THE TOPOLOGICAL SPACE OF PRE-ORDERS ON ABELIAN GROUPS

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The spectrum of an Abelian lattice-ordered group (ℓ-group, for short) endowed with the hull-kernel topology has received wide attention in the literature, and has proved to be an essential tool in the study of Abelian ℓ-groups. In 2004, Sikora's paper *"Topology on the spaces of orderings of groups"* pioneered a different perspective on the study of the interplay between topology and ordered groups, that has led to applications to both orderable groups and algebraic topology. The basic construction in Sikora's paper is the definition of a topology on the set of total orders on a given orderable group. The topology is then proved compact, Hausdorff, and zero-dimensional.

I will present ongoing joint work with Vincenzo Marra, where we compare and connect the spectral topology to the topology on orderings in the Abelian case, aiming to combine them in the study of ordered groups.

For G a partially ordered Abelian group, $F(G)$ is the Abelian ℓ-group free over G. We start by proving that for an unperforated (equivalently, isolated) partially ordered Abelian group G, the set $\mathscr{P}(G)$ of pre-orders on G and the set $\operatorname{Spec} F(G)$ of prime ideals of $F(G)$ ordered by inclusion are order-isomorphic. If we endow $\mathscr{P}(G)$ with a topology extending Sikora's construction, the order-isomorphism that we exhibit restricts to a homeomorphism between the subspace $\mathscr{O}(G)$ of orders on G and the subspace $\operatorname{Min} F(G)$ of minimal prime ideals of $F(G)$.

If time allows, we will discuss some applications, and possibilities for further research.

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