PRETOPOSES AND TOPOLOGICAL REPRESENTATIONS

LUCA REGGIO

I will explain how, under mild assumptions, every pretopos $\mathbf{X}$ with enough points admits a topological representation, that is a faithful functor

$$S\colon \mathbf{X} \to \mathbf{Top}$$

into the category of topological spaces and continuous maps. Pretoposes are *exact* categories (i.e., regular categories in which every equivalence relation is effective) that are *extensive* (i.e., finite sums are disjoint and pullback stable). These two properties can be thought of as the *algebraic* and *spatial* sides, respectively, of a pretopos. An example of pretopos is the category $\mathbf{KH}$ of compact Hausdorff spaces and continuous maps.

The topological representation $S\colon \mathbf{X} \to \mathbf{Top}$ lands in $\mathbf{KH}$ precisely when $\mathbf{X}$ is *filtral*. The latter is a condition on certain posets of subobjects, and it roughly asserts that the I-fold copower of the terminal object behaves like the Stone-Čech compactification of the discrete space I. The (dual of the) notion of filtrality has its origins in universal algebra, in the work of Magari [1].

This leads to the following characterisation of the category of compact Hausdorff spaces: *up to equivalence,* $\mathbf{KH}$ *is the unique non-trivial well-powered pretopos that is well-pointed, admits all coproducts, and is filtral.* In the talk I will explain all the notions involved and I will give an idea of the main constructions. If time allows I will discuss the possibility of adapting this result to the category of compact ordered spaces, or to the point-free setting.

This is joint work with Vincenzo Marra.

References

[1] R. Magari. Varietà a quozienti filtrali. *Ann. Univ. Ferrara Sez. VII (N.S.)*, 14:5–20, 1969.

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