

## TOWARD A GENERAL STONE-GELFAND DUALITY

LUCA REGGIO

I will report on joint ongoing work with Vincenzo Marra on an abstract algebraic formulation of dualities *à la* Stone-Gelfand. This project arose in connection with the study of the algebraic dual of the category  $\mathbf{KHaus}$  of compact Hausdorff spaces and continuous maps.

In 1969 Duskin showed that  $\mathbf{KHaus}^{\text{op}}$  is equivalent to some variety  $\mathbf{V}$  of possibly infinitary algebras. It is not difficult to see that  $\mathbf{V}$  cannot be finitary, and in previous work we have provided a finite equational axiomatisation of such an infinitary variety. A careful analysis of the latter variety leads to the study of a class of (possibly infinitary) varieties satisfying appropriate universal algebraic conditions. If the variety  $\mathbf{W}$  belongs to this class, we will explain how to define — in a completely natural manner — an adjunction between  $\mathbf{W}$  and the dual of the category of Hausdorff spaces and continuous maps. If, additionally, the initial object of  $\mathbf{W}$  satisfies a compactness-like algebraic property, the adjunction restricts to either compact Hausdorff spaces or Stone spaces.

In closing, we discuss how further research may complete these results to a classification theorem for equationally definable classes of algebras equivalent to either Boolean algebras or to  $\mathbf{V}$ .

*E-mail address:* `luca.reggio@liafa.univ-paris-diderot.fr`

IRIF, UNIVERSITÉ PARIS DIDEROT AND UNIVERSITÉ SORBONNE PARIS CITÉ, FRANCE