Modal logics of polytopes – what we know so far

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We will present results of an ongoing project investigating modal logics that arise from interpreting modal formulas as piecewise linear subsets (polytopes) of an Euclidean space $\mathbb{R}^n$. The modal diamond can be interpreted as the topological closure, or as the limit set operator. Thus we get logics $\boldsymbol{PC_n}$ and $\boldsymbol{PD_n}$ for each $n \geq 1$. We will talk about our knowledge so far of these logics.

Quite a lot is known for $n \leq 2$, however some observations can be made in general as well. For example, it turns out that spatial dimension is expressible modally, since the boundary $\mathbb{C}P \setminus P$ of a polytope is always a polytope of a strictly less dimension. It follows that all of these logics have finite height and by Segerberg's theorem, have the finite model property.

The finite axiomatization of these logics boils down semantically to identifying the finite set of `forbidden' Kripke frames, to which non-frames of the logic in question are reducible. Such sets are known in cases when $n \leq 2$. For $n = 3$ interesting connections to graph theory and combinatorial geometry show up in attempting to identify the forbidden configurations. We will discuss these connections in our talk.

This is a joint work with members of Esakia seminar.