Free constructions and quotients of d-frames

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(joint work with Aleš Pultr and Achim Jung)

Abstract:
Our main motivation for studying bitopological spaces (also called bispaces) comes from the fact that many known mathematical structures are naturally bitopological; although this often might not be mentioned explicitly.

To name a few, partially ordered spaces, such as real line or Priestley spaces, can be equipped with two topologies of upper and lower opens, respectively. Less obvious examples arise from spectral spaces (or, more generally, from stably compact spaces) with the second topology being the de Groot dual of the original topology. Often some seemingly non-topological structures, such as DCPOs, also can be equipped with two topologies.

The theory of bispaces serves as a neat organising tool since often a bitopological reorganisation of known facts leads to shorter and more explanatory proofs. We study the structures called d-frames which play the role of algebraic duals of bispaces the same way frames are algebraic duals of (mono)topological spaces. The carrier sets of d-frames are two frames L_+ and L_- equipped with two relations $con \subseteq L_+ \times L_-$ and $tot \subseteq L_+ \times L_-$, to express two fundamental situations: when two open sets are disjoint and when they together cover the space, respectively.

Because of their mixed algebraic and relational nature, we cannot reuse any general framework to describe free constructions of d-frames from generators and relations. An extra complexity of d-frame free constructions also comes from the fact that some axioms of d-frames are not expressed in algebraic terms. We overcome this difficulty by an iterative technique which will be the main content of my talk.

It turns out that our construction is fairly versatile and many of the free frame constructions can be easily adapted to the context of d-frames. Lastly, we also show how to form quotients of d-frames.

References:

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