

Duality and canonicity for Boolean algebra with a relation

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In this work we study Boolean algebras (BAs) with a binary relation satisfying some properties. In particular, we introduce categories of BA with an operator relation (**BAOR**) which preserves finite joins in each coordinate, and BA with a dual operator relation (**BADOR**) which preserves finite meets in each coordinate. The maps in these categories are Boolean homomorphisms preserving the relation. It turns out that well-known algebras such as de Vries algebras, contact algebras, lattice subordinations and Boolean proximity lattices are examples of objects in these categories. Using the characteristic map of the relations, we show that the category **BAOR** (resp. **BADOR**) is isomorphic to the category of BA with binary operators (resp. dual operators) into the 2-element BA $\mathbf{2}$. This allows us to import results from the theory of BAOs into our setting.

We show that both finite **BAOR** and **BADOR** are dual to the category of Kripke frames with weak p-morphisms. In the infinite case, we show that both **BAOR** and **BADOR** are dual to the category of Stone spaces with closed binary relations and continuous maps. We also define canonical extensions of **BAOR** and **BADOR**, and show that Sahlqvist inequalities are canonical.