Topology and Measure in Logics for Point-Free Space

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Abstract

Physical space, as we typically conceive of it, is composed of dimensionless points. But philosophers have for a long time wondered whether points are in some sense too idealized. On an alternative conception of space, the parts of space are extended regions, and each region has a proper part. Formal logics describing region-based theories of space were studied in, *e.g.*, Vakarelov [2006] and Balbiani *et. al.* [2007], where both a relational and topological semantics for these logics is given. Vakarelov [2006] shows that the region based logic $\mathbb{L}^{min}_{cont}$ is complete for the topological semantics. The purpose of the present paper is two-fold. First, we study completeness of $\mathbb{L}^{min}_{cont}$ and simple extensions of that logic for certain important topological spaces: the real line, Cantor space, and the rationals. Second, we show how we can interpret these logics in the Lebesgue measure algebra (or algebra of Borel subsets of the real line modulo sets of measure zero) together with a contact relation that was first discussed in Arntzenius [2004]. The main result of the present work is that the logic $\mathbb{L}^{min}_{cont} + (Con)$ (also studied in Vakarelov [2006]) is complete for the Lebesgue measure algebra as well as for the algebra of regular closed subsets of the real line with measure zero boundaries. Along the way to obtaining that result, we also obtain completeness results for several specific topological spaces.