BANDS, SKEW LATTICES, AND SHEAVES

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In [1], we proved that the category of left-handed strongly distributive skew lattices with zero and proper homomorphisms is dually equivalent to a category of sheaves over local Priestley spaces. This provides a non-commutative version of classical Priestley duality and generalizes Stone duality for skew Boolean algebras [2,5]. From the point of view of skew lattices, Leech showed in [6] that any strongly distributive skew lattice can be embedded in the skew lattice of partial functions on some set with the operations being given by restriction and so-called override. Our duality shows that there is a canonical choice for this embedding. Conversely, from the point of view of sheaves over Boolean spaces, our results show that skew lattices correspond to Priestley orders on these spaces and that skew lattice structures are naturally appropriate in any setting involving sheaves over Priestley spaces.

While strongly distributive skew lattices are quite restrictive, bands, that is, idempotent semigroups, are ubiquitous and play a role in many parts of mathematics. In on-going joint work with Clemens Berger, we reconsider the duality of [1] from a wider point of view. We show that the comprehensive factorisation for posetal categories of [3] lifts to regular bands, giving rise to the representation of so-called normal bands as presheaves over semilattices, essentially an early result of Kimura [4]. The non-commutative Stone duality for left-handed strongly distributive skew lattices of [1] may then be seen as a special case of this wider representation theory and one can explore intermediate classes.

References

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