Bisimulation games and formula depth

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Introduction

- Ehrenfeucht-Fraïssé games are well known in classical model theory, they are used to study elementary equivalence.
- Bisimulation games are their analogues for Kripke models and modal and intermediate logic.

Bisimulation games have been used

- for constructing bisimulations
- for study of expressive power of logical languages
- for completeness proofs in modal logic
- for proofs of local finiteness in modal and intermediate logics (and more exactly, for classifying formulas).

Modal propositional language

N-modal formulas are built from a countable set of proposition letters $PL=\{p_1,p_2,...\}$ using boolean connectives and unary modal connectives $\Box_1 ,..., \Box_N$;as usual $\Diamond_i = \neg \Box_i \neg$

If N=1 we denote the modalities just by $\Box$ and $\Diamond$.

The modal depth md(A) is defined by induction:

$md(p_i)=0$, $md(\neg A)=md(A)$,

$md(A \vee B)= md(A \wedge B) = max(md(A),md(B))$,

$md(\Box_i A)=md(A)+1$

Intuitionistic propositional language

Intutionistic formulas are built from $PL=\{p_1,p_2,...\}$ and the

connectives $\wedge, \vee, \rightarrow, \bot$.

$\neg A := A \rightarrow \bot$

The implication depth di(A) is defined by induction:

$di(p_i) = di(\bot) = 0$,

$di(A \vee B) = di(A \wedge B) = max(di(A), di(B))$,

$di(A \rightarrow B) = max(di(A), di(B)) + 1$.

Logics-1

An *N-modal logic* is a set of N-modal formulas L such that:

- L contains all boolean tautologies
- L is closed under Modus Ponens: if A, $A \rightarrow B \in L$, then $B \in L$.
- L is closed under Substitution:

  if $A(p_1,\ldots,p_n) \in L$, then $A(B_1,\ldots,B_n)$ (for any formulas $B_1,\ldots,B_n$)

- if $A \in L$, then $\Box_i A \in L$
- $\Box_i(A \rightarrow B) \rightarrow (\Box_i A \rightarrow \Box_i B) \in L$

The *minimal logic* $\mathbf{K}_N$ is the smallest such set; **K** denotes $\mathbf{K}_1$.

Logics-2

An *intermediate logic* is a set of intuitionistic formulas L such that:

- L contains all intuitionstic axioms
- L is closed under Modus Ponens: if A, $A \to B \in L$, then $B \in L$.
- L is closed under Substitution:

if $A(p_1,\ldots,p_n) \in L$, then $A(B_1,\ldots,B_n)$ (for any formulas $B_1,\ldots,B_n$)

- L is consistent

The smallest intermediate logic is intuitionistic (**H**), the largest is classical (**CL**).

Formula depth-1

$L\lceil k$ denotes the restriction of a logic L to formulas in variables $p_1,...,p_k$. The sets $L\lceil k$ are called *weak logics*

The *modal depth of a formula* A *in a* (maybe weak) *modal logic* L

$md_L(A) := \min\{md(B) \mid L \vdash A \leftrightarrow B\}$

The *implication depth of a formula* A *in an intermediate logic* L

$di_L(A) := \min\{di(B) \mid L \vdash A \leftrightarrow B\}$

The *modal / implication depth of a logic L*

$md(L) := \max\{md_L(A) \mid$ A is in the language of L$\}$

$di(L) := \max\{di_L(A) \mid$ A is an intuitionistic formula$\}$

Formula depth-2

Trivial examples:

di(**H**) = ∞, md(**K**)= ∞

di(**CL**) = 1

md(**K**+$\Box\bot$) = md(**K**+ p$\leftrightarrow\Box$p)= 0.

A nontrivial (well-known) example:

md(**S5**) = 1

Kripke frames and models-1

An N-modal Kripke frame is a nonempty set with N binary relations $F = (W, R_1, \ldots, R_N)$.

An *intuitionistic Kripke frame* is a poset $F = (W, \leq)$.

A valuation in F is a function $\theta: PL \to 2^W$ (so $\theta(p_i) \subseteq W$).

(F, θ) is a *Kripke model* over F.

In *intuitionistic Kripke models* $\theta(p_i)$ should be $\leq$-stable:

$x \in \theta(p_i)$ & $x \leq y \Rightarrow y \in \theta(p_i)$

In *k-weak Kripke models* only $p_1, \ldots, p_k$ are evaluated.

Kripke frames and models-2

The inductive truth definition for the modal case ($M,x \models A$)

- $M,x \models p_i$ iff $x \in \theta(p_i)$
- $M,x \models \Box_i A$ iff $\forall y(xR_i y \Rightarrow M,y \models A)$
- $M,x \models \Diamond_i A$ iff $\exists y(xR_i y \;\&\; M,y \models A)$

A formula A is valid in a frame F (in symbols, $F \models A$) if A is true at all points in every Kripke model over F.

Kripke frames and models-3

The inductive truth definition for the intuitionistic

case $(M,x \Vdash A)$

- $M,x \Vdash p_i$ iff $x \in \theta(p_i)$
- $M,x \Vdash A \vee B$ iff $(M,x \Vdash A$ or $M,x \Vdash B)$
- $M,x \Vdash A \wedge B$ iff $(M,x \Vdash A$ and $M,x \Vdash B)$
- $M,x \Vdash A \rightarrow B$ iff $\forall y \geq x\ (M,y \Vdash A \Rightarrow M,y \Vdash B)$

Then

- $M,x \Vdash \neg A$ iff $\forall y \geq x\ M,y \nVdash A$

A formula A is valid in a frame F (in symbols, $F \Vdash A$) if A is true at all points in every intuitionistic Kripke model over F.

Kripke frames and models-4

Canonical model theorem

For any modal or intermediate logic L (weak or not) there exists the *canonical model* M_L such that

- for any A in the language of L

$$M_L \vDash (\Vdash)\ A \text{ iff } L \vdash A$$

- M_L is *distinguishable* :

two points x,y satisfy the same formulas iff x=y.

Tabularity and FMP

Kripke complete logics

L(F) := { A | F ⊨ A} (the *logic of a frame* F).

L($\mathcal{C}$) := $\bigcap$ {**L**(F)|F ∈ $\mathcal{C}$} (the *logic of a class of frames* $\mathcal{C}$).

- If F is finite, **L**(F) is called *tabular* (or *finite*)
- If $\mathcal{C}$ consists of finite frames, **L**($\mathcal{C}$) has the finite model property (FMP). Or:

L has the FMP iff L is an intersection of tabular logics.

Proposition ('Harrop's theorem') If L is finitely axiomatizable and has the FMP, then L is decidable.

Bisimulation games-1

Origin: Colin Stirling (1995) <<
n-bisimulations by Johan Van Benthem (1989) <<
n-equivalence by Kit Fine (1974)

Def For a k-weak Kripke model $M=(W,R_1,\ldots,R_N,\theta)$ consider the *0-equivalence* relation between points

$$x \equiv_0 y := \forall j \leq k \, (M,x \vDash p_j \Leftrightarrow M,y \vDash p_j)$$

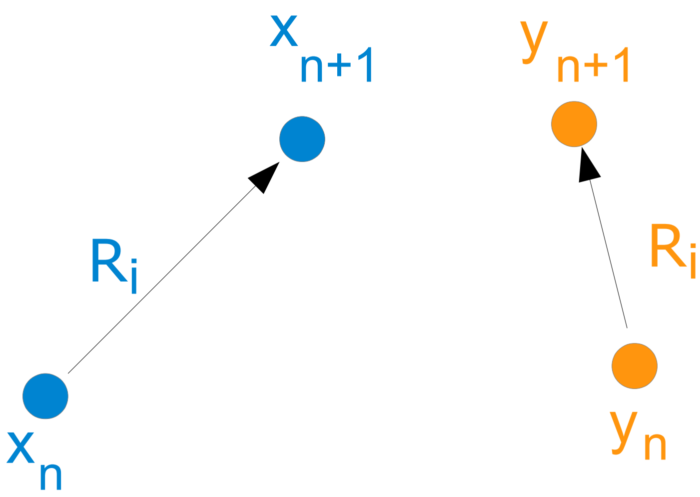
Given M and two points $x_0 \equiv_0 y_0$ we can play the *r-round bisimulation game* $BG_r(M,x_0,y_0)$.

Players: Spoiler (Abelard) vs Duplicator (Éloïse).

Remark More generally, bisimulation games can be defined for two Kripke models M,M' and points $x_0 \in M$, $y_0 \in M'$. We do not need this in our talk.

Bisimulation games-2

The initial position in $BG_r(M,x_0,y_0)$ is (x_0,y_0).



<u>Round (n+1)</u>

- Spoiler chooses i, x_{n+1} [or y_{n+1}] such that $x_n R_i x_{n+1}$ [$y_n R_i y_{n+1}$]
- Duplicator chooses y_{n+1} [x_{n+1}] such that $y_n R_i y_{n+1}$ [$x_n R_i x_{n+1}$] and $x_{n+1} \equiv_0 y_{n+1}$

- A player loses if he/she cannot move.
- Duplicator wins after r rounds.

Bisimulation games-3

Def Formula and game *n-equivalence* relations (on M)

- $x \equiv_n y$:= for any $A(p_1,\ldots,p_k)$ of modal depth $\leq n$

  $M,x \vDash A \Leftrightarrow M,y \vDash A$
- $x \sim_n y$:= Duplicator has a winning strategy in $BG_n(M,x,y)$

Main Theorem on finite bisimulation games (Stirling, 1995)

$$\equiv_n \;=\; \sim_n$$

- The same theorem holds for the intuitionistic case.

Local tabularity-1

Def A logic L is *locally tabular* *(or* *locally finite)*

if for any k there are finitely many formulas in $p_1,\ldots,p_k$ up to equivalence in L.

Equivalent definitions:

- L is locally tabular if all its weak fragments $L\lceil k$ are tabular.
- The variety of L-algebras is *locally finite* : every finitely generated L-algebra is finite
- For every finite k, the free k-generated L-algebra (the *Lindenbaum algebra* of $L\lceil k$) is finite
- Every weak canonical model $M_{L\lceil k}$ is finite.

Local tabularity-2

Finite modal (implication) depth ⇒

local tabularity ⇒ fmp

- The first implication is easy: there are finitely many k-formulas of bounded depth up to equivalence in the basic modal or intuitionistic logic.
- The second one is well-known: a locally tabular logic is complete w.r.t. its weak canonical frames

The second implication is not revertible: plenty of examples (**K, S4, H** etc.)

PROBLEM. *Does every locally tabular modal or intermediate logic have a finite formula depth?*

The problem seems difficult. Conjecture: no.

Formula depth and games-1

In every Kripke model there is a decreasing sequence

$\equiv_0 \supseteq \equiv_1 \ldots$ Put $\equiv_\infty := \bigcap_n \equiv_n$

Lemma 1 In a weak Kripke model every relation $\equiv_n$ induces a finite partition ($W/\equiv_n$ is finite).

Lemma 2 $x \equiv_\infty y$ iff for any $A(p_1,\ldots,p_k)$ $(M,x \vDash A \Leftrightarrow M,y \vDash A)$

Lemma 3 (distingushability) In canonical models:

$x \equiv_\infty y$ iff $x=y$.

Stabilization lemma (modal case)

If $\equiv_n = \equiv_{n+1}$ in every $M_{L\lceil k}$ (bisimulation games *stabilize at round n*), then $md(L) \leq n$.

Stabilization lemma (intuitionistic case) If $\equiv_n = \equiv_{n+1}$ in every $M_{L\lceil k}$, then $di(L) \leq n+1$.

Formula depth and games-2

Proof of modal Stabilization lemma

For every x in $M_{L\lceil k}$, put

$$B_x := \bigwedge \{C \mid x \vDash C,\ md(C) \leq n\}$$

Then B_x defines x. So for any k-formula A

$$M_{L\lceil k} \vDash A \leftrightarrow \bigvee \{B_x \mid x \vDash A\},$$

and the disjunction is actually finite.

By Canonical model theorem

$$L \vdash A \leftrightarrow \bigvee \{B_x \mid x \vDash A\}.\ \text{QED}$$

Formula depth and games-3

<u>Stabilization lemma (intuitionistic case)</u> If $\equiv_n = \equiv_{n+1}$ in every $M_{L\lceil k}$, then $di(L\lceil k) \leq n+1$.

Proof. Similar to the modal case, but now we need

$$B_x := \bigwedge \{D \mid x \Vdash D,\ di(D) \leq n\},$$

$$C_x := \bigvee \{D \mid x \nVdash D,\ di(D) \leq n\}.$$

Then $y \nVdash B_x \to C_x$ iff $y \leq x$. So for any k-formula A

$$M_{L\lceil k} \vDash A \leftrightarrow \bigwedge \{B_x \to C_x \mid x \nVdash A\}.$$

Hence $L \vdash A \leftrightarrow \bigwedge \{B_x \to C_x \mid x \nVdash A\}$. QED

Formula depth and games-4

Normal forms in intuitionistic logic

The previous proof allows us to present every intuitionistic formula in the normal form, as a conjunction of `characteristic formulas' (cf. [Ghilardi, 1992]). This is an analogue to Hintikka theorem for classical FOL.

Depth 1 Characteristic k-formulas are $B_J \to C_J$, where

$$B_J := \bigwedge \{p_i \mid i \in J\},\ C_J := \bigvee \{p_i \mid i \notin J\},$$

for $J \subseteq \{1,...,k\}$.

Depth n+1 Characteristic k-formulas are $B_J \to C_J$, where

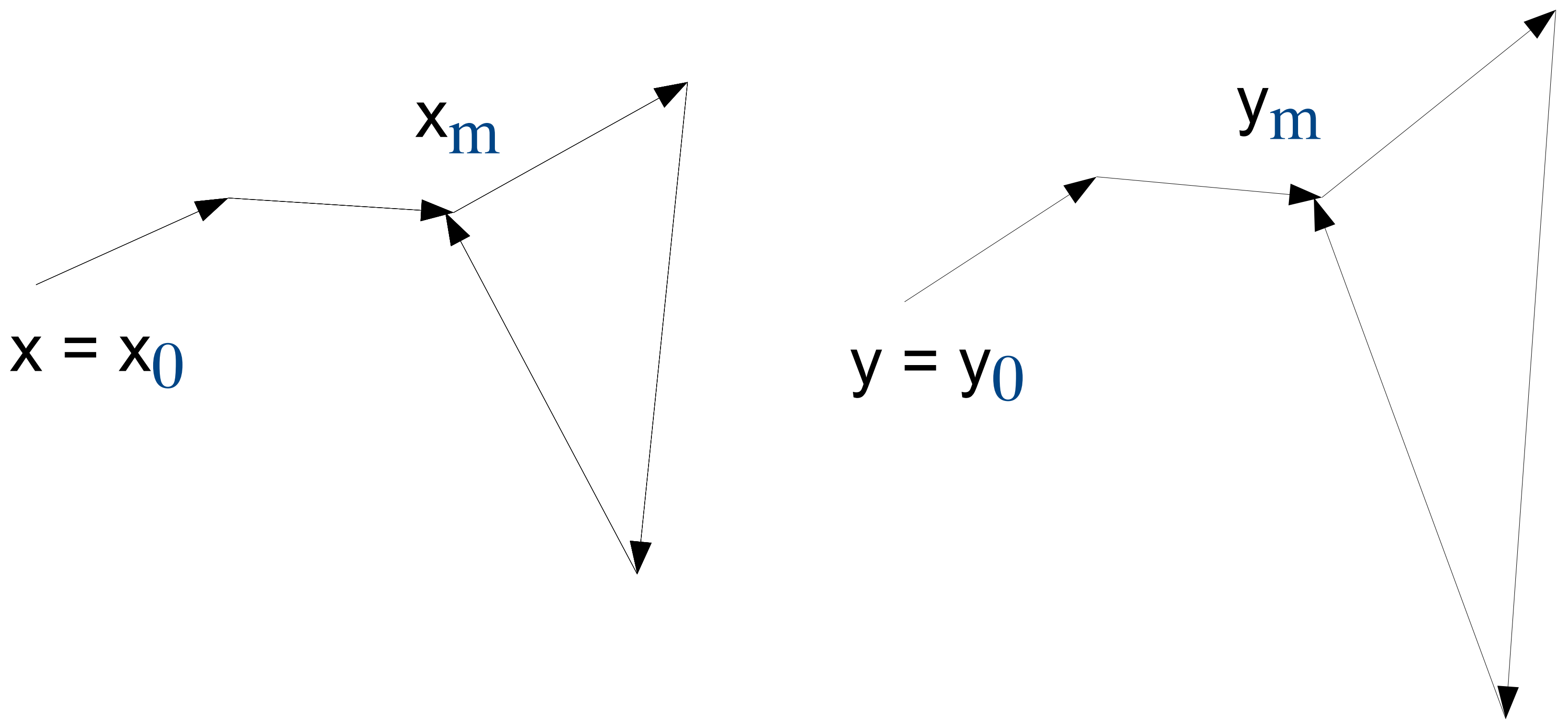
$$B_J := \bigwedge \{D_i \mid i \in J\},\ C_J := \bigvee \{D_i \mid i \notin J\},$$

wher $D_1,...,D_m$ are all characteristic formulas of depth n,

$J \subseteq \{1,...,m\}$.

Formula depth and games-5

Lemma on repeating positions Suppose in a Kripke model M $x \equiv_n y$ and the Duplicator has a winning strategy s in $BG_n(M,x,y)$ such that every play controlled by s has at least two repeating positions. Then $x \equiv_{n+1} y$.



Formula depth and games-6

tabularity $\Rightarrow$ finite formula depth

Theorem If F is finite, then $md(L(F)) \leq |F|^2+1$.

Proof: The Pigeonhole principle gives repeating positions.

Remark In many cases we have a better (linear) upper bound.

Examples of finite depth-1

$$md(\mathbf{K} + \Box^n\bot) = n\text{-}1$$

and more generally,

$$md(\mathbf{K}_N + \Box^n\bot) = n\text{-}1$$

where

$$\Box A := \Box_1 A \wedge \ldots \wedge \Box_N A.$$

The axiom $\Box^n\bot$ forbids paths of length n in Kripke frames:

$x_1Rx_2\ldots Rx_n$, where $R = R_1 \cup\ldots\cup R_N$

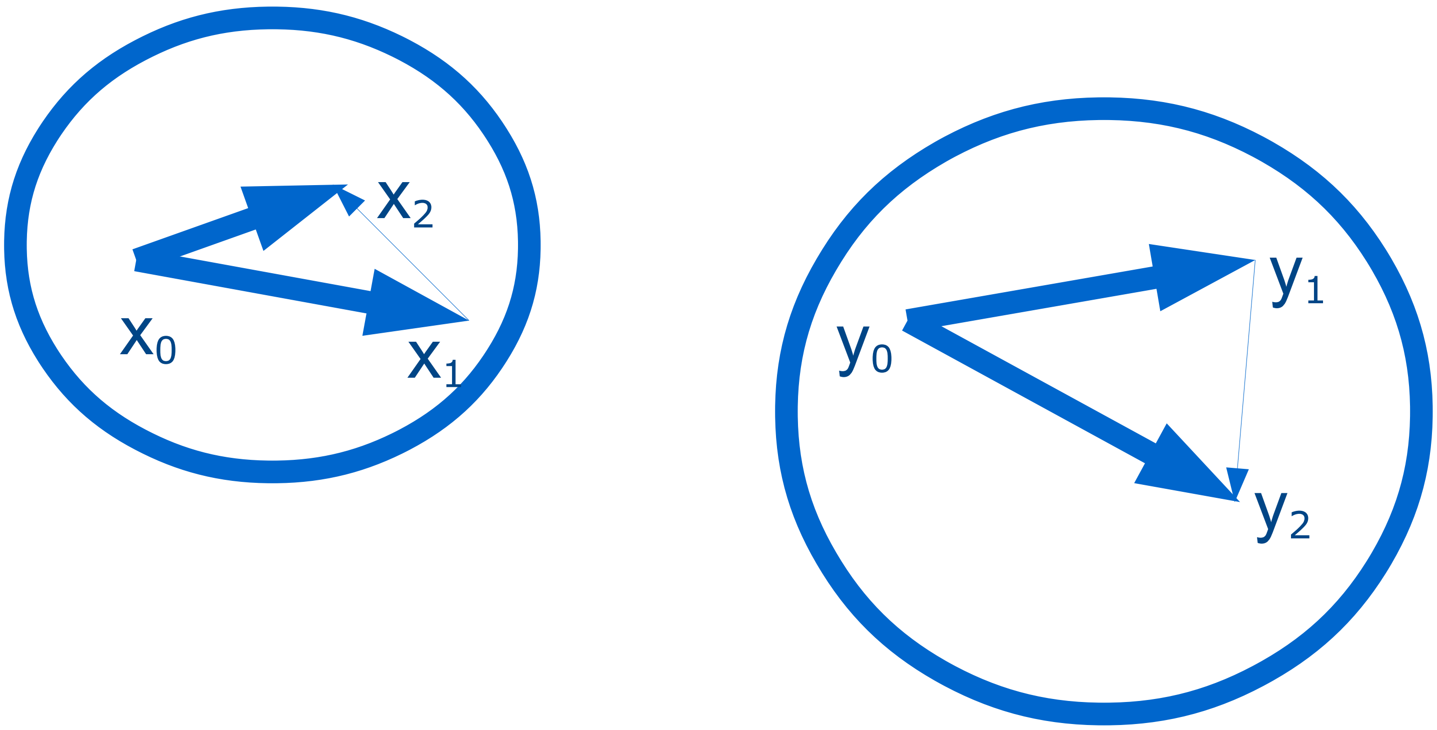
Proof. For the upper bound: every play of a bisimulation game contains at most (n-1) rounds. For the lower bound: $md_L(\Box^{n-1}\bot) = n\text{-}1$.

An earlier result: $\mathbf{K}_N + \Box^n\bot$ is locally tabular (Gabbay & Sh, 1998; a routine proof by induction).

Examples of finite depth-2

md(**S5**) = 1 (a well-known fact)

Proof. If Duplicator can win the 1-game, she can win the 2-game



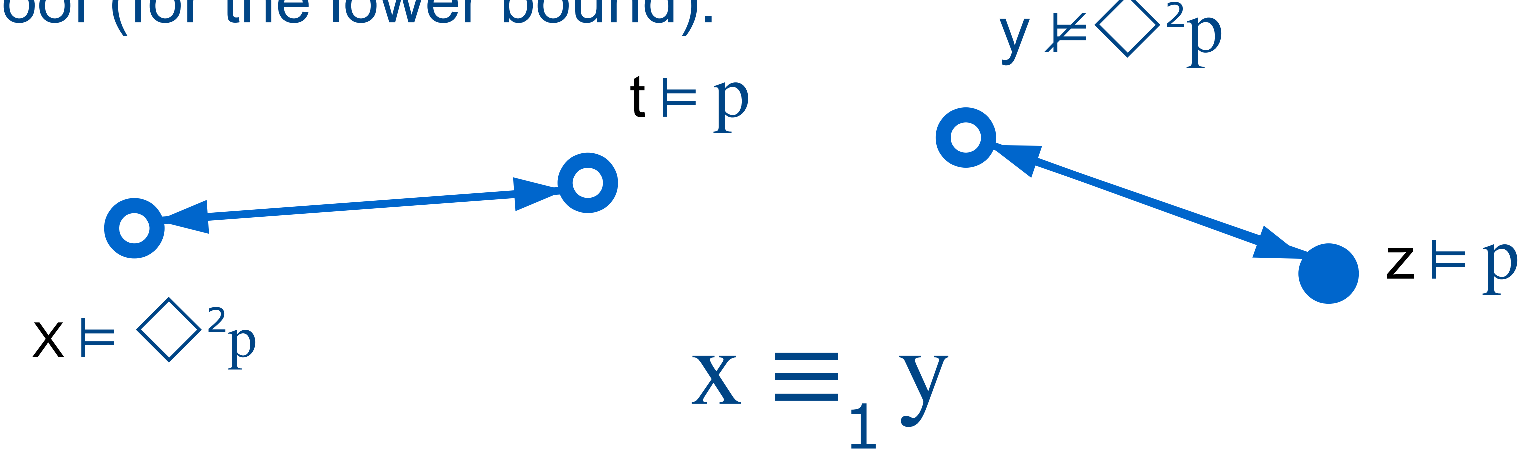
Examples of finite depth-3

md(**DL**) = 2

DL is the *difference logic*

$$\mathbf{DL} = \mathbf{K} + \Diamond\Box p \to p + \Diamond\Diamond p \to p \vee \Diamond p$$

- **DL** is complete w.r.t inequality frames $(W, \neq_W)$.
- Arbitary **DL-**frames are obtained from **S5-**frames (equivalence frames) by making some points irreflexive.
- Proof (for the lower bound):



Examples of finite depth-4

For the upper bound we have to examine games in canonical models

<u>Lemma</u> In $M_{DL\lceil k}$ $x \equiv_0 y$ & xRy implies $x \equiv_1 y$.

Proof. Duplicator's responses for the moves of Spoiler are:

S: (x,z) (with $z \neq x,y$) D: (y,z)

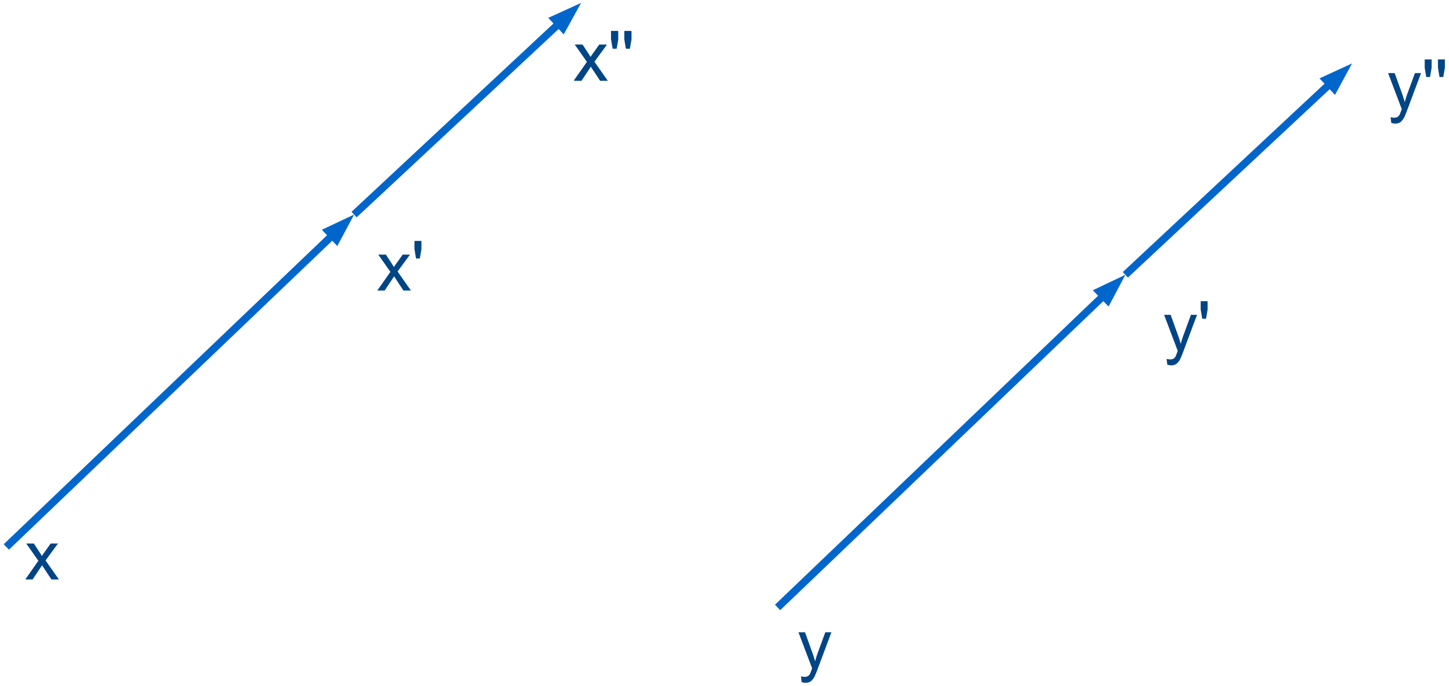
S: (x,x) D: (y,x)

S: (x,y) D: (y,x)

They lead to 0-equivalent points. QED

Examples of finite depth-5

Now in the general case suppose $x \equiv_2 y$ in $M_{DL\lceil k}$. We have to show that $x \equiv_3 y$. Let us start playing a 2-round game, so we have $x' \equiv_1 y'$, and we have to show $x' \equiv_2 y'$.

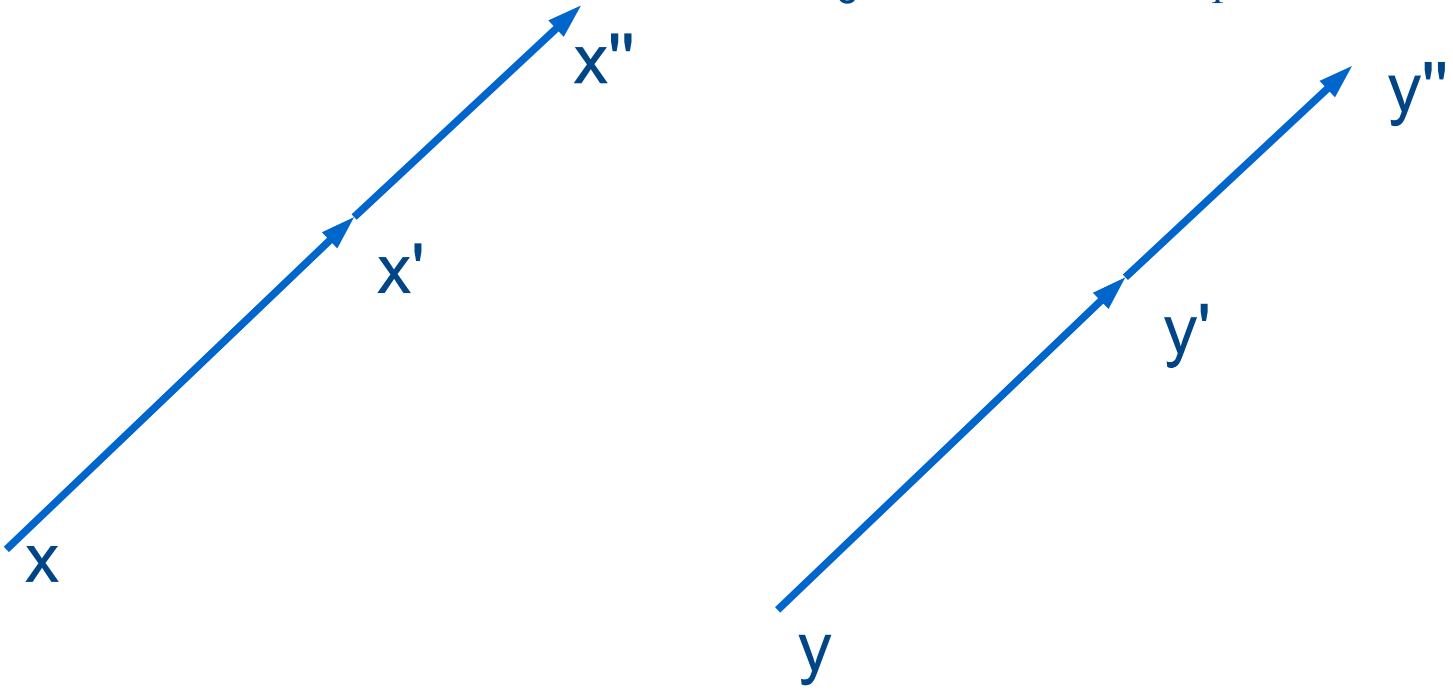


Examples of finite depth-6

Consider the next Spoiler's move (x',x'').

(a) x'' = x. The Duplicator responds with y''=y.

(b) x'' $\neq$ x, x'' $\not\equiv_0$ x'. Then xRx'', and (x,x'') can be regarded as the first move in the 2-round game. For the response (y,y'') we have y'Ry'' (since y' $\neq$ y'', otherwise x'' $\equiv_0$ x') and x'' $\equiv_1$ y''.



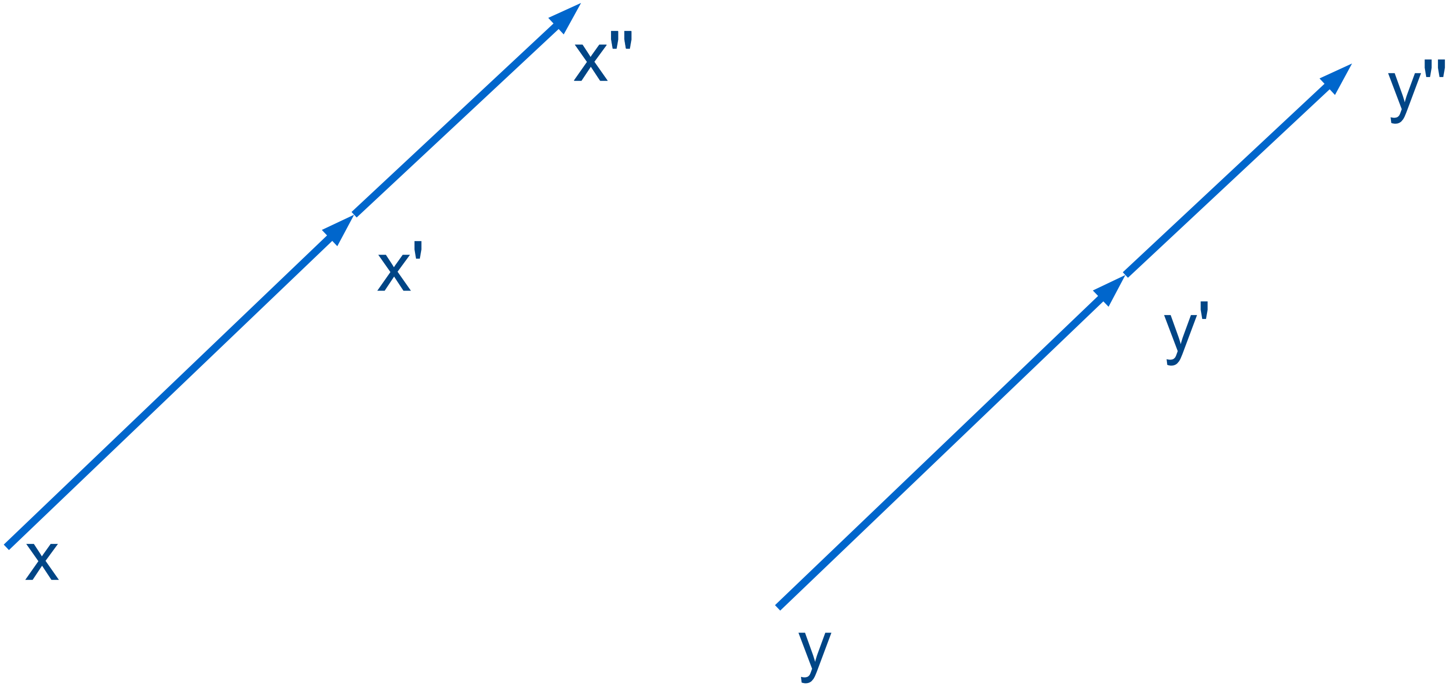
Examples of finite depth-7

(c) $x'' \neq x$, $x'' \equiv_0 x'$. There is a response (y', y''), with $x'' \equiv_0 y''$.

So $y'' \equiv_0 y'$ by the transitivity of $\equiv_0$.

Now by Lemma $x'' \equiv_1 x'$ and $y'' \equiv_1 y'$; thus

$x'' \equiv_1 y''$ by the transitivity of $\equiv_1$. QED.



Examples of finite depth-8

$$di(\mathbf{H}+ibd_n) \leq 2n\text{-}1$$

In posets ibd_n forbids *chains of length n+1* : $x_1<x_2...<x_{n+1}$.

$$ibd_1 = p_1 \vee \neg p_1,$$

$$ibd_{n+1} = p_{n+1} \vee (p_{n+1} \rightarrow ibd_n).$$

Def Intermediate logics of finite transitive depth: extensions of $\mathbf{H}+ibd_n$ are of depth $\leq$ n-1 (or of height $\leq$ n).

Theorem (Kuznetsov – Komori) These logics are locally tabular.

Proof of the upper bound: by induction we show that $x \equiv_k y$ implies $x \equiv_{k+1} y$ whenever depth(x)+depth(y) $\leq$ k.

So the bisimulation game stabilizes at 2n-2.

Examples of finite depth-9

$md(\mathbf{Grz}+bd_n) \le 2n\text{-}2,$

$md(\mathbf{Grz3}+bd_n) = n\text{-}1$

Grz is the logic of finite partial orders,

Grz3 is the logic of finite chains.

In transitive Kripke frames bd_n forbids *chains of clusters of length n+1* : $x_1Rx_2 \ldots Rx_{n+1}$, where $\neg x_iRx_{i+1}$ for each i.

$$bd_n = \neg\Diamond(Q_1 \wedge \Diamond(Q_2 \wedge \ldots \wedge \Diamond Q_{n+1})),$$

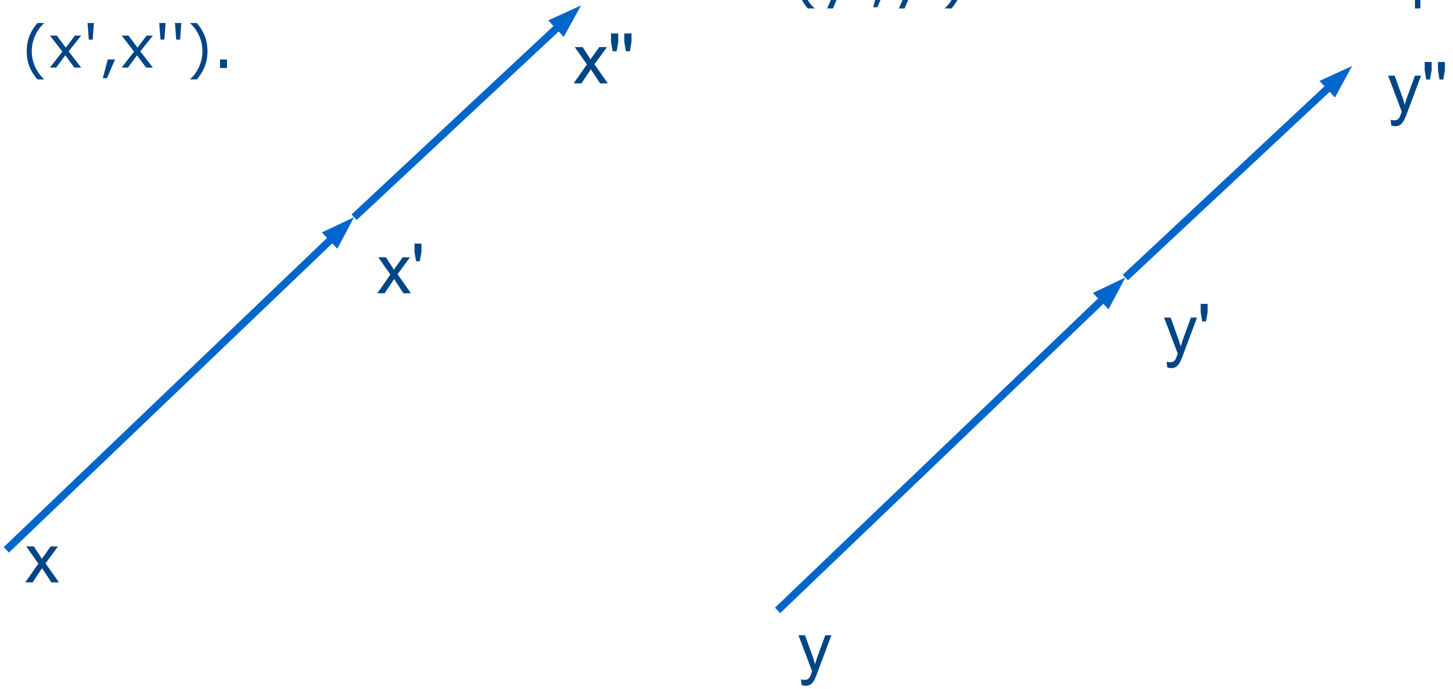
$$Q_i = p_i \wedge \bigwedge\{\neg\Diamond p_j \mid 1 \le j < i\}.$$

Grz3 + bd_n = **L**(n-element chain)

Examples of finite depth-10

di(**LC**) = 2, where **LC =** H+ $(p \to q) \vee (q \to p)$ is the intermediate logic of arbitrary chains.

Proof. $x \equiv_1 y$ implies $x \equiv_2 y$, since $x' \equiv_0 y'$ implies $x' \equiv_1 y'$: we can ignore the first move. If the 1-round game response for (x,x'') is (y,y'') with y''<y, then $x'' \equiv_0 y''$, and $y'' \equiv_0 y'$ as the model in intuitionistic. So (y',y') can be the response for (x',x'').



Examples of finite depth-11

md(**Grz**+bd_2) =2

(0,1 show the truth values of p)

Here $x \equiv_1 y$, but $x \not\equiv_2 y$: Duplicator wins after 1 round. Spoiler wins after 2 rounds.

A distinguishing formula is $\Box\Diamond p$. So it has depth 2 in **Grz**+bd_2

But note that md(**Grz3**+bd_2) = 1 and

Grz3+bd_2 $\vdash \Box\Diamond p \leftrightarrow (\Box p \lor (\neg p \land \Diamond p))$.

Examples of finite depth-12

di(**LC**+ibd$_2$) = di(**LC**)= 2, while di(**H**+ibd$_2$) =3:

As in the modal case, $x \equiv_1 y$, but $x \not\equiv_2 y$:

$x \not\Vdash \neg\neg p$, $y \Vdash \neg\neg p$

Note that di($\neg\neg p \to p$)=3 in **H**+ibd$_2$

But di($\neg\neg p \to p$)=1 in **LC**+ibd$_2$: it is equivalent to ($p \vee \neg p$).

Examples of finite depth-13

$$md(\mathbf{K4}+bd_n) \leq 4n - 3$$

Theorem (Segerberg 1971;Maksimova 1975) For L ⊇ **K4**

L is locally tabular iff L is of finite transitive depth.

Def L is of *finite transitive depth* if $L \vdash bd_n$ for some n.

Corollary For extensions of **K4** local tabularity is equivalent to finite modal depth.

PROBLEM (Chagrov) Find a description of local tabularity for extensions of **K**.

Examples of finite depth-14

If $md(L) = m$, then $md([\mathbf{K}+\Box^n\bot, L]) \le (m+1)n-1$

Def. The commutative join (commutator)

$[L_1, L_2] := L_1 * L_2$ (the fusion) +

$\blacksquare_j \Box_i p \leftrightarrow \Box_i \blacksquare_j p$ (commutation axioms)

$\blacklozenge_j \Box_i p \rightarrow \blacksquare_j \Diamond_i p$ (Church-Rosser axioms)

Tabularity criterion-1

Theorem (Chagrov 1994)

L is tabular iff L $\vdash \alpha_n \wedge \mathrm{Alt}_n$ for some n.

The formulas α_n , Alt_n correspond to universal conditions on frames:

- α_n forbids simple paths of length n:

  $x_1 R x_2 \ldots R x_n$, where all the x_i are different.

- Alt_n forbids n-branching: $xRx_1, \ldots, xRx_n$, , where all the x_i are different.

Tabularity criterion-2

$$\alpha_n = \neg\Diamond(P_1 \wedge \Diamond(P_2 \wedge \ldots \Diamond(P_{n-1} \wedge \Diamond P_n)\ldots)),$$

$$Alt_n = \neg(\Diamond P_1 \wedge \Diamond P_2 \wedge \ldots \wedge \Diamond P_n),$$

where

$$P_i = \neg p_i \wedge \bigwedge\{p_j \mid 1 \leq j \leq n,\ j \neq i\}.$$

Theorems on local tabularity-1

1. Every logic $\mathbf{K}_N + \alpha_n$ (Chagrov's formula) is locally tabular.

(This theorem was conjectured in 1994 by Chagrov.)

The proof does not give the FMD. To reach a repeating position, Duplicator should keep track of all possible returns.

So she plays her own stronger game:

at the position (x,y) at every stage not only

$x \equiv_0 y$, but for any $m<n$, $i \leq N$

there is a return m steps back from x along R_i iff

there is a return m steps back from y along R_i .

This is actually a bisimulation game in another model.

As it stabilizes at n, we obtain the local tabularity.

Theorems on local tabularity-2

2. The logics [$\mathbf{K}_N + \alpha_n$,$\mathbf{K}_{N'} + \Box^n\bot$], [$\mathbf{K}_N + \alpha_n$,**S5**] are locally tabular.

Remark. In general products and commutative joins do not preserve local tabularity, a counterexample is [**S5**,**S5**] = $\mathbf{S5}^2$ (Tarski).
Theorem [N.Bezhanishvili, 2002] $\mathbf{S5}^2$ is pre-locally tabular.
Probably, there exists a game-theoretic proof.

THANK YOU!

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Logics

- **K** = **L**(all frames)
- **K4** := **K** + $\Diamond\Diamond p \to \Diamond p$ = **L**(all transitive frames)
- **S4** := **K4** + $p \to \Diamond p$ = **L**(all transitive reflexive frames)

  = **L**(all partial orders)

- **Grz** := **S4** + $\neg(p \wedge \Box(p \to \Diamond(\neg p \wedge \Diamond p)))$

  = **L**(all finite partial orders)

- **Grz3** := **Grz** + $\Diamond p \wedge \Diamond q \to \Diamond(p \wedge \Diamond q) \vee \Diamond(q \wedge \Diamond p)$

  = **L**(all finite chains)

- **S5** := **S4** + $\Diamond\Box p \to p$ = **L**(all equivalence frames)

  = **L**(all universal frames [clusters])

All these logics have the FMP, so they are decidable.