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HEAT TRANSFER OF THE STEADY MAGNETOHYDRODYNAMIC FLOW OF A CONDUCTING FLUID IN THE NEIGHBORHOOD OF AN INFINITE POROUS PLATE WITH REGARD FOR A STRONG MAGNETIC FIELD

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Abstract. Using the method of successive approximations, by means of Green's function and small parameter, we study heat transfer under steady magnetohydrodynamic flow of conducting fluid in the neighborhood of an infinite porous plate with regard for a strong magnetic field for large values of suction velocity.

Physical characteristics of heat transfer are represented in the form of infinite series with respect to a small parameter. Recurrence relations are given which allow one to write out the solutions in any approximation. The first two approximations are found explicitly.

რეზიუმე. მიმდევრობითი მიახლოების მეთოდით გრინის ფუნქციისა და მცირე პარამეტრის დახმარებით შესაწავლილია უსასრულო ფოროვანი ფირფიტის მახლობლობაში გამტარი სითხის მაგნიტოჰიდროდინამიკური სტაციონარული დინების სითბოგადაცემა ძლიერი მაგნიტური ველის გათვალისწინებით გამოჟონვის სიჩქარის დიდი მნიშვნელობებისათვის.

სითბოგადაცემის ფიზიკური მახასიათებლები მცირე პარამეტრის მიხედვით წარმოდგენილია უსასრულო მწკრივების სახით. ჩაწერილია რეკურენტული თანაფარდობანი, რომლებიც საშუალებას იძლევა ამოვწეროთ ამონახსნები ნებისმიერ მიახლოებაში. ცხადი სახით ნაპოვნია პირველი ორი მიახლოება.

In [1] we studied the motion of a rotating porous plate in a conducting fluid in the presence of a weak magnetic field with regard for heat transfer. A similar problem has been investigated in [2] under simultaneous rotation of a permeable plate and surroundings with different angular velocities in the presence of a weak magnetic field with regard for heat transfer.

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In [3] we investigated a conducting fluid flow in the neighborhood of an infinite porous plate under simultaneous rotation of the plate and surroundings with different angular velocities and strong magnetic field for large suction velocity.

In the present paper, just as in the previous ones, using the method of successive approximations, by means of Green's function and small parameter we study heat transfer of steady MHD-flow of conducting fluid in the neighborhood of an infinite porous plate under simultaneous rotation of the plate and surroundings with different angular velocities, taking into account a strong magnetic field for large values of suction velocity.

To solve the problem, the use will be made of the equation of energy balance for axially symmetric flow in terms of the cylindrical coordinates (r, φ, z) ([4]):

$$\rho c_p\Big(v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\Big)=\lambda\Big[\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}\Big]+$$
$$+\mu\Big\{2\Big(\frac{\partial v_r}{\partial r}\Big)^2+2\Big(\frac{v_r}{r}\Big)^2+2\Big(\frac{\partial v_z}{\partial z}\Big)^2+\Big(\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r}\Big)^2+\Big(\frac{\partial v_\varphi}{\partial z}\Big)^2+$$
$$+\Big(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\Big)^2\Big\}, \tag{1}$$

where v_r, v_φ, v_z are the components of flow velocity, T is temperature, c_p is heat capacity for constant pressure, λ is heat conductivity, and μ is viscosity.

Temperature distribution is sought in the form

$$T(r,z)=T_1(z)+r^2T_2(z). \tag{2}$$

Such a temperature distribution has been first used by I.A. Kibel [5]. Here T_1 and T_2 are the functions of one variable z.

To solve the problem, it is advisable to make the change of variables

$$\begin{cases} v_r=r\omega_0 f(\xi), & v_\varphi=r\omega_0 q(\xi), \quad v_z=\sqrt{v\omega_0}\,[g(\xi)-v_w], \\ & T_1=\dfrac{\mu\omega_0\nu v_w}{\lambda}\,\tau_1(\xi), \quad T_2=\dfrac{\mu\omega_0^2 v_w}{\lambda}\,\tau_2(\xi) \\ z=\sqrt{\dfrac{\nu}{\omega_0}}\,\xi, & u_w=\sqrt{\nu\omega_0}v_w \end{cases} \tag{3}$$

where ω_0 is the characteristic angular velocity, u_w is the suction velocity, ν is kinematic viscosity, f, q, g, τ_1, τ_2 are dimensionless values, and v_w is dimensionless value characterizing the suction velocity.

Taking in equation (1) into account the relations (2) and (3) and introducing the variable $\eta=v_w\xi$, we obtain the following system of equations:

$$\begin{cases} \tau_2''+P_r\tau_2'=\varepsilon[P_r g\tau_2'-q'^2-f'^2]+\varepsilon^2(2P_r f\tau_2), \\ \tau_1''+P_r\tau_1'=\varepsilon[P_r(g\tau_1')-2g'^2]-\varepsilon^2(4\tau_2)-\varepsilon^3(4f^2). \end{cases} \tag{4}$$

where $P_r = \dfrac{\mu c_p}{\lambda}$ is the Prandtle number and $\varepsilon = \dfrac{1}{v_w}$, which for a large suction will be infinitely small; primes denote derivatives with respect to the variable η.

To solve equation (1), we have the following boundary conditions:

$$\begin{cases} z = 0, & T_1 = T_w, \quad T_2 = 0, \\ z = \infty, & T_1 = T_\infty, \quad T_2 = 0, \end{cases}$$

which under our assumptions take the form

$$\begin{cases} \eta = 0; & \tau_1 = \tau_w, \quad \tau_2 = 0, \\ \eta = \infty; & \tau_1 = \tau_\infty, \quad \tau_2 = 0. \end{cases} \tag{5}$$

Here T_w is the plate temperature, and τ_w is the dimensionless value characterizing the plate temperature.

The solution of the problem (1)-(5) can be reduced by means of the Green's function to the solution of the system of integro-differential equations

$$\begin{cases} \tau_2 = & \varepsilon \int\limits_0^\infty [P_r g \tau_2' - q'^2 - f'^2] G(\eta, \zeta) d\zeta + \varepsilon^2 \int\limits_0^\infty (2P_r f \tau_2) G(\eta, \zeta) d\zeta, \\ \tau_1 = & \varepsilon \int\limits_0^\infty [P_r (g\tau_1') - 2g'^2] G(\eta, \zeta) d\zeta - \\ & -\varepsilon^2 \int\limits_0^\infty (4\tau_2) G(\eta, \zeta) d\zeta - \varepsilon^3 \int\limits_0^\infty (4f^2) G(\eta, \zeta) d\zeta, \end{cases} \tag{6}$$

where $G(\eta, \zeta)$ is the Green's function of the problem

$$G'' + P_r G' = 0,$$
$$G(0) = 0, \quad G(\infty) = 0.$$

It has the form

$$G = \begin{cases} G_1 = \dfrac{\overline{e}^{P_r \eta} - 1}{P_r}, & 0 \leq \eta < \eta, \\ G_2 = \dfrac{1 - e^{P_r \zeta}}{P_r} \overline{e}^{P_r \eta}, & \zeta < \eta < \infty. \end{cases}$$

A solution of the system (6) is sought in the form of series with respect to a small parameter ε:

$$\begin{cases} \tau_1 = \tau_{10} + \sum\limits_{i=1}^\infty \varepsilon^{i+2} \tau_{1i}, \\ \tau_2 = \sum\limits_{i=0}^\infty \varepsilon^{4i+5} \tau_{2i}. \end{cases} \tag{7}$$

Substituting expansions of the functions f, q, g into series with respect to a small parameter ε,

$$f=\sum_{i=0}^{\infty}\varepsilon^{4i+2}f_i,\quad q=\sum_{i=0}^{\infty}\varepsilon^{4i}q_i,\quad g=\sum_{i=0}^{\infty}\varepsilon^{4i+3}g_i$$

defined in [3], and the series (7) into the system (6) and then aquating the coefficients for identical powers ε, we obtain the following recurrence formulas:

$$\tau_{10}=B(\eta),$$

$$\tau_{12j}=\int_0^{\infty}\Big[P_r\sum_{\alpha=0}^{j-1}g_\alpha\tau'_{12(j-\alpha-1)}\Big]G(\eta,\zeta)d\zeta,\quad (j\geq 1),$$

for even indices of the function τ_1;

$$\tau_{11}=\int_0^{\infty}(-4\tau_{20})G(\eta,\zeta)d\zeta,$$

$$\tau_{i2j+1}=\int_0^{\infty}\Big[\sum_{\alpha=0}^{j-1}(P_rg_\alpha\tau'_{12j-2\alpha-1}-2g'_\alpha g'_{j-\alpha-1}-4f_\alpha f_{j-\alpha-1})-$$
$$-4\tau_{2j}\Big]G(\eta,\zeta)d\zeta,\quad (j\geq 1),$$

for odd indices of the function tau_1;

$$\tau_{20}=\int_0^{\infty}(-q_0'^2)G(\eta,\zeta)d\zeta,$$

$$\tau_{2j}=\int_0^{\infty}\Big[\sum_{\alpha=0}^{j-1}(2P_rf_\alpha\tau_{2j-\alpha-1}+P_rg_\alpha\tau'_{2j-\alpha-1}-f'_\alpha f'_{j-\alpha-1})-$$
$$-\sum_{\alpha=0}^{j}q'_\alpha q'_{j-\alpha}\Big]G(\eta,\zeta)d\zeta,\quad (j\geq 1),$$

for the function τ_2, where $B(\eta)$ is a solution of the problem

$$B''(\eta)+P_rB'(\eta)=0,$$
$$B(0)=\tau_w,\quad B(\infty)=\tau_\infty.$$

The first two approximations τ_{10}, τ_{11}, τ_{20}, τ_{21} have the form

$$\tau_{10}=(\tau_w-\tau_\infty)\overline{e}^{P_r\eta}+\tau_\infty,$$
$$\tau_{11}=\frac{4n^2(\omega_1-\omega_2)^2}{P_r(P_r-2n)}\Big[\frac{P_r}{2n(P_r-2n)}(e^{-2n\eta}-e^{-P_r\eta}-\eta e^{-P_r\eta})\Big],$$

$$\begin{aligned}
\tau_{20} =& \frac{n^2(\omega_1-\omega_2)^2}{P_r-2n}(e^{-2n\eta}-e^{-P_r\eta}),\\
\tau_{21} =& -\frac{D_1}{4n(P_r-4n)}(e^{-4n\eta}-e^{-P_r\eta})-\\
&-\Big[\frac{D_2}{3n(P_r-3n)}-\frac{(P_r-6n)D_8}{9n^2(P_r-3n)^2}\Big](e^{-3n\eta}-e^{-P_r\eta})\\
&+\Big[\frac{D_3}{2n(P_r-2n)}-\frac{(P_r-4n)D_9}{4n^2(P_r-2n)^2}\Big](e^{-2n\eta}-e^{-P_r\eta})+\\
&+\Big[\frac{D_4}{n(P_r+n)}-\frac{(P_r+2n)D_7}{n^2(P_r+n)^2}\Big](e^{-(P_r+n)\eta}-e^{-P_r\eta})-\\
&-\frac{D_5}{2n(P_r+2n)}(e^{-(P_r+2n)\eta}-e^{-P_r\eta})+D_6\eta e^{-P_r\eta}+\frac{D_7}{n(P_r+n)}\times\\
&\times\eta e^{-(P_r+n)\eta}+\frac{D_8}{3n(P_r-3n)}\eta e^{-3n\eta}-\frac{D_9}{2n(P_r-2n)}\eta e^{-2n\eta}.
\end{aligned}$$

Here we introduce the following notation:

$$\begin{aligned}
D_1 =& \frac{(\omega_1-\omega_2)^4}{3n-1}\Big[\frac{2n(2n+1)P_r}{P_r-2n}-\frac{5n-1}{(3n-1)(4n-1)}\Big],\\
D_2 =& \frac{4\omega_2(\omega_1-\omega_2)^3}{2n-1}\Big[\frac{2(2n-1)(4n-1)}{(3n-1)^2}+\frac{n(n-2)}{P_r-2n}\Big]+\\
&+\frac{2(\omega_1-\omega_2)^4}{3n-1}\Big[\frac{2}{3n-1}-\frac{n(2n+1)P_r}{P_r-2n}\Big],\\
D_3 =& \frac{4\omega_2(\omega_1-\omega_2)^3}{(2n-1)}\Big[\frac{(8n-3)(4n^2+n-1)-(2n-1)(3n-1)}{(2n-1)(3n-1)^2}+\frac{2nP_r}{P_r-2n}\Big]+\\
&+\frac{(\omega_1-\omega_2)^4}{(3n-1)}\Big[\frac{10n-3}{(3n-1)(4n-1)}-\frac{2nP_r}{P_r-2n}\Big]+\frac{4(4n-1)\omega_2^2(\omega_1-\omega_2)^2}{(2n-1)^3},\\
D_4 =& \frac{2P_r(\omega_1-\omega_2)^3}{P_r-2n}\Big[\frac{2\omega_2(P_r-n)^2}{2n-1}+\frac{(P_r+n^2)(\omega_1-\omega_2)}{3n-1}\Big],\\
D_5 =& \frac{P_r(P_r+4n^2)(\omega_1-\omega_2)^2}{(3n-1)(P_r-2n)},\\
D_6 =& \frac{P_r(\omega_1-\omega_2)^3}{P_r-2n}\Big[\frac{4\omega_2}{2n-1}+\frac{\omega_1-\omega_2}{3n-1}\Big],\\
D_7 =& \frac{4nP_r(P_r+n^2)\omega_2(\omega_1-\omega_2)^3}{(2n-1)(P_r-2n)},\\
D_8 =& \frac{4n\omega_2(\omega_1-\omega_2)^3}{2n-1}\Big[\frac{n(n+2)P_r}{P_r-2n}-\frac{2}{3n-1}\Big],\\
D_9 =& \frac{8n\omega_2(\omega_1-\omega_2)^2}{(2n-1)^2}\Big[\frac{n\omega_2}{2n-1}+(\omega_1-\omega_2)\Big],
\end{aligned}$$

where ω_1 and ω_2 are, respectively, the angular velocities of rotation of the plate and surroundings, and $n = \frac{\sqrt{1+4k^2}+1}{2}$, $k^2 = \frac{m^2}{v_w^2}$, $m^2 = \frac{\sigma B_0^2}{\rho\omega_0}$.

For the coefficient of heat transfer we obtain the expression

$$N = -\frac{rv_w}{\tau_w}\Big\{(\tau_\infty - \tau_w)P_r + \frac{2n\varepsilon^3(\omega_1-\omega_2)^2}{P_r} + \frac{r^2\omega_0}{\nu}\Big[\varepsilon^5 n^2(\omega_1-\omega_2)^2 - \\ -\varepsilon^9\Big(\frac{3D_1+4D_2-6D_3}{12n} + \frac{D_4}{P_r+n} - \frac{D_5}{P_r+2n} - D_6 + \\ + \frac{D_7}{(P_r+n)^2} - \frac{4D_8-9D_9}{36n^2}\Big)\Big]\Big\}.$$

The above-obtained solutions show how the suction velocity, parameter of magnetic interaction and angular velocities of rotation of the plate and fluid affect physical characteristics of heat transfer.

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