TOPOLOGICAL COMPLETENESS OF MODAL LOGICS FOR SPACES CONSTRUCTED FROM TREES

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Abstract: As relational structures, trees play an important role in modal logic. We extend this role by exhibiting a uniform construction of topologies on trees. The construction has finite and countably infinite variants. The finite variant applied to trees with countable branching yields a new perspective on a collection of known topological completeness results regarding some metrizable spaces. The countably infinite variant applied to trees with uncountable branching leads to new topological completeness results for extremally disconnected, and hence nonmetrizable, Tychonoff spaces. We obtain these results within Zermelo-Fraenkel Set Theory with the Axiom of Choice by utilizing embeddings of spaces arising from trees into either the Čech-Stone compactification of a suitably large discrete space or the Gleason cover of a large enough power of the closed real unit interval. In particular, we show that each of the logics $\mathsf{S4.2}$, $\mathsf{S4.1.2}$, $\mathsf{Grz.2}$, and $\mathsf{Grz.2}_n$ ($n \geq 1$) is complete with respect to some extremally disconnected Tychonoff space.