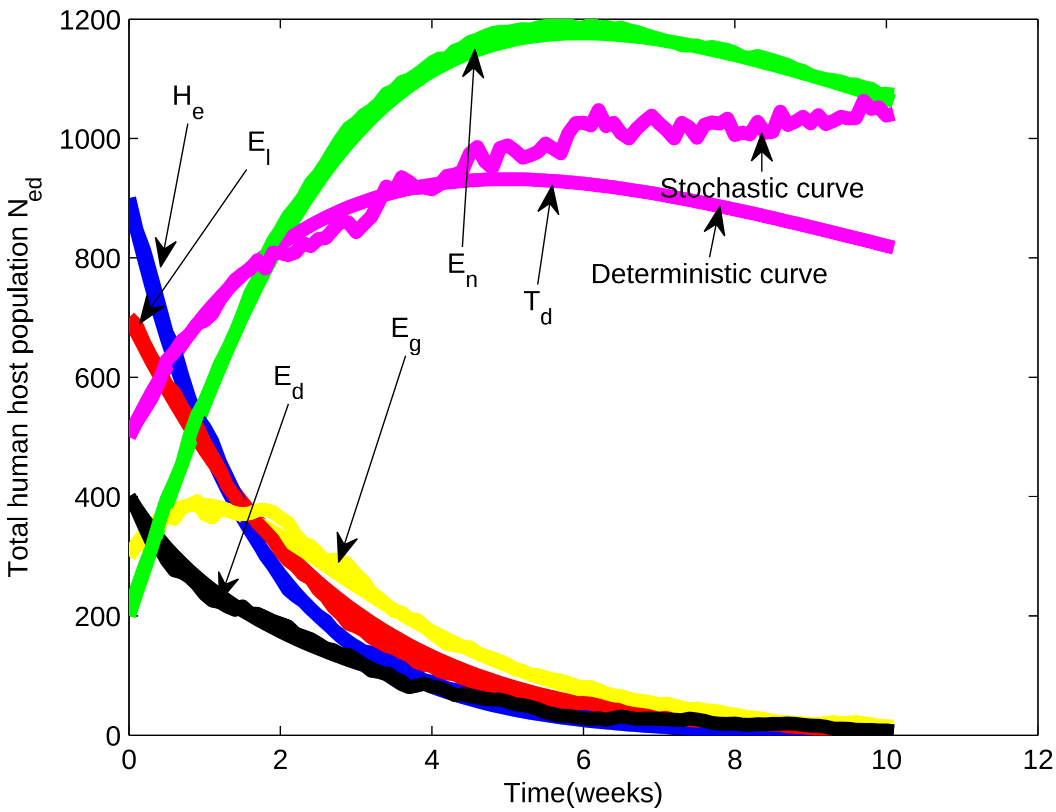

FIGURE 4. Behavior of the stochastic and deterministic model solutions with fixed parameters in Table 1 together with initial starts $H_e = 900, E_l = 700, E_g = 350, E_d = 400, E_n = 200$ and $T_d = 550$

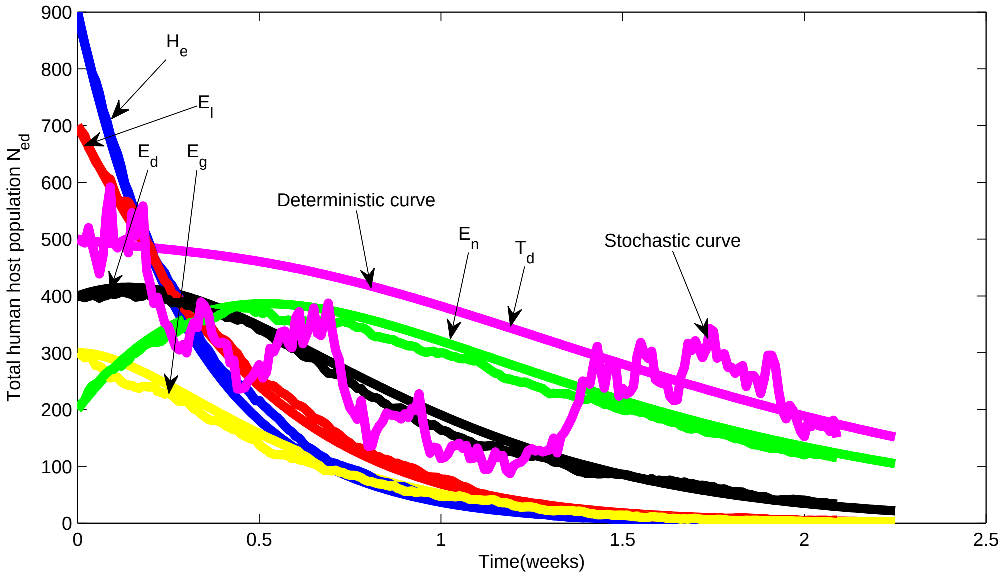

FIGURE 5. Behavior of the stochastic and deterministic model solutions with varied parameters in Table 1 together with initial starts $H_e = 900, E_l = 700, E_g = 350, E_d = 400, E_n = 200$ and $T_d = 550$

5. Conclusion

In conclusion, our study has developed a stochastic model employing ordinary differential equations to elucidate the incidence of Erectile Dysfunction (ED) in men, considering both unhealthy lifestyles and genetic factors. The model not only establishes the existence, uniqueness, and positive global solutions but also incorporates transition probabilities, drift, and diffusion matrices. Through the application of the E-M method, we have generated approximate solutions, revealing the better performance of stochastic curves over deterministic ones, particularly in the presence of environmental noise. Our simulations provide valuable insights into the dynamics of ED, emphasizing the importance of timely interventions. The simulations illustrates the declining trend of individuals at risk H_e, the decreasing prevalence of ED due to unhealthy lifestyles E_l upon disclosure and treatment, and the transient increase followed by a decline in cases associated with genetic factors E_g. Concurrently, there is a concerning rise in those withholding their ED status E_n due to the fear of stigmatization, underscoring the need for targeted strategies to address this trend. Further simulations support our conclusions by demonstrating the rapid convergence of stochastic curves, especially with increased parameter values for initial conditions, and by highlighting the pronounced impact of stigmatization fears on individuals choosing not to disclose their ED status. Overall, our findings contribute to a comprehensive understanding of ED disease dynamics and provide insights into potential strategies for its effective management. Addressing the psychological barriers associated with stigmatization emerges as a crucial aspect of enhancing interventions and minimizing the long-term prevalence of undisclosed ED cases.

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