Proceedings of A. Razmadze
Mathematical Institute
Vol. **169** (2015), 133–140

ON SLIGHTLY CONTINUOUS MULTIFUNCTIONS VIA GENERALIZED TOPOLOGY

B. ROY

Abstract. In this paper, the notion of upper (lower) slightly (μ, σ)-continuous multifunctions has been introduced. Some characterizations of these types of multifunctions have been given. Several properties of such multifunctions are also obtained.

რეზიუმე. სტატიაში შემოღებულია ზედა (ქვედა) მცირედ (μ, σ)-უწყვეტი მულტიფუნქციები. მიღებულია ასეთი ტიპის ფუნქციების გარკვეული დახასიათება და დადგენილია მათი ზოგიერთი თვისება.

1. Introduction

One of the most important area in the theory of classical point set topology is continuity of functions and multifunctions as they are important tools for studying properties of spaces and for constructing new spaces from previously existing ones. Several weaker forms of continuous functions have been introduced and studied by different mathematicians. In [7] A. Kanibir and I. L. Reilly introduced upper (lower) semi generalized continuous multifunctions by using the concept of generalized topology. Similar types of functions have also been studied by C. Boonpok [2]. Such a generalized topology was first introduced by A. Cs*ászár*. We first recall some notions defined in [3]. Let X be a non-empty set, $\exp X$ denotes the power set of X. We call a class $\mu \subsetneqq \exp X$ a generalized topology [3], (briefly, GT) if $\varnothing \in \mu$ and union of elements of μ belongs to μ. A set X, with a GT μ on it is said to be a generalized topological space (briefly, GTS) and is denoted by (X, μ). For a GTS (X, μ), the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. For $A \subsetneqq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open

---

2010 *Mathematics Subject Classification.* 54C08, 54C60.
Key words and phrases. Upper (lower) slightly (μ, σ)-continuous multifunctions, μ-connectedness, (μ, σ)-clopen graph.

sets contained in A, i.e., the largest μ-open set contained in A (see [3, 4] for details).

It is easy to observe that i_μ and c_μ are idempotent and monotonic, where $\gamma : \exp X \to \exp X$ is said to be idempotent iff $A \subseteqq B \subseteqq X$ implies $\gamma(\gamma(A)) = \gamma(A)$ and monotonic iff $\gamma(A) \subseteqq \gamma(B)$. It is also well known from [4, 5] that if μ is a GT on X and $A \subseteqq X$, $x \in X$, then $x \in c_\mu(A)$ iff $x \in M \in \mu \Rightarrow M \cap A \neq \varnothing$ and $c_\mu(X \setminus A) = X \setminus i_\mu(A)$.

Throughout the paper, we shall use (X, μ) to mean a generalized topological space and (Y, σ) will denote a topological space. For a subset A, $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ denote the closure and interior of A respectively. For a multifunction $F : (X, \mu) \to (Y, \sigma)$, the upper and lower inverse of a set A of Y are denoted by $F^+(A)$ and $F^-(A)$ and defined by $F^+(A) = \{x \in X : F(x) \subseteqq A\}$ and $F^-(A) = \{x \in X : F(x) \cap A \neq \varnothing\}$. Also here $\mu(x) = \{U \in \mu : x \in U\}$.

2. Slightly (μ, σ)-Continuous Multifunctions

Definition 2.1. A multifunction $F : (X, \mu) \to (Y, \sigma)$ is said to be

(a) upper slightly (μ, σ)-continuous if for each point $x \in X$ and each clopen set V in Y containing $F(x)$, there exists $U \in \mu(x)$ in X such that $F(U) \subseteqq V$.

(b) lower slightly (μ, σ)-continuous if for each point $x \in X$ and each clopen set V in Y with $F(x) \cap V \neq \varnothing$, there exists $U \in \mu(x)$ in X such that $F(u) \cap V \neq \varnothing$ for each $u \in U$.

Theorem 2.2. *For a multifunction $F : (X, \mu) \to (Y, \sigma)$ the followings are equivalent:*

(a) *F is upper slightly (μ, σ)-continuous;*

(b) *$F^+(V)$ is μ-open for each clopen set V of Y;*

(c) *$F^-(V)$ is μ-closed for each clopen set V of Y.*

Proof. (a) $\Rightarrow$ (b): Let V be any clopen set in Y and $x \in F^+(V)$. Then $F(x) \subseteqq V$. Thus by (a), there exists $U \in \mu(x)$ in X such that $F(U) \subseteqq V$. Thus $x \in U \subseteqq F^+(V)$ and hence $x \in i_\mu(F^+(V))$. Therefore, $F^+(V) \subseteqq i_\mu(F^+(V))$ i.e., $F^+(V)$ is μ-open.

(b) $\Rightarrow$ (c): Let V be a clopen set of Y. Then $Y \setminus V$ is clopen in Y. Then by (b), $X \setminus F^-(V) = F^+(Y \setminus V) = i_\mu(F^+(Y \setminus V)) = X \setminus c_\mu(F^-(V))$. Thus $F^-(V) = c_\mu(F^-(V))$ i.e., $F^-(V)$ is μ-closed.

(c) $\Rightarrow$ (b): This follows from the fact that $F^-(Y \setminus B) = X \setminus F^+(B)$ for any subset B of Y.

(b) $\Rightarrow$ (a): Let $x \in X$ and V be any clopen set of Y containing $F(x)$. Then $x \in F^+(V) = i_\mu(F^+(V))$. Thus there exists $U \in \mu(x)$ such that $U \subseteqq F^+(V)$. Therefore, there exists a μ-open set U in X containing x such that $F(U) \subseteqq V$. Thus F is upper slightly (μ, σ)-continuous. $\square$

Theorem 2.3. *For a multifunction $F : (X, \mu) \to (Y, \sigma)$ the followings are equivalent:*
(a) *F is lower slightly (μ, σ)-continuous;*
(b) *$F^-(V)$ is μ-open for each clopen set V of Y;*
(c) *$F^+(V)$ is μ-closed for each clopen set V of Y.*

Proof. (a) $\Rightarrow$ (b): Let V be a clopen set of Y and $x \in F^-(V)$. Then $F(x) \cap V \neq \varnothing$ and hence by (a) there exists $U \in \mu(x)$ such that $F(u) \cap V \neq \varnothing$ for each $u \in U$. Therefore, we have $U \subseteq F^-(V)$ and hence $x \in U \subseteq i_\mu(F^-(V))$. Thus $F^-(V) \subseteq i_\mu(F^-(V))$ i.e., $F^-(V)$ is μ-open.

(b) $\Rightarrow$ (c): Let V be a clopen set in Y. Then $Y \setminus V$ is clopen in Y and by (b), we have $X \setminus F^+(V) = F^-(Y \setminus V) = i_\mu(F^-(Y \setminus V)) = X \setminus c_\mu(F^+(V))$. Thus $F^+(V)$ is μ-closed.

(c) $\Rightarrow$ (a): Let x be any point of X and V be any clopen set in Y such that $F(x) \cap V \neq \varnothing$. Then $x \in F^-(V)$ and hence $x \notin X \setminus F^-(V) = F^+(Y \setminus V)$. As $Y \setminus V$ is clopen in Y, by (c) we have $x \notin c_\mu(F^+(Y \setminus V))$. Thus there exists $U \in \mu(x)$ such that $U \cap F^+(Y \setminus V) = \varnothing$; hence $U \subseteq F^-(V)$. Thus $F(u) \cap V \neq \varnothing$ for each $u \in U$. Therefore F is lower slightly (μ, σ)-continuous. □

Definition 2.4. A topological space (X, τ) is said to be extremally disconnected (in short, E.D.) if closure of each open set in X is open in X.

Theorem 2.5. *Let (Y, σ) be an extremally disconnected space. For a multifunction $F : (X, \mu) \to (Y, \sigma)$ the followings are equivalent:*
(a) *F is upper slightly (μ, σ)-continuous;*
(b) *$c_\mu(F^-(V)) \subseteq F^-(\mathrm{cl}(V))$ for each open set V in Y;*
(c) *$F^+(\mathrm{int}(K)) \subseteq i_\mu(F^+(K))$ for each closed set K in Y.*

Proof. (a) $\Rightarrow$ (b): Let V be an open set in Y. Then $\mathrm{cl}(V)$ is clopen in Y (as Y is E.D.). By Theorem 2.2, $F^-(\mathrm{cl}(V)) = c_\mu(F^-(\mathrm{cl}(V)))$ and $F^-(V) \subseteq F^-(\mathrm{cl}(V))$. Thus $c_\mu(F^-(V)) \subseteq c_\mu(F^-(\mathrm{cl}(V))) = F^-(\mathrm{cl}(V))$. So $c_\mu(F^-(V)) \subseteq F^-(\mathrm{cl}(V))$.

(b) $\Rightarrow$ (c): Let K be any closed set in Y. Put $V = Y \setminus K$. Then V is an open set in Y. Then $X \setminus i_\mu(F^+(K)) = c_\mu(X \setminus F^+(K)) = c_\mu(F^-(V)) \subseteq F^-(\mathrm{cl}(V))$ (by (b)) $= F^-(Y \setminus \mathrm{int}(K)) = X \setminus F^+(\mathrm{int}(K))$. Thus we have $F^+(\mathrm{int}(K)) \subseteq i_\mu(F^+(K))$.

(c) $\Rightarrow$ (a): Let x be any point of X and V be a clopen set in Y containing $F(x)$. Then by (c) we have $x \in F^+(\mathrm{int}(V)) = F^+(V) \subseteq i_\mu(F^+(V))$. Therefore, there exists $U \in \mu(x)$ such that $U \subseteq F^+(V)$. Thus there exists a μ-open set U in X containing x such that $F(U) \subseteq V$. Therefore, F is upper slightly (μ, σ)-continuous. □

Theorem 2.6. *Let (Y, σ) be an extremally disconnected space. For a multifunction $F : (X, \mu) \to (Y, \sigma)$ the followings are equivalent:*
(a) *F is lower slightly (μ, σ)-continuous;*

(b) $c_\mu(F^+(V)) \subseteqq F^+(\mathrm{cl}(V))$ *for each open set* V *in* Y*;*
(c) $F^-(\mathrm{int}(K)) \subseteqq i_\mu(F^-(K))$ *for every closed set* K *in* Y*.*

Proof. The proof is similar to that of Theorem 2.5. □

Lemma 2.7 ([8])**.** *For a topological space* (Y, σ)*, the followings are equivalent:*
(a) (Y, σ) *is extremally disconnected;*
(b) *The closure of every semi-open set of* (Y, σ) *is open;*
(c) *The closure of every pre-open set of* (Y, σ) *is open;*
(d) *The closure of every* β*-open set of* (Y, σ) *is open.*

Lemma 2.8. *Let* (Y, σ) *be an extremally disconnected space. For a multifunction* $F : (X, \mu) \to (Y, \sigma)$ *the followings are equivalent:*
(a) F *is upper slightly* (μ, σ)*-continuous;*
(b) $c_\mu(F^-(V)) \subseteqq F^-(\mathrm{cl}(V))$ *for each semi-open* (*resp. pre-open,* β*-open*) *set* V *in* Y*;*
(c) $F^+(\mathrm{int}(K)) \subseteqq i_\mu(F^+(K))$ *for each semi-closed* (*resp. pre-closed,* β*-closed*) *set* K *in* Y*.*

Proof. The proof is similar to that of Theorem 2.5 and it follows from Theorem 2.2 and Lemma 2.7. □

Theorem 2.9. *Let* (Y, σ) *be an extremally disconnected space. For a multifunction* $F : (X, \mu) \to (Y, \sigma)$ *the followings are equivalent:*
(a) F *is lower slightly* (μ, σ)*-continuous;*
(b) $c_\mu(F^+(V)) \subseteqq F^+(\mathrm{cl}(V))$ *for each semi-open* (*resp. pre-open,* β*-open*) *set* V *in* Y*;*
(c) $F^-(\mathrm{int}(K)) \subseteqq i_\mu(F^-(K))$ *for each semi-closed* (*resp. pre-closed,* β*-closed*) *set* K *in* Y*.*

Proof. The proof is similar to that of Theorem 2.6 and it follows from Theorem 2.3 and Lemma 2.7. □

Definition 2.10. A multifunction $F : (X, \mu) \to (Y, \sigma)$ is said to be
(a) upper (μ, σ)-continuous (resp. upper almost (μ, σ)-continuous, upper weakly (μ, σ)-continuous) if for each point $x \in X$ and each open set V of Y containing $F(x)$, there exists a μ-open set U in X containing x such that $F(U) \subseteqq V$ (resp. $F(U) \subseteqq \mathrm{int}(\mathrm{cl}(V))$, $F(U) \subseteqq \mathrm{cl}(V)$).
(b) lower (μ, σ)-continuous (resp. lower almost (μ, σ)-continuous, lower weakly (μ, σ)-continuous) if for each point $x \in X$ and each open set V of Y with $F(x) \cap V \neq \varnothing$, there exists a μ-open set U in X containing x such that $F(u) \cap V \neq \varnothing$ (resp. $F(u) \cap \mathrm{int}(\mathrm{cl}(V)) \neq \varnothing$, $F(u) \cap \mathrm{cl}(V) \neq \varnothing$) for each $u \in U$.

Theorem 2.11. *If a multifunction* $F : (X, \mu) \to (Y, \sigma)$ *is upper weakly* (μ, σ)*-continuous, then it is upper slightly* (μ, σ)*-continuous.*

Proof. Let $x \in X$ and V be a clopen set in Y containing $F(x)$. Since F is upper weakly (μ, σ)-continuous, there exists a μ-open set U containing x such that $F(U) \subsetneqq \text{cl}(V) = V$. So F is upper slightly (μ, σ)-continuous. □

Theorem 2.12. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is lower weakly (μ, σ)-continuous, then it is lower slightly (μ, σ)-continuous.*

Proof. Similar to that of Theorem 2.11. □

Example 2.13. Let $X = Y = \{a, b, c\}$, $\mu = \{\varnothing, \{a\}, \{a, b\}, X\}$, $\sigma = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$. It can be shown that the function $F : (X, \mu) \to (Y, \sigma)$ defined by $F(a) = \{b, c\}$, $F(b) = F(c) = \{a\}$ is upper slightly (μ, σ)-continuous, but not upper weakly (μ, σ)-continuous.

Lemma 2.14. *A multifunction $F : (X, \mu) \to (Y, \sigma)$ is upper almost (μ, σ)-continuous (resp. lower almost (μ, σ)-continuous) if and only if for each regular open set V containing $F(x)$ (resp. $V \cap F(x) \neq \varnothing$) there exists a μ-open set U containing x such that $F(U) \subsetneqq V$ (resp. $F(u) \cap V \neq \varnothing$ for each $u \in U$).*

Theorem 2.15. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is upper slightly (μ, σ)-continuous and (Y, σ) is extremally disconnected then F is upper almost (μ, σ)-continuous.*

Proof. Let $x \in X$ and V be any regular open set of Y containing $F(x)$. Then V is a clopen set (as Y is extremally disconnected [see [10], Lemma 5.6]). Since F is upper slightly (μ, σ)-continuous, there exists a μ-open set U in X containing x such that $F(U) \subsetneqq V$. Thus by Lemma 2.14, F is upper almost (μ, σ)-continuous. □

Theorem 2.16. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is lower slightly (μ, σ)-continuous and (Y, σ) is extremally disconnected, then F is lower almost (μ, σ)-continuous.*

Proof. Similar to that of Theorem 2.15. □

Definition 2.17. A topological space (X, τ) is said to be 0-dimensional [14] if each point of X has a base consisting of clopen sets.

Definition 2.18. A topological space (X, τ) is said to be mildly compact [13] or slightly compact [10] if every clopen cover of X admits a finite subcover. A subset A of X is called mildly compact relative to X if every cover of A by clopen subsets of X has a finite subcover.

Theorem 2.19. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is upper slightly (μ, σ)-continuous and (Y, σ) is 0-dimensional and $F(x)$ is mildly compact relative to Y for each $x \in X$, then F is upper (μ, σ)-continuous.*

Proof. Let $x \in X$ and V be any open set of (Y, σ) containing $F(x)$. Then by 0-dimensionality of (Y, σ), for each $y \in F(x)$ there exists a clopen set G_y such that $y \in G_y \subsetneqq V$. Since $F(x)$ is mildly compact relative to Y, there exists a finite number of points $y_1, y_2, \ldots, y_n \in F(x)$ such that G_{y_i} is clopen in Y for each i and $F(x) \subsetneqq \cup\{G_{y_i} : i = 1, 2, \ldots, n\} \subsetneqq V$. Let $G = \cup\{G_{y_i} : i = 1, 2, \ldots, n\}$. Then G is clopen in Y and $F(x) \subsetneqq G \subsetneqq V$. Since F is upper slightly (μ, σ)-continuous, there exists a μ-open set U with $x \in U$ such that $F(U) \subsetneqq G \subsetneqq V$. Thus F is upper (μ, σ)-continuous. □

Theorem 2.20. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is lower slightly (μ, σ)-continuous and (Y, σ) is 0-dimensional, then F is lower (μ, σ)-continuous.*

Proof. Let $x \in X$ and V be an open set in Y such that $F(x) \cap V \neq \varnothing$. Then there exists a clopen set V_x such that $V_x \cap F(x) \neq \varnothing$ and $V_x \subsetneqq V$. Since F is lower slightly (μ, σ)-continuous and $V_x \cap F(x) \neq \varnothing$ there exists $U \in \mu(x)$ such that $F(u) \cap V_x \neq \varnothing$ for each $u \in U$. Thus there exists $U \in \mu(x)$ such that $F(u) \cap V \neq \varnothing$ for each $u \in U$ (as $V_x \subsetneqq V$). Therefore F is lower (μ, σ)-continuous. □

The clopen subsets of a topological space (X, τ) forms a base for a topology on X. This topology is called ultra-regularization [9] of τ and is denoted by τ_u. A topological space (X, τ) is said to be ultra-regular [6] if $\tau = \tau_u$.

Theorem 2.21. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is upper slightly (μ, σ)-continuous and (Y, σ) is ultra-regular and $F(x)$ is mildly compact relative to Y for each $x \in X$, then F is upper (μ, σ)-continuous.*

Proof. Similar to that of Theorem 2.19. □

Theorem 2.22. *If a multifunction $F : (X, \mu) \to (Y, \sigma)$ is lower slightly (μ, σ)-continuous and (Y, σ) is ultra-regular, then F is lower (μ, σ)-continuous.*

Proof. Similar to that of Theorem 2.20. □

3. Properties of Upper (Lower) Slightly (μ, σ)-Continuous Multifunctions

Definition 3.1. A GTS (X, μ) is said to be μ-connected [12] if X can not be written as union of two non-empty μ-open sets.

Theorem 3.2. *Let $F : (X, \mu) \to (Y, \sigma)$ be an upper (lower) slightly (μ, σ)-continuous surjection. If (X, μ) is μ-connected and $F(x)$ is connected for each $x \in X$, then (Y, σ) is connected.*

Proof. If possible let (Y, σ) be not connected. Then there exists a pair of disjoint open sets U and V such that $Y = U \cup V$. Since $F(x)$ is connected, for each $x \in X$ either $F(x) \subseteqq U$ or $F(x) \subseteqq V$. If $x \in F^+(U \cup V)$, then $F(x) \in U \cup V$ and hence $x \in F^+(U) \cup F^+(V)$. Also, as F is surjective there exist $x, y \in X$ such that $F(x) \subseteqq U$, $F(y) \subseteqq V$ hence $x \in F^+(U)$ and $y \in F^+(V)$. Thus $F^+(U) \cup F^+(V) = F^+(U \cup V) = X$, $F^+(U) \cap F^+(V) = F^+(U \cap V) = \varnothing$ and $F^+(U) \neq \varnothing \neq F^+(V)$.

If F is upper slightly (μ, σ)-continuous then since U and V are clopen, by Theorem 2.2 $F^+(U)$ and $F^+(V)$ are μ-clopen in X- a contradiction to the fact that X is μ-connected. If F is lower slightly (μ, σ)-continuous then by Theorem 2.3 we can have a similar contradiction. □

Definition 3.3. A multifunction $F : (X, \mu) \to (Y, \sigma)$ is said to have a (μ, σ)-clopen graph if for each $(x, y) \in X \times Y \setminus G(F)$, there exist a μ-open set U in X containing x and a clopen set V in Y containing y such that $(U \times V) \cap G(F) = \varnothing$.

Lemma 3.4. *A multifunction $F : (X, \mu) \to (Y, \sigma)$ has a (μ, σ)-clopen graph if and only if for each $(x, y) \in X \times Y \setminus G(F)$, there exist a μ-open set U in X containing x and a clopen set V in Y containing y such that $F(U) \cap V = \varnothing$.*

Definition 3.5. A topological space (X, τ) is said to be ultra-Hausdorff [13] if for each pair of distinct points $x, y \in X$, there exist disjoint pair of clopen sets U and V such that $x \in U$ and $y \in V$.

Theorem 3.6. *If $F : (X, \mu) \to (Y, \sigma)$ is an upper slightly (μ, σ)-continuous multifunction such that $F(x)$ is mildly compact relative to Y for each $x \in X$ and (Y, σ) is ultra-Hausdorff, then $G(F)$ is (μ, σ)-clopen.*

Proof. Suppose that $(x_0, y_0) \in X \times Y \setminus G(F)$. Then $y_0 \notin F(x_0)$. Since Y is ultra-Hausdorff, for each $y \in F(x_0)$ there exist clopen sets G_y and H_y in Y containing y and y_0 respectively, such that $G_y \cap H_y = \varnothing$. Then the family $\{G_y : y \in F(x_0)\}$ is a clopen cover of $F(x_0)$. Since $F(x_0)$ is mildly compact relative to Y, there exists a finite number of points $y_1, y_2, \ldots, y_n$ in $F(x_0)$ such that $F(x_0) \subseteqq \cup\{G_{y_i} : i = 1, 2, \ldots, n\} = G$ (say). Let $H = \cap\{H_{y_i} : i = 1, 2, \ldots, n\}$. Then G and H both are clopen in Y such that $F(x_0) \subseteqq G$, $y_0 \in H$ and $G \cap H = \varnothing$. Since F is upper slightly (μ, σ)-continuous, there exists a μ-open set U in X containing x_0 such that $F(U) \subseteqq G$. Thus $F(U) \cap H = \varnothing$. Hence by Lemma 3.4, $G(F)$ is (μ, σ)-clopen. □

Definition 3.7. For any subset A of a GTS (X, μ), the μ-frontier [11] of A is denoted by $Fr_\mu(A)$ and defined by $Fr_\mu(A) = c_\mu(A) \cap c_\mu(X \setminus A)$.

Theorem 3.8. *The set of all points $x \in X$ at which a multifunction $F : (X, \mu) \to (Y, \sigma)$ is not upper (lower) slightly (μ, σ)-continuous is identical with the union of μ-frontier of the upper (resp. lower) inverse image of clopen sets containing (resp. meeting) $F(x)$.*

Proof. We shall prove the theorem when F is upper slightly (μ, σ)-continuous. The case for lower slightly (μ, σ)-continuous can be shown in a similar fashion. Suppose that F is not upper slightly (μ, σ)-continuous at $x \in X$. Then there exists a clopen set V in Y containing $F(x)$ such that $U \cap (X \setminus F^+(V)) \neq \varnothing$ for each μ-open set U containing x. Then $x \in c_\mu(X \setminus F^+(V))$. On the other hand, we have $x \in F^+(V) \subseteqq c_\mu(F^+(V))$. Hence $x \in Fr_\mu(F^+(V))$.

Conversely, suppose that F is upper slightly (μ, σ)-continuous at $x \in X$. Let V be any clopen set in Y containing $F(x)$. Then there exists a μ-open set U in X containing x such that $U \subseteqq F^+(V)$; hence $x \in i_\mu(F^+(V))$. Therefore, $x \notin Fr_\mu(F^+(V))$ for each clopen set V in Y containing $F(x)$. □

References

1. T. Bânzaru, More remarks on some forms of continuity for multifunctions. *Bul. tiin. Univ. Politeh. Timics Ser. Mat. Fiz.* **44(58)** (1999), No. 2, 34–39.
2. C. Boonpok, On upper and lower $\beta(\mu_X, \mu_Y)$-continuous multifunctions. *Int. J. Math. Math. Sci.* 2012, Art. ID 931656.
3. *Á.* Császár, Generalized topology, generalized continuity. *Acta Math. Hungar.* **96** (2002), No. 4, 351–357.
4. *Á.* Császár, Generalized open sets in generalized topologies. *Acta Math. Hungar.* **106** (2005), No. 1-2, 53–66.
5. *Á.* Császár, δ- and θ-modifications of generalized topologies. *Acta Math. Hungar.* **120** (2008), No. 3, 275–279.
6. R. L. Ellis, A non-Archimedean analog of the Tietze-Urysohn extension theorem. *Indag. Math.* **70** (1967), 332–333.
7. A. Kanibir and I. L. Reilly, Generalized continuity for multifunctions. *Acta Math. Hungar.* **122** (2009), No. 3, 283–292.
8. T. Noiri, Characterizations of extremally disconneced spaces. *Indian J. Pure Appl. Math.* **19** (1988), No. 4, 325–329.
9. T. Noiri, Slightly β-continuous functions. *Internat. J. Math. Math. Sci.* **28** (2001), No. 8, 469–478.
10. M. C. Pal and P. Bhattacharyya, Faint precontinous functions. *Soochow J. Math.* **21** (1995), No. 3, 273–289.
11. B. Roy, On faintly continuous functions via generalized topology. *Chinese Math. J.* 2013 (2013), Article ID 412391.
12. R. -X. Shen, A note on generalized connectedness. *Acta Math. Hungar.* **122** (2009), No. 3, 231–235.
13. R. Staum, The algebra of bounded continuous functions into a nonarchimedean field. *Pacific J. Math.* **50** (1974), 169–185.
14. S. Willard, General topology. *Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont.*, 1970.

(Received 02.12.2014)

Author's address:

Department of Mathematics, Women's Christian College
6, Greek Church Row, Kolkata-700 026, India
E-mail: bishwambhar_roy@yahoo.co.in