Duality and Bounded Bisimulations: old and new applications

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[Recent new results come from joint work with Luigi Santocanale]

We recall the duality for finitely presented Heyting algebras from [1] and its applications to intuitionistic propositional calculus (Pitts theorem, definability of difference, characterization of projectivity, etc.). We show, as a new application [2], a semantic proof of Ruitemburg Theorem [3].

Ruitemburg Theorem can be formulated as follows. For a given intuitionistic propositional formula A and a propositional variable x occurring in it, define the infinite sequence of formulae $\{A_i\}_{i\geq 1}$ by letting A_1 be A and A_{i+1} be $A(A_i/x)$. Ruitenburg's Theorem says that the sequence $\{A_i\}_{i\geq 1}$ (modulo logical equivalence) is ultimately periodic with period 2, i.e. there is $N \geq 0$ such that $A_{N+2} \leftrightarrow A_N$ is provable in intuitionistic propositional calculus.

References

[1] S. Ghilardi, L. Santocanale, "Ruitenburg's Theorem via Duality and Bounded Bisimulations", arXiv:1804.06130v1, 2018.

[2] S. Ghilardi, M. Zawadowski, "Sheaves, Games, and Model Completions", Kluwer 2002.

[3] W. Ruitenburg, "On the period of sequences $a^{n(p)}$ in intuitionistic propositional calculus", The Journal of Symbolic Logic 49 (1984), pp. 892–899.