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ON ONE CLASS OF THREE-DIMENSIONAL PROBLEMS OF ELASTICITY THEORY FOR PLATES

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Abstract. A special problem of elasticity theory is solved for plates, when one of its facial surfaces is free from stresses, but the displacements of its points are known. A similar problem arises particularly, when studying tectonics of the earth plates by means of the data of seismic stations and GPS systems. Using the asymptotic method of solution of singularly perturbed differential equations, a general asymptotic solution formulated by the non-classical boundary value problem, is constructed. For the cases, when the displacements of the facial surface points of the plate are described by polynomials in tangential coordinates, the exact mathematical solutions are obtained. It is shown that the solution of the determined classical boundary value problem corresponds to the solution of the formulated non-classical boundary value problem.

რეზიუმე. დრეკადობის თეორიის ამოცანა ფირფიტისათვის ამოხსნილია, როდესაც მისი საზღვრის ერთი მხარე თავისუფალია დატვირთვისაგან, ხოლო გადაადგილებები ცნობილია. ასიმპტოტური მეთოდის გამოყენებით სინგულარულად შეშფოთებული დიფერენციალური განტოლებებისათვის აგებულია ზოგადი ასიმპტოტური ამონახსნები არაკლასიკურ სასაზღვრო პირობებში. იმ შემთხვევებისათვის, როდესაც ფირფიტის ზედაპირის წერტილთა გადაადგილებები აღწერილია პოლინომებით ტანგენციალურ კოორდინატებში, აგებული ზუსტი მათემატიკური ამონახსნები შეესაბამება ფუნდამენტურ არაკლასიკურ სასაზღვრო ამოცანების ამონახსნებს.

1. Introduction

Modern science revealed the real reasons for the rise in strong earthquakes.They are basically connected with tectonics of the Earth plates (≈95% of the earthquakes) [1,2]. The existence of the thick net of seismostations and modern GPS systems permit us to follow the behaviour of lithospheric plates of the Earth and its separate parts. Having at hand the

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geological structure of location (layering, density), it is possible, on the basis of the data of seismostations and GPS systems, to observe the behaviour of stress-strained state of the corresponding layered packet.

Towards this end, it is, first of all, necessary to have a solution of the special problem of elasticity theory for plate-like layered packet, when on the facial surface of the packet the corresponding stresses tensor components are equal to zero, but the displacement vector components of the points of the same surface coinciding with the data of seismostations and GPS systems are known. The corresponding boundary value problem of elasticity is non-classical, since the conditions are given on one surface and their number unlike the classical boundary value problem is more than three (six). On the other hand, proceeding from physical considerations, this problem is well defined and, as we shall see below, it is correct mathematically, as well. Moreover, there is always the classical boundary value problem for the packet (plate), the solution of which is that of the corresponding non-classical boundary value problem.

Here we consider non-classical three-dimensional problems for one-layered orthotropic plate. The corresponding approach may be spread to layered plates allowing one to consider dynamical problems as well.

2. Basic Equations and Formulation of the Boundary Value Problem

It is required to find the solution of the equations and relations of three-dimensional problem of elasticity theory in the area $D = \{(x, y, z) : 0 \le x \le a, 0 \le y \le b, -h \le z \le h, h << \ell, \quad \ell = \min(a, b)\}$ occupied by orthotropic plate-equilibrium equations, elasticity relations [3,4], under the boundary conditions

$$\sigma_{xz}(x, y, h) = 0, \ \sigma_{yz}(x, y, h) = 0, \ \sigma_{zz}(x, y, h) = 0, \tag{1}$$

$$u(x, y, h) = u^+(x, y), \quad \mathrm{v}(x, y, h) = \mathrm{v}^+(x, y), \quad w(x, y, h) = w^+(x, y), \tag{2}$$

where σ_{ij}, (u, v, w) are the components correspondingly of the stress tensor and the displacement vector, u^+, v^+, w^+ are the well-known functions of C^n, $\forall n$ class. The condition on the lateral surface of the plate will not be defined concretely so far, the rise of the boundary layer, just as in the classical boundary value problems, is conditioned by them [5].

In order to solve the formulated boundary value problem in the equations and correlations of the elasticity theory, we pass to dimensionless coordinates and displacements:

$$x = \ell\xi, \ \ y = \ell\eta, \ \ z = h\zeta, \ \ U = u/\ell, \ \ V = \mathrm{v}/\ell, \ \ W = w/\ell. \tag{3}$$

As a result, we have the following singularly perturbed by small parameter $\varepsilon = h/\ell$ system:

Equilibrium equations

$$
\begin{aligned}
&\frac{\partial \sigma_{xx}}{\partial \xi}+\frac{\partial \sigma_{xy}}{\partial \eta}+\varepsilon^{-1}\frac{\partial \sigma_{xz}}{\partial \zeta}+\ell F_x(\ell\xi,\ell\eta,h\zeta)=0,\\
&\frac{\partial \sigma_{xy}}{\partial \xi}+\frac{\partial \sigma_{yy}}{\partial \eta}+\varepsilon^{-1}\frac{\partial \sigma_{yz}}{\partial \zeta}+\ell F_y(\ell\xi,\ell\eta,h\zeta)=0,\\
&\frac{\partial \sigma_{xz}}{\partial \xi}+\frac{\partial \sigma_{yz}}{\partial \eta}+\varepsilon^{-1}\frac{\partial \sigma_{zz}}{\partial \zeta}+\ell F_z(\ell\xi,\ell\eta,h\zeta)=0.
\end{aligned}
\tag{4}
$$

Elasticity correlations

$$
\begin{aligned}
&\frac{\partial U}{\partial \xi}=a_{11}\sigma_{xx}+a_{12}\sigma_{yy}+a_{13}\sigma_{zz},\quad \frac{\partial V}{\partial \eta}=a_{12}\sigma_{xx}+a_{22}\sigma_{yy}+a_{23}\sigma_{zz},\\
&\varepsilon^{-1}\frac{\partial W}{\partial \zeta}=a_{13}\sigma_{xx}+a_{23}\sigma_{yy}+a_{33}\sigma_{zz},\quad \frac{\partial U}{\partial \eta}+\frac{\partial V}{\partial \xi}=a_{66}\sigma_{xy},\\
&\frac{\partial W}{\partial \xi}+\varepsilon^{-1}\frac{\partial U}{\partial \zeta}=a_{55}\sigma_{xz},\quad \frac{\partial W}{\partial \eta}+\varepsilon^{-1}\frac{\partial V}{\partial \zeta}=a_{44}\sigma_{yz},
\end{aligned}
\tag{5}
$$

where a_{ij} are the constants of elasticity, F_x, F_y, F_z are the components of volume forces.

3. General Solution of the Internal Problem

The solution of the singularly perturbed system (4), (5) is combined from the solutions of the internal problem I^{int} and the boundary layer I_b [5-7]

$$I=I^{\text{int}}+I_b. \tag{6}$$

The solution of the internal problem will be sought in the form

$$\sigma_{ij}^{\text{int}}=\varepsilon^{-1+s}\sigma_{ij}^{(s)},\ \ (U^{\text{int}},V^{\text{int}},W^{\text{int}})=\varepsilon^{s}(U^{(s)},V^{(s)},W^{(s)}),\ \ s=\overline{0,N}, \tag{7}$$

where $s=\overline{0,N}$ means summing up by umbral index s by integral values from zero to the number of approximations N. Substituting (7) into (4), (5) and equalizing in each equation the corresponding coefficients at small parameter, we have the system

$$
\begin{aligned}
&\frac{\partial \sigma_{xx}^{(s-1)}}{\partial \xi}+\frac{\partial \sigma_{xy}^{(s-1)}}{\partial \eta}+\frac{\partial \sigma_{xz}^{(s)}}{\partial \zeta}+F_x^{(s)}=0,\\
&\frac{\partial \sigma_{xy}^{(s-1)}}{\partial \xi}+\frac{\partial \sigma_{yy}^{(s-1)}}{\partial \eta}+\frac{\partial \sigma_{yz}^{(s)}}{\partial \zeta}+F_y^{(s)}=0,\\
&\frac{\partial \sigma_{xz}^{(s-1)}}{\partial \xi}+\frac{\partial \sigma_{yz}^{(s-1)}}{\partial \eta}+\frac{\partial \sigma_{zz}^{(s)}}{\partial \zeta}+F_z^{(s)}=0,\\
&\frac{\partial U^{(s-1)}}{\partial \xi}=a_{11}\sigma_{xx}^{(s)}+a_{12}\sigma_{yy}^{(s)}+a_{13}\sigma_{zz}^{(s)},\\
&\frac{\partial V^{(s-1)}}{\partial \eta}=a_{12}\sigma_{xx}^{(s)}+a_{22}\sigma_{yy}^{(s)}+a_{23}\sigma_{zz}^{(s)},
\end{aligned}
\tag{8}
$$

$$\frac{\partial W^{(s)}}{\partial \zeta} = a_{13}\sigma_{xx}^{(s)} + a_{23}\sigma_{yy}^{(s)} + a_{33}\sigma_{zz}^{(s)},$$
$$\frac{\partial U^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s-1)}}{\partial \xi} = a_{66}\sigma_{xy}^{(s)},$$
$$\frac{\partial W^{(s-1)}}{\partial \xi} + \frac{\partial U^{(s)}}{\partial \zeta} = a_{55}\sigma_{xz}^{(s)}, \quad \frac{\partial W^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s)}}{\partial \zeta} = a_{44}\sigma_{yz}^{(s)},$$
$$F_x^{(0)} = \varepsilon^2 \ell F_x, \;\; F_x^{(s)} = 0, \;\; s \neq 0, \;\; (x,y,z), \quad Q^{(m)} \equiv 0 \;\text{ at }\; m < 0.$$

From system (8) follows

$$\begin{aligned}
&\sigma_{xz}^{(s)} = \sigma_{xz0}^{(s)}(\xi,\eta) + \sigma_{xz*}^{(s)}(\xi,\eta,\zeta), \quad (x,y,z),\\
&U^{(s)} = a_{55}\zeta\sigma_{xz0}^{(s)} + u_0^{(s)}(\xi,\eta) + u_*^{(s)}(\xi,\eta,\zeta),\\
&V^{(s)} = a_{44}\zeta\sigma_{yz0}^{(s)} + \mathrm{v}_0^{(s)}(\xi,\eta) + \mathrm{v}_*^{(s)}(\xi,\eta,\zeta),\\
&W^{(s)} = \tfrac{A_{33}}{A_{11}}\zeta\sigma_{zz0}^{(s)} + w_0^{(s)}(\xi,\eta) + w_*^{(s)}(\xi,\eta,\zeta),\\
&\sigma_{xy}^{(s)} = \frac{1}{a_{66}}\left[\frac{\partial U^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s-1)}}{\partial \xi}\right],\\
&\sigma_{xx}^{(s)} = -\frac{A_{23}}{A_{11}}\sigma_{zz0}^{(s)} + \sigma_{xx*}^{(s)}(\xi,\eta,\zeta), \quad \sigma_{yy}^{(s)} = -\frac{A_{13}}{A_{11}}\sigma_{zz0}^{(s)} + \sigma_{yy*}^{(s)}(\xi,\eta,\zeta),\\
&A_{11} = a_{11}a_{22} - a_{12}^2, \;\; A_{13} = a_{11}a_{23} - a_{12}a_{13}, \;\; A_{23} = a_{22}a_{13} - a_{12}a_{23},\\
&A_{33} = a_{33}A_{11} - a_{13}A_{23} - a_{23}A_{13}.
\end{aligned} \tag{9}$$

For each s the values with a star are the well-known functions, if the previous approximations are built and are calculated by the formulae

$$\begin{aligned}
&\sigma_{xz*}^{(s)} = -\int_0^{\zeta}\left[F_x^{(s)} + \frac{\partial \sigma_{xx}^{(s-1)}}{\partial \xi} + \frac{\partial \sigma_{xy}^{(s-1)}}{\partial \eta}\right] d\zeta, \qquad (x,y;\xi,\eta),\\
&\sigma_{zz*}^{(s)} = -\int_0^{\zeta}\left[F_z^{(s)} + \frac{\partial \sigma_{xz}^{(s-1)}}{\partial \xi} + \frac{\partial \sigma_{yz}^{(s-1)}}{\partial \eta}\right] d\zeta,\\
&u_*^{(s)} = \int_0^{\zeta}\left[a_{55}\sigma_{xz*}^{(s)} - \frac{\partial W^{(s-1)}}{\partial \xi}\right] d\zeta, \quad \mathrm{v}_*^{(s)} = \int_0^{\zeta}\left[a_{44}\sigma_{yz*}^{(s)} - \frac{\partial W^{(s-1)}}{\partial \eta}\right] d\zeta,\\
&w_*^{(s)} = \int_0^{\zeta}\left[a_{13}\sigma_{xx*}^{(s)} + a_{23}\sigma_{yy*}^{(s)} + a_{33}\sigma_{zz*}^{(s)}\right] d\zeta,\\
&\sigma_{xx*}^{(s)} = \frac{1}{A_{11}}\left[a_{22}\frac{\partial U^{(s-1)}}{\partial \xi} - a_{12}\frac{\partial V^{(s-1)}}{\partial \eta} - A_{23}\sigma_{zz*}^{(s)}\right],\\
&\sigma_{yy*}^{(s)} = \frac{1}{A_{11}}\left[a_{11}\frac{\partial V^{(s-1)}}{\partial \eta} - a_{12}\frac{\partial U^{(s-1)}}{\partial \xi} - A_{13}\sigma_{zz*}^{(s)}\right].
\end{aligned} \tag{10}$$

Solution (7), (9) contains six yet unknown functions $\sigma_{xz0}^{(s)}$, $\sigma_{yz0}^{(s)}$, $\sigma_{zz0}^{(s)}$, $u_0^{(s)}$, $\mathrm{v}_0^{(s)}$, $w_0^{(s)}$, which are uniquely determined in the process of satisfaction of boundary conditions (1), (2). Using (7), (9), (10) and satisfying these conditions, we have

$$
\begin{aligned}
&\sigma_{xz0}^{(s)}(\xi,\eta) = -\sigma_{xz*}^{(s)}(\xi,\eta,1), \quad (x,y)\\
&\sigma_{zz0}^{(s)}(\xi,\eta) = -\sigma_{zz*}^{(s)}(\xi,\eta,1)\\
&u_0^{(s)}(\xi,\eta) = u^{+(s)} + a_{55}\sigma_{xz*}^{(s)}(\xi,\eta,1) - u_*^{(s)}(\xi,\eta,1)\\
&\mathrm{v}_0^{(s)}(\xi,\eta) = \mathrm{v}^{+(s)} + a_{44}\sigma_{yz*}^{(s)}(\xi,\eta,1) - \mathrm{v}_*^{(s)}(\xi,\eta,1)\\
&w_0^{(s)}(\xi,\eta) = w^{+(s)} + \frac{A_{33}}{A_{11}}\sigma_{zz*}^{(s)}(\xi,\eta,1) - w_*^{(s)}(\xi,\eta,1)\\
&u^{+(0)} = u^+/\ell, \ \ u^{+(s)} = 0, \ s \neq 0, \ \ (u,\mathrm{v},w)
\end{aligned}
\tag{11}
$$

Using (11), formulae (9) will be of the form

$$
\begin{aligned}
\sigma_{xz}^{(s)} &= \sigma_{xz*}^{(s)}(\xi,\eta,\zeta) - \sigma_{xz*}^{(s)}(\xi,\eta,1), \qquad (x,y),\\
\sigma_{zz}^{(s)} &= \sigma_{zz*}^{(s)}(\xi,\eta,\zeta) - \sigma_{zz*}^{(s)}(\xi,\eta,1),\\
U^{(s)} &= u^{+(s)} + a_{55}(1-\zeta)\sigma_{xz*}^{(s)}(\xi,\eta,1) + u_*^{(s)}(\xi,\eta,\zeta) - u_*^{(s)}(\xi,\eta,1),\\
V^{(s)} &= \mathrm{v}^{+(s)} + a_{44}(1-\zeta)\sigma_{yz*}^{(s)}(\xi,\eta,1) + \mathrm{v}_*^{(s)}(\xi,\eta,\zeta) - \mathrm{v}_*^{(s)}(\xi,\eta,1),\\
W^{(s)} &= w^{+(s)} + \frac{A_{33}}{A_{11}}(1-\zeta)\sigma_{zz*}^{(s)}(\xi,\eta,1) + w_*^{(s)}(\xi,\eta,\zeta) - w_*^{(s)}(\xi,\eta,1),\\
\sigma_{xx}^{(s)} &= -\frac{A_{23}}{A_{11}}(\sigma_{zz*}^{(s)}(\xi,\eta,\zeta) - \sigma_{zz*}^{(s)}(\xi,\eta,1)) +\\
&\quad + \frac{1}{A_{11}}\left[a_{22}\frac{\partial U^{(s-1)}}{\partial\xi} - a_{12}\frac{\partial V^{(s-1)}}{\partial\eta}\right],\\
\sigma_{yy}^{(s)} &= -\frac{A_{13}}{A_{11}}(\sigma_{zz*}^{(s)}(\xi,\eta,\zeta) - \sigma_{zz*}^{(s)}(\xi,\eta,1)) +\\
&\quad + \frac{1}{A_{11}}\left[a_{11}\frac{\partial V^{(s-1)}}{\partial\eta} - a_{12}\frac{\partial U^{(s-1)}}{\partial\xi}\right],\\
\sigma_{xy}^{(s)} &= \frac{1}{a_{66}}\left(\frac{\partial U^{(s-1)}}{\partial\eta} + \frac{\partial V^{(s-1)}}{\partial\xi}\right).
\end{aligned}
\tag{12}
$$

So, by formulae (7), (10), (12) the solution of the internal problem will be fully determined. This solution, as a rule, will not satisfy the boundary conditions on the lateral surface of the plate. These conditions are satisfied with the help of the solution for the boundary layer, which is built and united with the solution of the internal problem by the procedure described in [5].

4. Connection with the Solution of Classical Mixed Boundary Value Problem

It is reasonable to put a question whether there is the classical boundary value problem whose solution coincides with that of the non-classical boundary value problem (1), (2). In order to answer this question we calculate the values of the displacements in problem (1), (2) at $\zeta = -1$. According to formulae (12) we have

$$
\begin{aligned}
&U^{(s)}(\zeta = -1) = u^{-(s)} = \\
&\quad = u^{+(s)} + 2a_{55}\sigma^{(s)}_{xz*}(\xi,\eta,1) + u^{(s)}_*(\xi,\eta,-1) - u^{(s)}_*(\xi,\eta,1), \\
&V^{(s)}(\zeta = -1) = \mathrm{v}^{-(s)} = \\
&\quad = \mathrm{v}^{+(s)} + 2a_{44}\sigma^{(s)}_{yz*}(\xi,\eta,1) + \mathrm{v}^{(s)}_*(\xi,\eta,-1) - \mathrm{v}^{(s)}_*(\xi,\eta,1), \\
&W^{(s)}(\zeta = -1) = w^{-(s)} = \\
&\quad = w^{+(s)} + \frac{2A_{33}}{A_{11}}\sigma^{(s)}_{zz*}(\xi,\eta,1) + w^{(s)}_*(\xi,\eta,-1) - w^{(s)}_*(\xi,\eta,1).
\end{aligned} \tag{13}
$$

Therefore,

$$
\begin{aligned}
&\left(\frac{u}{\ell}\right)_{\zeta=-1} = \varepsilon^s u^{-(s)}, \quad \left(\frac{\mathrm{v}}{\ell}\right)_{\zeta=-1} = \varepsilon^s \mathrm{v}^{-(s)}, \\
&\left(\frac{w}{\ell}\right)_{\zeta=-1} = \varepsilon^s w^{-(s)}, \quad s = \overline{0,N}.
\end{aligned} \tag{14}
$$

Now consider the problem: find in the domain D a solution of the equations and correlations of the three-dimensional problem of elasticity under the classical mixed conditions (1) and (14). Having solved this problem by the asymptotic method, in the internal problem we shall again have a general solution (9). The satisfaction of the conditions (1) results in the correlations

$$
\begin{aligned}
&\sigma^{(s)}_{xz0}(\xi,\eta) = -\sigma^{(s)}_{xz*}(\xi,\eta,1), \quad (x,y), \\
&\sigma^{(s)}_{zz0}(\xi,\eta) = -\sigma^{(s)}_{zz*}(\xi,\eta,1)
\end{aligned} \tag{15}
$$

by virtue of which we have

$$
\begin{aligned}
&\sigma^{(s)}_{xz} = \sigma^{(s)}_{xz*}(\xi,\eta,\zeta) - \sigma^{(s)}_{xz*}(\xi,\eta,1), \quad (x,y), \\
&\sigma^{(s)}_{zz} = \sigma^{(s)}_{zz*}(\xi,\eta,\zeta) - \sigma^{(s)}_{zz*}(\xi,\eta,1).
\end{aligned} \tag{16}
$$

Using (9), satisfy conditions (14). According to (9), (13)–(15), we have

$$
\begin{aligned}
U^{(s)}(\zeta = -1) &= -a_{55}\sigma^{(s)}_{xz0} + u^{(s)}_0(\xi,\eta) + u^{(s)}_*(\xi,\eta,-1) = u^{-(s)} = \\
&= u^{+(s)} + 2a_{55}\sigma^{(s)}_{xz*}(\xi,\eta,1) + u^{(s)}_*(\xi,\eta,-1) - u^{(s)}_*(\xi,\eta,1).
\end{aligned} \tag{17}
$$

Analogous formulae can be obtained for $V^{(s)}(\zeta = -1)$, $W^{(s)}(\zeta = -1)$. From these formulae follow

$$
\begin{aligned}
&u^{(s)}_0(\xi,\eta) = u^{+(s)} + a_{55}\sigma^{(s)}_{xz*}(\xi,\eta,1) - \\
&\qquad -u^{(s)}_*(\xi,\eta,1), \ (u,\mathrm{v};x,y,;a_{55},a_{44}), \\
&w^{(s)}_0(\xi,\eta) = w^{+(s)} + \frac{A_{33}}{A_{11}}\sigma^{(s)}_{zz*}(\xi,\eta,1) - w^{(s)}_*(\xi,\eta,1),
\end{aligned} \tag{18}
$$

which coincide with the corresponding formulae (11).

Substituting the found values $u^{(s)}_0, \mathrm{v}^{(s)}_0, w^{(s)}_0$ into formulae (9), we obtain formulae (12) for the sought values.

In this way the solution of non-classical problem (1), (2) corresponds to the solution of classical boundary value problem (1), (14). They not only correspond, but also coincide.

5. Mathematically Exact Solutions of the Internal Problem

If functions u^+, v^+, w^+ are polynomials, the iteration process cuts and mathematically exact problem in the internal problem (for the layer) is obtained. As the illustration of the above told, we find the solution of non-classical boundary value problem (1), (2) at

$$\begin{aligned} u^+ &= \ell(a_1 + a_2\xi + a_3\eta), \\ \mathrm{v}^+ &= \ell(b_1 + b_2\xi + b_3\eta), \\ w^+ &= \ell(c_1 + c_2\xi + c_3\eta), \end{aligned} \tag{19}$$

and at the absence of the volume forces.

Using formulae (10)–(12) at $s = 0$ we have

$$\begin{aligned} &\sigma_{xz*}^{(0)} = 0, \ \ \sigma_{yz*}^{(0)} = 0, \ \ \sigma_{zz*}^{(0)} = 0, \ \ u_*^{(0)} = 0, \ \mathrm{v}_*^{(0)} = 0, \\ &\sigma_{xz}^{(0)} = \sigma_{yz}^{(0)} = \sigma_{zz}^{(0)} = 0, \ \ \sigma_{xx*}^{(0)} = 0, \ \ \sigma_{yy*}^{(0)} = 0, \\ &\sigma_{xx}^{(0)} = \sigma_{yy}^{(0)} = \sigma_{xy}^{(0)} = 0, \\ &U^{(0)} = a_1 + a_2\xi + a_3\eta, \ \ V^{(0)} = b_1 + b_2\xi + b_3\eta, \ \ W^{(0)} = c_1 + c_2\xi + c_3\eta. \end{aligned} \tag{20}$$

At $s = 1$ we have

$$\begin{aligned} &\sigma_{xz*}^{(1)} = \sigma_{yz*}^{(1)} = \sigma_{zz*}^{(1)} = 0, \quad \sigma_{xz}^{(1)} = \sigma_{yz}^{(1)} = \sigma_{zz}^{(1)} = 0, \\ &\sigma_{xy}^{(1)} = \frac{1}{a_{66}}(a_3 + b_2), \ \ \sigma_{xx*}^{(1)} = \frac{1}{A_{11}}(a_{22}a_2 - a_{12}b_3) = \sigma_{xx}^{(1)}, \\ &\sigma_{yy*}^{(1)} = \frac{1}{A_{11}}(a_{11}b_3 - a_{12}a_2) = \sigma_{yy}^{(1)}, \\ &u_*^{(1)} = -c_2\zeta, \ \ \mathrm{v}_*^{(1)} = -c_3\zeta, \ \ w_*^{(1)} = \frac{1}{A_{11}}(A_{23}a_2 + A_{13}b_3)\zeta, \\ &U^{(1)} = c_2(1-\zeta), \ \ V^{(1)} = c_3(1-\zeta), \\ &W^{(1)} = -\frac{1}{A_{11}}(A_{23}a_2 + A_{13}b_3)(1-\zeta). \end{aligned} \tag{21}$$

The iteration process cuts at $s = 2$, i.e. $Q^{(s)} \equiv 0$ at $s \geq 2$.

As a result, according to (7), (19)–(21) we have the following mathematically exact solution

$$\begin{aligned} &u = \ell(a_1 + a_2\xi + a_3\eta) + hc_2(1-\zeta), \\ &\mathrm{v} = \ell(b_1 + b_2\xi + b_3\eta) + hc_3(1-\zeta), \\ &w = \ell(c_1 + c_2\xi + c_3\eta) - \frac{h}{A_{11}}(A_{23}a_2 + a_{13}b_3)(1-\zeta), \\ &\sigma_{xz} = 0, \ \ \sigma_{yz} = 0, \ \ \sigma_{zz} = 0, \ \ \sigma_{xy} = \frac{1}{a_{66}}(a_3 + b_2), \\ &\sigma_{xx} = \frac{1}{A_{11}}(a_{22}a_2 - a_{12}b_3), \ \ \sigma_{yy} = \frac{1}{A_{11}}(a_{11}b_3 - a_{12}a_2). \end{aligned} \tag{22}$$

From Weierstrass theorem the arbitrary continuous function may be approximated by a polynomial, hence for a wide class of functions u^+, v^+, w^+ it is possible to obtain a sufficiently exact solution.

Finally, it should be noted that by using the above-mentioned method of solution of the non-classical boundary value problem it is possible to find solutions for layered plates too, and to consider the dynamic problems, as well.

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