Modal logic with the difference modality of topological T_0-spaces

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03.07.2018

History

Theorem (McKinsey–Tarski, 1944)

$S4$ is the logic of all topological spaces.

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S4 is the logic of any dense-in-itself separable metrizable space when interpreting $\Box$ as interior.

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Increase expressive power:

- derivational interpretation

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Increase expressive power:

- derivational interpretation
- derivational interpretation + universal modality

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We denote $[\neq]A \wedge A$ by $[\forall]A$.

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A *normal bimodal logic* is a subset of the formulas $L \subseteq \mathcal{ML}_2$ such that

1. L contains all the classical tautologies:

2. L contains the modal axioms of normality:

$$\Box(p \to q) \to (\Box p \to \Box q),$$
$$[\neq](p \to q) \to ([\neq]p \to [\neq]q);$$

3. L is closed with respect to the following inference rules:

$$\frac{A \to B,\, A}{B} \text{ (MP)},$$
$$\frac{A}{\Box A},\ \frac{A}{[\neq]A} (\to \Box, \to [\neq]),$$
$$\frac{A}{[B/p]A} \text{ (Sub)}.$$

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We introduce the notation for the following logics:

- $S4 = K_1 + T_\Box + 4_\Box$
- $S4D = K_2 + T_\Box + 4_\Box + D_\Box + B_D + 4^-_D$

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- $S4 = K_1 + T_\Box + 4_\Box$
- $S4D = K_2 + T_\Box + 4_\Box + D_\Box + B_D + 4_D^-$
- $S4DT_0 = S4D + AT_0$

Topological semantics

A topological model on a topological space $\mathbb{X} := (X, \Omega)$ is the pair $(\mathbb{X}, V)$, where $V : PV \to P(X)$ (the set of all subsets). The truth of a formula ϕ at a point x of the topological model $\mathcal{M} = (\mathbb{X}, V)$ (notation: $\mathcal{M}, x \vDash \phi$) is defined by induction:

- $\mathcal{M}, x \vDash p \Leftrightarrow x \in V(p)$
- $\mathcal{M}, x \nvDash \bot$
- $\mathcal{M}, x \vDash \phi \to \psi \Leftrightarrow \mathcal{M}, x \nvDash \phi$ or $\mathcal{M}, x \vDash \psi$
- $\mathcal{M}, x \vDash \Box\phi \Leftrightarrow \exists U \in \Omega(x \in U$ and $\forall y \in U(\mathcal{M}, y \vDash \phi)$
- $\mathcal{M}, x \vDash [\neq]\phi \Leftrightarrow \forall y \neq x(\mathcal{M}, y \vDash \phi)$

Topological semantics

- ϕ is true in a model $\mathcal{M}$: $\mathcal{M} \vDash \phi \Leftrightarrow \forall x \in X \ (\mathcal{M}, x \vDash \phi)$
- ϕ is valid in $\mathbb{X}$: $\mathbb{X} \vDash \phi \Leftrightarrow \forall V \ (\mathbb{X}, V \vDash \phi)$.
- *Logic* of a class of topological spaces $\mathcal{C}$
  $L(\mathcal{C}) = \{\phi \mid \forall \mathbb{X} \in \mathcal{C} \ \mathbb{X} \vDash \phi\}$

Lemma

Let $\mathbb{X} = (X, \Omega)$ be a topological space then $\mathbb{X} \vDash AT_0$ iff $\mathbb{X}$ is a T_0 space.

Definition

We call logic L complete with respect to a class of topological spaces $\mathcal{C}$ if $L(\mathcal{C}) = L$.

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- $M, x \vDash p \Leftrightarrow x \in V(p)$, for $p \in PV$
- $M, x \nvDash \bot$
- $M, x \vDash \phi \rightarrow \psi \Leftrightarrow M,\ x \nvDash \phi$ or $M,\ x \vDash \psi$
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Let L be a modal logic. A frame F is called an L-frame if $L \subseteq L(F)$.

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$S4DT_0$ logic has countable frame property (c.f.p.).

Cones

Let $F = (W, R_1, ..., R_n)$ be a frame, and let $x \in W$.

$R_i(x) = \{y \mid xR_iy\}$, $R_i^{-1}(x) = \{y \mid yR_ix\}$. Let $U \subseteq W$, then $R_i(U) = \bigcup_{x \in U} R_i(x)$, $R_i^{-1}(U) = \bigcup_{x \in U} R_i^{-1}(x)$.

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For $x \in W$, $W^x \rightleftharpoons \{y \mid xS^*y\}$.
The frame $F^x = (W^x, R_1|_{W^x}, ..., R_n|_{W^x})$ is called cone. If F is an L-frame, then the F^x is called the L-cone.

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Lemma

Let $F = (W, R_1, R_2, ..., R_n)$ be a Kripke frame, then

$$L(F) = \bigcap_{x \in W} L(F^x).$$

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Let $F = (W, R, R_D)$ be an S4D-cone, then:

$$F \vDash AT_0 \Longleftrightarrow \forall x, y \in W(xRy \wedge yRx \Longrightarrow xR_Dx \vee yR_Dy)$$

Spaces with selected points

Let $F = (W, R)$ be an $S4$-frame, then the set of subsets $T = \{U \subseteq W \mid R(U) \subseteq U\}$ defines a topology on the set W. Topological space (W, T) is denoted by $Top(F)$.

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Topological space with a binary relation $(\mathbb{X}, R)$. Consider $\Box$ is interpreted in the same way as in topological semantics, and $[\neq]$ as in Kripke semantics. If the reflexive closure of relation R is the universal relation (i.e. $R \cup Id_W = W \times W$), then the relation can be characterized by the set of all irreflexive points, which we call *selected points*.

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Now let $F = (W, R, R_D)$ be a $S4D$-cone. We define a *space with selected points* $Top_D(F) \rightleftharpoons (Top(F), A)$, where $A = \{v \mid \neg v R_D v\}$.

Spaces with selected points

Lemma

Let $F = (W, R, R_D)$ be a S4D-cone and (F, V) a model, then

$$L(F) = L(Top_D(F)).$$

p-morphism

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A map between topological spaces with selected points $\mathcal{X} = (\mathbb{X}, A_{\mathbb{X}})$ and $\mathcal{Y} = (\mathbb{Y}, A_{\mathbb{Y}})$ is called a p-morphism if it is a p-morphism of topological spaces $f : \mathbb{X} \twoheadrightarrow \mathbb{Y}$, and

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Lemma

Let $\mathcal{X} = (\mathbb{X}, A_{\mathbb{X}})$ and $\mathcal{Y} = (\mathbb{Y}, A_{\mathbb{Y}})$ be topological spaces with selected points and $f : \mathbb{X} \twoheadrightarrow \mathbb{Y}$ be a p-morphism. Then

$$L(\mathbb{X}) \subseteq L(\mathbb{Y}).$$

Topological completeness of $S4DT_0$ and sketch proof

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I. The cone is a cluster without R_D-irreflexive points.

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II. The cone is a cluster with a R_D-irreflexive point.
III. General case.

f.m.p.

Definition

A logic L has the *finite model property* if $L = L(\mathcal{C})$, where $\mathcal{C}$ is a class of finite frames.

Definition

Let us consider a frame $F = (W, R_1, R_2)$ and an equivalence relation $\sim$ on W. A frame $F/{\sim} = (W/{\sim}, R_1/{\sim}, R_2/{\sim})$ is said to be a *minimal filtration* of F through $\sim$, if for $U_1, U_2 \in W/{\sim}$ and $i = 1, 2$

$$U_1 R_i/{\sim} U_2 \Leftrightarrow \exists u \in U_1 \, \exists v \in U_2 \, uR_i v$$

f.m.p.

Definition

Let $M = (W, R_1, R_2, V)$ be a Kripke model, Φ a set of bimodal formulas closed under subformulas. For $x \in W$ let $\Phi(x) := \{A \in \Phi | M, x \vDash A\}$. Two worlds $x, y \in W$ are called Φ-*equivalent in* M (notation: $x \equiv_\Phi y$) if $\Phi(x) = \Phi(y)$.

We say that the equivalence $\sim$ agrees with a set Φ if $\sim \subseteq \equiv_\Phi$.

Lemma

If a formula ϕ is satisfiable in model M over a frame F and the equivalence $\sim$ agrees with a set of all subformulas of ϕ, then ϕ is satisfiable in $F/\sim$.

f.m.p.

A partition of the set W is a family of disjoint subsets of W whose union is W. If $\mathbb{A}$ and $\mathbb{B}$ are partitions of a set W and each element of $\mathbb{A}$ is a subset of one element from $\mathbb{B}$, then we say $\mathbb{A}$ is a refinement of $\mathbb{B}$. We denote by $\sim_{\mathbb{A}}$ the equivalence relation whose set of classes coincides with $\mathbb{A} : \mathbb{A} = W/{\sim_{\mathbb{A}}}$. We write $F_{\mathbb{A}}$ and $R_{\mathbb{A}}$ instead of $F/{\sim_{\mathbb{A}}}$ and $R/{\sim_{\mathbb{A}}}$.

Definition

A class of frames $\mathcal{C}$ *admits minimal filtration* if for each frame $F = (W, R, R_D) \in \mathcal{C}$ and for each finite partition $\mathbb{A}$ of W, there is a finite refinement $\mathbb{B}$ of $\mathbb{A}$, such that $F_{\mathbb{B}} \in \mathcal{C}$.

Lemma

If $\mathcal{C}$ admits minimal filtration, then $L(\mathcal{C})$ has the finite model property.

Result and sketch proof

Theorem

$S4DT_0$ has the finite model property.

THANK YOU!