Proceedings of A. Razmadze
Mathematical Institute
Vol. **148** (2008), 35–43

ON s-CLOSED SPACES VIA GRILLS

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Abstract. The s-closed spaces as introduced by Mayo and Noiri [11] in terms of semi-open sets, have already been investigated by many topologists from different approaches and orientations. In this paper an investigation of s-closedness is being continued by use of a different technique viz. via grills.

რეზიუმე. მაიოსა და ნოირის [11] მიერ შემოღებულია s-ჩაკეტილი სივრცეები ნახევრად ღია სიმრავლეების ტერმინებში. ასეთი სივრცეები შესწავლილია სხვადასხვა მიმართულებით. ნაშრომში s-ჩაკეტილობა გამოკვლეულია მესერების სხვადასხვა ტექნიკის მეშვეობით.

1. Introduction

It was Thompson [15] who introduced the concept of S-closed spaces by using the notion of semi-open sets of Levine [10] and closure of such sets. Crossley and Hildebrand [4] introduced the notion of semiclosure of a subset of a topological space; Mayo and Noiri [11] used it along with semi-open sets to initiate the concept of s-closed spaces and also derived certain characterizations. Subsequently s-closedness has been studied extensively by many researchers. Ganster and Reilly [9] have established that every infinite topological space can be embedded as a closed subspace of a connected S-closed space which is not s-closed. It has been observed from the literature that many researchers have worked on this concept by adopting a number of approaches and also from different angles. Different techniques, mainly covers, nets, filters and filterbase etc. are employed so far to study the said type of spaces. The purpose of this paper is to continue the investigations of the concept of s-closedness through a new approach, viz. by means of grills. For this purpose we require to establish certain concepts concerning grills and then apply this general discussion to the intended study of s-closed spaces in the subsequent section.

2000 *Mathematics Subject Classification.* 54D20, 54D99.
Key words and phrases. Grills, s-closed spaces, $s(\theta)$-closure, $s(\theta)$-adherence and $s(\theta)$-convergence of grills.

Throughout this paper, a space X or simply X will mean a topological space (X, τ) and $\mathcal{P}(X)$ will stand for the power set of X. For a set A in X, $int\,A$ and clA will denote respectively the interior and closure of A.

A subset A of X is said to be semi-open [10] if $A \subseteq clintA$, the complement of a semi-open set is called semi-closed [5]. $SO(X)$ and $SO(x)$ will denote the collections of semi-open sets in X and semi-open sets containing x in X respectively. The semiclosure [4] and semi-θ-closure [11] of $A \subseteq X$ are denoted by $sclA$, $sintA$ and $s(\theta)$-clA respectively and are defined by $sclA = \{x \in X : \forall\ U \in SO(x),\ U \cap A \neq \Phi\}$, $sintA = \{x \in X : \exists\ U \in SO(x), U \subseteq A\}$ and $s(\theta)\text{-}clA = \{x \in X : \forall\, U \in SO(x),\ sclU \cap A \neq \Phi\}$. A set A is semi-θ-closed if $A = s(\theta)$-clA, the complement of such a set is known as semi-θ-open. If $A \in SO(X)$, then clA, $sclA$ and $s(\theta)$-clA are all members of $SO(X)$. For this paper, we require to consider the following results.

Result 1.1. [11, 12] (a) Semiclosure and semi-θ-closure are both idempotent operators.

(b) For any semi-open set U in X, $sclU = s(\theta)$-clU.

Result 1.2. For any subset A of X, $s(\theta)\text{-}clA = \cap\{sclU : A \subseteq U \in SO(X)\}$.

Proof. Indeed, for any $U \in SO(X)$ with $A \subseteq U$ we have, $s(\theta)$-$clA \subseteq s(\theta)$-$clU = sclU$ (by Result 1.1(b)). So, L.H.S. $\subseteq$ R.H.S. Next let, $x \notin s(\theta)$-clA. Then for some $V \in SO(x)$, $sclV \cap A = \Phi$, which implies that $A \subseteq X \setminus sclV = W(\text{say}) \in SO(x)$ such that $sclW = scl(X \setminus sclV) = X \setminus sclV$ (since sclV is semi-open) $\subseteq X \setminus V$. Hence $x \notin sclW$. Thus $x \notin$ R.H.S. $\square$

2. Grills And Associated Concepts

In 1947, Choquet [3] first introduced the concept of grills. Afterwards Thron [16] investigated extensively the theory and structure of grills and many other researchers (e.g. [2]) worked on it in detail to study the closure spaces and proximity spaces. The role of grills is remarkable in the study of proximity spaces and in discussing the compacifications [2]. Let us recall the definition of grills from [3].

Definition 2.1. [3] A nonempty collection $\mathcal{G}$ of subsets of X is called a grill on X if

(i) $\Phi \notin \mathcal{G}$,

(ii) $A \in \mathcal{G}, A \subseteq B \Rightarrow B \in \mathcal{G}$,

(iii) $A \bigcup B \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$.

In this section, we propose to discuss the theory of grills and a few allied concepts to some extent so that the notion of s-closedness can be studied in the subsequent section with these concepts as prerequisite tools. Thus our prime objective is to develop the theory of grills and to observe the interplay between grills and filters as required for this paper.

Definition 2.2. A grill $\mathcal{G}$ on a space X is said to

(a) s-adhere (resp. $s(\theta)$-adhere) at a point x of X, if for each $U \in SO(x)$ and each $G \in \mathcal{G}$, $U \cap G \neq \Phi$ (resp. $sclU \cap G \neq \Phi$).

(b) s-converge (resp. $s(\theta)$-converge) to $x \in X$ if to each $U \in SO(x)$, there corresponds some $G \in \mathcal{G}$ such that $G \subseteq U$ (resp. $G \subseteq sclU$).

Remark 2.3. It is clear from the above definition that a grill $\mathcal{G}$ on a space X is s-convergent (resp. $s(\theta)$-convergent) to a point $x \in X$ iff $\mathcal{G}$ contains the collection $SO(x)$ (resp.$\{sclU : U \in SO(x)\}$).

Definition 2.4. [11] A filter (or a filterbase) $\mathcal{F}$ on a space X is said to s(θ)-adhere at a point x of X (resp. $s(\theta)$-*converge* to x in X), if for each $U \in SO(x)$ and for each $F \in \mathcal{F}$, $F \cap sclU \neq \Phi$ (resp. to each $U \in SO(x)$, there corresponds $F \in \mathcal{F}$ such that $F \subseteq sclU$).

Let us now recall a few terminologies and results from the literature which are required for the rest of the section.

Definition 2.5. [16] For any grill (or a filter) $\mathcal{G}$ on a space X, the section of $\mathcal{G}$, denoted by sec $\mathcal{G}$ is given by sec $\mathcal{G} = \{A \subseteq X : A \cap G \neq \Phi, \forall G \in \mathcal{G}\}$.

Theorem 2.6. [16] (a) *For any grill (filter) $\mathcal{G}$ on a space X, sec $\mathcal{G}$ is a filter (resp. grill) on X.*

(b) *If $\mathcal{F}$ and $\mathcal{G}$ are respectively a filter and a grill on a space X, with $\mathcal{F} \subseteq \mathcal{G}$, then there is an ultrafilter $\mathcal{U}$ on X such that $\mathcal{F} \subseteq \mathcal{U} \subseteq \mathcal{G}$.*

Theorem 2.7. *If a grill $\mathcal{G}$ on a topological space X $s(\theta)$-adhere at some point $x \in X$, then $\mathcal{G}$ is $s(\theta)$-convergent to x.*

Proof. Let $\mathcal{G}$ be a grill on a topological space X, which $s(\theta)$-adheres at $x \in X$. Then for each $U \in SO(x)$ and each $G \in \mathcal{G}$, $sclU \cap G \neq \Phi$ and so, $sclU \in$ sec $\mathcal{G}$, for each $U \in SO(x)$ and hence $X \setminus sclU \notin \mathcal{G}$. Then, $sclU \in \mathcal{G}$ (as $\mathcal{G}$ is a grill on X and $X \in \mathcal{G}$) for each $U \in SO(x)$. Hence $\mathcal{G}$ must $s(\theta)$-converge to x. □

The following example shows that the $s(\theta)$-convergence of a grill does not imply its $s(\theta)$-adherence.

Example 2.8. Let $X = \{a, b, c\}$, $\tau = \{\Phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\mathcal{G} = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Then it is easy to verify that $\mathcal{G}$ is a grill on X and τ is a topology on X such that $SO(X) = \tau$. Now $scl\{a\} = \{a, c\}$, $scl\{b\} = \{b\}$, $scl\{a, b\} = X$ and $scl\{a, c\} = \{a, c\}$. So, $\{sclU : U \in SO(X)\} = \{\{a, c\}, \{b\}, X\} \subseteq \mathcal{G}$. Hence, $\mathcal{G}$ $s(\theta)$-converges to each point x of X but does not $s(\theta)$-adhere at any point $x \in X$.

Let us consider the following notations to be used in the sequel.

Notation 2.9. Let X be topological space. Then for any $x \in X$, we take the following notations
(a) $\mathcal{G}(s(\theta), x) = \{A \subseteq X : x \in s(\theta)\text{-}clA\}$.
(b) sec $\mathcal{G}(s(\theta), x) = \{A \subseteq \mathcal{G} : A \cap G \neq \Phi, \forall G \in \mathcal{G}(s(\theta), x)\}$.

Theorem 2.10. *A grill $\mathcal{G}$ on a space X, $s(\theta)$-adheres at a point $x \in X$, iff $\mathcal{G} \subseteq \mathcal{G}(s(\theta), x)$.*

Proof. A grill $\mathcal{G}$ on a space X, $s(\theta)$-adheres at $x \in X \Rightarrow sclU \cap G \neq \Phi$, $\forall$ $U \in SO(x)$ and $\forall$ $G \in \mathcal{G} \Rightarrow x \in s(\theta)\text{-}clG$, $\forall$ $G \in \mathcal{G} \Rightarrow G \in \mathcal{G}(s(\theta), x)$, $\forall$ $G \in \mathcal{G} \Rightarrow \mathcal{G} \subseteq \mathcal{G}(s(\theta), x)$.

Conversely, let $\mathcal{G} \subseteq \mathcal{G}(s(\theta), x)$. Then for all $U \in SO(x)$ and for all $G \in \mathcal{G}$, $x \in s(\theta)\text{-}clG$. So, for all $U \in SO(x)$ and for all $G \in \mathcal{G}$, $sclU \cap G \neq \Phi$. Hence, $\mathcal{G}$ $s(\theta)$-adheres at x. □

Theorem 2.11. *A grill $\mathcal{G}$ on a topological space X is $s(\theta)$-convergent to a point $x \in X$ iff sec $\mathcal{G}(s(\theta), x) \subseteq \mathcal{G}$*

Proof. Let $\mathcal{G}$ be a grill on X, $s(\theta)$-converging to $x \in X$. Then for each $U \in SO(x)$ there exists $G \in \mathcal{G}$ such that $G \subseteq sclU$ and hence $sclU \in \mathcal{G}$ for each $U \in SO(x) \cdots (i)$.

Now, $B \in$ sec $\mathcal{G}(s(\theta), x) \Rightarrow X \setminus B \notin \mathcal{G}(s(\theta), x) \Rightarrow x \notin s(\theta)\text{-}cl(X \setminus B) \Rightarrow$ there exists $U \in SO(x)$ such that $sclU \cap (X \setminus B) = \Phi \Rightarrow sclU \subseteq B$, where $U \in SO(x) \Rightarrow B \in \mathcal{G}$ (by (i)).

So, sec $\mathcal{G}(s(\theta), x) \subseteq \mathcal{G}$.

Conversely, let if possible, $\mathcal{G}$ do not $s(\theta)$-converge to x. Then for some $U \in SO(x)$, $sclU \notin \mathcal{G}$ and hence $sclU \notin$ sec $\mathcal{G}(s(\theta), x)$. Thus for some $A \in \mathcal{G}(s(\theta), x)$, $A \cap sclU = \Phi \cdots (ii)$. But $A \in \mathcal{G}(s(\theta), x) \Rightarrow x \in s(\theta)$-$clA \Rightarrow sclU \cap A \neq \Phi$ contradicting (ii). □

3. s-Closedness Through Grills

Our purpose in this section is to do some investigations concerning s-closedness of a topological space, on the basis of the discussion in the last section involving grills. We begin with the definition of an s-closed space.

Definition 3.1. [11] A non-empty subset A of a topological space X is said to be s-closed relative to X if for each cover $\{U_\alpha : \alpha \in \Lambda\}$ of A by semi-open sets of X, there exists a finite subset Λ_0 of Λ such that $A \subseteq \bigcup_{\alpha \in \Lambda_0} sclU_\alpha$. If in addition $A = X$, the space X is called an s-closed space.

Some of the various known characterizations of an s-closed space are given as follows.

Theorem 3.2. [11,12,14] *For a topological space (X,τ), the following are equivalent.*

(a) *(X,τ) is s-closed.*

(b) *Every maximal filterbase $s(\theta)$-converges to some point of X.*

(c) *Every filterbase $s(\theta)$-adheres at some point of X.*

(d) *For every family $\{V_\alpha : \alpha \in \Lambda\}$ of semiclosed subsets of X such that $\bigcap\limits_{\alpha\in\Lambda} V_\alpha = \Phi$, there exists a finite subset Λ_0 of Λ such that $\bigcap\limits_{\alpha\in\Lambda_0} sintV_\alpha = \Phi$.*

(e) *For every semi-θ-open cover $\{V_\alpha : \alpha \in \Lambda\}$ of X, there exists a finite subset Λ_o of Λ such that $X = \bigcup\limits_{\alpha\in\Lambda_0} V_\alpha$.*

From part (e) of the above theorem it follows at once that

Theorem 3.3. *For a topological space (X,τ), the following are equivalent:*

(a) *(X,τ) is s-closed.*

(b) *For every family $\{V_\alpha : \alpha \in \Lambda\}$ of semi-θ-closed subsets of X, such that $\bigcap\limits_{\alpha\in\Lambda} V_\alpha = \Phi$, there exists a finite subset Λ_0 of Λ such that $\bigcap\limits_{\alpha\in\Lambda_0} V_\alpha = \Phi$.*

Theorem 3.4. *A subset A of a topological space X is s-closed relative to X iff every grill $\mathcal{G}$ on X with $A \in \mathcal{G}$, $s(\theta)$-converges to a point in A.*

Proof. Let A be s-closed relative to X and $\mathcal{G}$ be a grill on X satisfying $A \in \mathcal{G}$ such that $\mathcal{G}$ does not $s(\theta)$-converge to any point of A. Then to each $a \in A$, there corresponds some $U_a \in SO(a)$ such that $sclU_a \notin \mathcal{G}$. Now, $\{U_a : a \in A\}$ is a cover of A by semi-open sets of X. Then $A \subseteq \bigcup\limits_{i=1}^{n} sclU_{a_i} = K$ (say) for some positive integer n. Since $\mathcal{G}$ is a grill, $K \notin \mathcal{G}$ and hence $A \notin \mathcal{G}$, which is a contradiction.

Conversely, let A be not s-closed relative to X. Then for some cover $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ of A by semi-open sets of X, $\mathcal{F} = \{A \setminus \bigcup\limits_{\alpha\in\Lambda_0} sclU_\alpha : \Lambda_0$ is a finite subset of $\Lambda\}$ is a filterbase on X. Then the family $\mathcal{F}$ can be extended to an ultrafilter $\mathcal{F}^*$ on X. Then $\mathcal{F}^*$ is a grill on X with $A \in \mathcal{F}^*$ (as each $F \in \mathcal{F}$, is a subset of A). Now for each $x \in A$, there must exist $\beta \in \Lambda$ such that $x \in U_\beta$, as $\mathcal{U}$ is a cover of A. Then for any $G \in \mathcal{F}^*$, $G \cap (A \setminus sclU_\beta) \neq \Phi$, so that $G \not\subseteq sclU_\beta$, for all $G \in \mathcal{G}$. Hence $\mathcal{F}^*$ cannot $s(\theta)$-converge to any point of A. This contradiction proves the desired result. □

X being the member of $\mathcal{G}$, the following theorem follows immediately from Theorem 3.4.

Theorem 3.5. *A topological space X is s-closed iff every grill on X is $s(\theta)$-convergent in X.*

Our intention now is to characterize s-closedness in terms of the concept of $s(\theta)$-adherence of grills. We see in the next theorem that such a characterization is possible for a suitable class of grills satisfying a certain condition.

Theorem 3.6. *A topological space X is s-closed iff every grill $\mathcal{G}$ on X satisfying the following condition C_1; $s(\theta)$-adheres in X, where C_1: For every finite subfamily $\{G_1, G_2, \ldots, G_n\}$ of $\mathcal{G}$, $\bigcap_{i=1}^{n} s(\theta)\text{-}clG_i \neq \Phi$.*

Proof. Let $\mathcal{U}$ be any ultrafilter on X. Then $\mathcal{U}$ is a grill on X and also for each finite subcollection $\{U_1, U_2, \ldots, U_n\}$ of $\mathcal{U}$, $\bigcap_{i=1}^{n} s(\theta)\text{-}clU_i \supseteq \bigcap_{i=1}^{n} U_i \neq \Phi$, so that $\mathcal{U}$ is a grill on X with the given condition. Hence by hypothesis, $\mathcal{U}$ $s(\theta)$-adheres. Consequently, by Theorem 3.2 the space X is s-closed.

Conversely, consider any grill $\mathcal{G}$ satisfying the condition C_1 on an s-closed space X. Now for any $A \subseteq X$, $s(\theta)\text{-}clA$ is semi-θ-closed. Thus $\{s(\theta)\text{-}clA$: $A \in \mathcal{G}\}$ is a collection of semi-θ-closed sets in X such that $\bigcap_{i=1}^{n} s(\theta)\text{-}clA_i \neq \Phi$, for any finite subcollection $A_1, A_2, \ldots, A_n$ of $\mathcal{G}$. Then by Theorem 3.3, $\bigcap_{A\in\mathcal{G}} s(\theta)\text{-}clA \neq \Phi$, i.e., there exists $x \in X$ such that $x \in s(\theta)\text{-}clA$ for all $A \in \mathcal{G}$. Hence, $\mathcal{G} \subseteq \mathcal{G}\ (s(\theta), x)$. So by Theorem 2.10, $\mathcal{G}$ $s(\theta)$-adheres at $x \in X$. □

An improved version of the necessity part of the above theorem is given as follows.

Theorem 3.7. *In an s-closed space X every grill $\mathcal{G}$ with the property that $s(\theta)\text{-}clA \cap s(\theta)\text{-}clB \neq \Phi$ for any subset $A, B \in \mathcal{G}$, $s(\theta)$-adheres in X.*

Proof. Let X be s-closed and let $\mathcal{G}$ be any grill on X with the given property such that $\mathcal{G}$ does not $s(\theta)$-adhere in X. Then for each $x \in X$, there exists $G_x \in \mathcal{G}$ such that $x \notin s(\theta)\text{-}clG_x = s(\theta)\text{-}cl(s(\theta)\text{-}clG_x)$ (by Result 1.1(a)). Thus there exists $U_x \in SO(x)$ such that $sclU_x \cap s(\theta)\text{-}clG_x = \Phi$ and hence $s(\theta)\text{-}clU_x \cap s(\theta)\text{-}clG_x = \Phi$ (by Result 1.1(b)). Since $s(\theta)\text{-}clG_x \in \mathcal{G}$ and $\mathcal{G}$ is a grill, $sclU_x = s(\theta)\text{-}clU_x \notin \mathcal{G}$. Now $\{U_x : x \in X\}$ is a cover of X by semi-open sets of X. So, by s-closedness of X, $X = \bigcup_{i=1}^{n} sclU_{x_i}$, for a finite subset $\{x_1, x_2, \ldots, x_n\}$ of X. It then follows that $X \notin \mathcal{G}$ (since $sclU_{x_i} \notin \mathcal{G}$, for $i = 1, 2, \ldots, n$) which is a contradiction. Hence $\mathcal{G}$ must $s(\theta)$-adhere in X. □

4. Semicompactness and s-Closedness

Charles Dorsett [7] introduced the notion of semicompact spaces. According to him a topological space X is semicompact if every cover of X by semi-open sets of X has a finite subcover. It is well known that every semicompact space is an s-closed space but not conversely. In this section we try to investigate for the condition under which the class of s-closed spaces may coincide with the class of semicompact spaces. We first derive a characterization of semicompactnessness in terms of grills.

Theorem 4.1. *A topological space X is semicompact iff every grill s-converges to some point x of X.*

Proof. Let $\mathcal{G}$ be a grill on a topological space X, such that $\mathcal{G}$ does not s-converge to any point $x \in X$. Then for each $x \in X$, there exists $U_x \in SO(x)$ with $U_x \notin \mathcal{G} \cdots (i)$.

As $\{U_x : x \in X\}$ is a cover of the semicompact space X by semi-open sets in X, there exist finitely many points $x_1, x_2, \ldots, x_n$ in X such that $X = \bigcup\limits_{i=1}^{n} U_{x_i} \notin \mathcal{G}$ (since $U_{x_i} \notin \mathcal{G}$ for $i = 1, 2, \ldots, n$) which is a contradiction.

Conversely, let every grill on X s-converge and if possible, let X be not semicompact. Then there exists a cover $\mathcal{U}$ of X by semi-open sets of X having no finite subcover.

Thus $\mathcal{F} = \{X \setminus \cup\, \mathcal{U}_o : \mathcal{U}_0$ is a finite subset of $\mathcal{U}\}$ is a filterbase on X. Then $\mathcal{F}$ is contained in an ultrafilter $\mathcal{F}_0$ and then $\mathcal{F}_0$ is a grill (since every ultrafilter is a grill) on X. By hypothesis $\mathcal{F}_0$ s-converges. Then for some $U \in \mathcal{U}$, $x \in U$ and hence $U \in \mathcal{F}_0$. Again $X \setminus U \in \mathcal{F} \subseteq \mathcal{F}_0$. Thus U and $X \setminus U$ both belong to $\mathcal{F}_0$, which is a filter. So, we arrive at a contradiction. Hence the theorem. □

It is known that in a space satisfying regularity, compactness become equivalent to certain weaker forms of it. We may thus consider a sort of regularity criterion, as given below, by the use of semi-open sets.

Definition 4.2. A topological space X is s-regular, if for each $x \in X$ and each $U \in SO(x)$ there exists $V \in SO(x)$, such that $sclV \subseteq U$.

Now it can be easily seen in a straightforward manner that in an s-regular space, there is no difference between s-closedness and semicompactness of a space, i.e., we have:

Theorem 4.3. *In an s-regular space, s-closed space is semicompact.*

Now we like to infer semicompactness from s-closedness with the assumption of a weaker condition than s-regularity. For this, we now define a sort of strictly weaker form of s-regularity in terms of grills, termed $s(\theta)$-regularity and prove the equivalence of the concepts of s-closedness and semicompactness in the presence of $s(\theta)$-regularity rather than s-regularity.

Definition 4.4. A topological space X is called $s(\theta)$-regular if every grill on X which $s(\theta)$-converges must s-converge (not necessarily to the same point).

Theorem 4.5. *Every s-regular space is $s(\theta)$-regular.*

Proof. Let $\mathcal{G}$ be a grill on an s-regular space X, such that it $s(\theta)$-converges to a point x of X. For each $U \in SO(x)$, there exists by s-regularity of X, some $V \in SO(x)$ such that $sclV \subseteq U$. By hypothesis, $sclV \in \mathcal{G}$. So, $U \in \mathcal{G}$, i.e., $\mathcal{G}$ s-converges to x, proving X to be $s(\theta)$-regular. □

The following example shows that a topological space may be $s(\theta)$-regular without being s-regular i.e., former one is strictly a weaker form of the latter one.

Example 4.6. Let $X = \{a, b, c\}$ and $\tau = \{\Phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. It can be easily verifed that τ is a topology on X and $SO(X, \tau) = \tau$. Now (X, τ) is semicompact (X being a finite set). Hence by Theorem 4.1 every grill on X s-converges. Then X is $s(\theta)$-regular. But the space is not s-regular. Indeed, $\{a\} \in SO(a)$, but $scl(\{a\}) = X \not\subseteq \{a\}$.

Now from the discussion so far, we are in a position to arrive at our desired conclusion as follows.

Theorem 4.7. *A semicompact space is s-closed and the converse is also true if X is $s(\theta)$-regular.*

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(Received 12.12.2007)

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