Reflection calculus and conservativity spectra

Lev Beklemishev

Strictly positive logics recently attracted attention both in the description logic and in the provability logic communities for their combination of efficiency and sufficient expressivity. The language of Reflection calculus **RC** consists of implications between formulas built up from propositional variables and constant `true' using only conjunction and diamond modalities which are interpreted in Peano arithmetic as restricted uniform reflection principles. We extend the language of **RC** by another series of modalities representing the operators associating with a given arithmetical theory T, its fragment axiomatized by all theorems of T of arithmetical complexity Π_n^0, for all $n > 0$. We note that such operators, in a strong sense, cannot be represented in the full language of modal logic.

Conservativity spectrum of an arithmetical theory is a sequence of ordinals characterizing the strength of its theorems of arithmetical complexity Π_1^0, Π_2^0, etc. We show that such sequences form a natural algebraic-type structure, a lower semilattice endowed with (infinitely many) monotone operators, and characterize this structure in several different ways. In particular, this structure is isomorphic to the Lindenbaum algebra of the variable-free fragment of the Reflection calculus with conservativity modalities.

____

Research reported in this work is supported by the Russian Science Foundation under project 16-11-10252