

THE MIXED PROBLEM OF THE THEORY OF
ELASTICITY FOR A RECTANGLE WEAKENED BY
UNKNOWN EQUIV-STRING HOLES

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ABSTRACT. In the present paper we investigate the mixed problem of the elasticity theory for a rectangle weakened by equiv-strong holes, when the tangential stresses on the outer boundary are equal to zero and the normal displacements are constant, while the tangential stresses on the holes are equal to zero and normal stresses are constant. We have to find the hole boundaries with the condition for the tangential normal stress to take constant values.

Using the methods of the theory of analytic functions, the solution is obtained in quadratures. The graphs of unknown contours are constructed in the case of one hole.

რეზიუმე. შესწავლილია დრეკადობის თეორიის შერეული ამოცანა უცნობი თანაბრადმოტყიცე ხვრელებით შესუსტებული მართკუთხედი-სათვის, როდესაც გარე საზღვარზე მხები ძაბვები ნულის ტოლია და ნორმალური გადაადგილებები მუდმივია, ხოლო ხვრელებზე მხები ძაბვებია ნულის ტოლი, ნორმალური ძაბვები კი - მუდმივია. უნდა მოიძებნოს ხვრელების საზღვრები იმ პირობით, რომ მათზე ტანგენციალური ნორმალური ძაბვა ღებულობდეს მუდმივ მნიშვნელობას.

ანალიზურ ფუნქციათა თეორიის მეთოდების გამოყენებით მიღებულია ამოხსნა კვადრატურებში. ერთი ხვრელის შემთხვევაში აგებულია საძიებელი კონტურების გრაფიკები.

Let a middle surface of an elastic homogeneous isotropic plate occupy on the plane $z = x + iy$ the $n + 1$ -connected domain whose outer boundary are straight lines $x = \pm x_0$ and $y = \pm y_0$, and inner boundary is a union of simple closed smooth contours (a rectangle with holes).

Suppose that on the entire boundary of the domain S the tangential stress $\tau_{xy} = 0$, the normal displacement on the rectangle sides take constant

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values, and the inner boundary is under the action of the constant normal stress $\sigma_n = p$.

Consider the problem: Find a stressed state of the plate, a form and mutual location of the hole contours so as the normal tangential stress σ on them to take constant value.

The problems of the theory of elasticity when the whole boundary or some of its part on the domain S occupied by the body is unknown for finite and infinite domains, have been considered by many authors (see, for e.g., [1]-[5]). These problems are easily solvable in the case of simply connected domains, but the solution of these problems for multiply connected domains presents great difficulties. For example, in the case of a finite doubly-connected domain the problem in [3] is reduced to the Riemann-Hilbert problem for a circular ring. In the other cases, allowing supplementary conditions (some kinds of geometric and force symmetries), these problems can be reduced to the problem of simply connected domains.

In the case under consideration we assume that the domain S is symmetric with respect to the ox -axis. This allows us to consider a part of the domain S lying in the upper half-plane. We denote it by S_1 (Fig. 1).

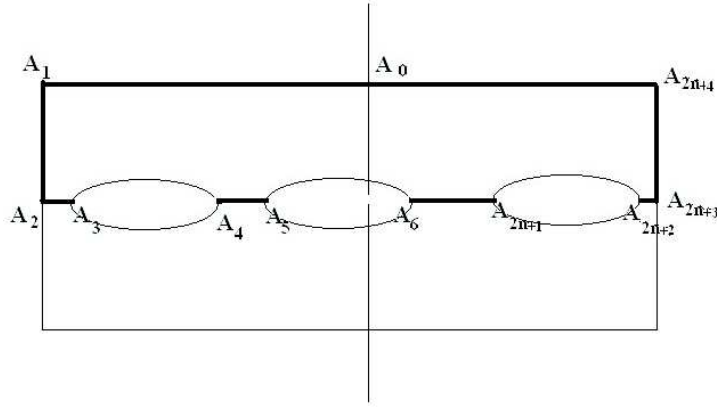


Fig. 1

Owing to the symmetry, for the domain S_1 we have the same boundary conditions as for S . The mathematical ground of this fact can be found in [8].

Thus for the domain S_1 we obtain the following boundary conditions:

$$\begin{aligned} \tau_{tn} = 0, \quad u_n = u(t) \quad \text{on} \quad L_1 + L_2, \\ \tau_{tn} = 0, \quad \sigma_n = p, \quad \sigma_t = k \quad \text{on} \quad L_0, \end{aligned} \quad (1)$$

where $u(t)$ is the given piecewise constant function, p is the given and k is an unknown constant. The value k will be defined later on. L_0 is a union of unknown arcs, L_1 and L_2 are the unions of segments which are parallel with respect to the ox - and oy -axes.

$$\begin{aligned} L_1 &= \bigcup_{k=0}^{n+2} A_{2k} A_{2k+1}, & L_2 &= \bigcup_{k=0}^1 A_{(2n+2)k+1} A_{(2k+2)k+2}, \\ L_0 &= \bigcup_{k=1}^n A_{2k+1} A_{2k+2}, & A_0 &= A_{2n+5} = y_0 i, \quad A_1 = -x_0 + y_0 i, \\ A_2 &= -x_0, \quad A_{2n+3} = x_0, \quad A_{2n+4} = x_0 + y_0 i \end{aligned}$$

(The points and their affixes are denoted by one and the same symbols).

As is known ([6]), the stressed state of the body is described by two analytic functions $\Phi(z)$ and $\Psi(z)$ which on the boundary of the domain satisfy the conditions

$$\Phi(t) + \overline{\Phi(t)} - e^{2i\alpha(t)} [\bar{t} \Phi'(t) + \Psi(t)] = \sigma_n - i\tau_{tn}, \quad (2)$$

$$\chi \Phi(t) - \overline{\Phi(t)} + e^{2i\alpha(t)} [\bar{t} \Phi'(t) + \Psi(t)] = 2\mu(u'_t + iu'_n), \quad (3)$$

$$4 \operatorname{Re} \Phi(t) = \sigma_n + \sigma_t, \quad (4)$$

where $\alpha(t)$ is the angle between the outer normal to the boundary of the domain S_1 at the point t and the ox -axis; ζ and μ are the elastic constants; $u'_t = \frac{du_t}{ds}$ and $u'_n = \frac{du_n}{ds}$. In our case, $u'_n = 0$; $\alpha(t) = 0$ or $\alpha(t) = \pi$, if $t \in L_2$, $\alpha(t) = \frac{\pi}{2}$ or $\alpha(t) = -\frac{\pi}{2}$, if $t \in L_1$, and $\alpha(t)$ is unknown if $t \in L_0$.

The functions $\Phi(z)$ and $\Psi(z)$ satisfy in the neighborhoods of the points A_k ($k = 1, 2, \dots, 2n+4$) the following conditions:

$$|\Phi(z)| < \frac{\operatorname{const}}{|z - c|^\delta}, \quad |\Psi(z)| < \frac{\operatorname{const}}{|z - c|^\delta}, \quad (5)$$

where $0 \leq \delta < 1$.

Let the function $z = \omega(\zeta)$ map conformally the upper half-plane of the complex variable $\zeta = \xi + i\eta$ onto the domain S_1 . Here we use the following notation: $\omega^{-1}(A_k) = a_k$ ($k = 0, 1, \dots, 2n+5$), $\omega^{-1}(L_k) = l_k$, $k = 0, 1, 2$. We may assume that $a_0 = -\infty$, $a_{2n+5} = +\infty$, $a_2 = -1$ and $a_{2n+3} = 1$.

Since the contours of the hole are smooth, the function $\omega(\zeta)$ in the neighborhoods of the points A_k ($k = 1, 2, \dots, 2n+4$) is representable in the form

$$\omega(\zeta) = (\zeta - c)^{\frac{1}{2}} \omega_0(\zeta) + \omega(c), \quad (6)$$

where $\omega_0(\zeta)$ is the function, holomorphic in the neighborhood of the point c , and $\omega_0(c) \neq 0$ (c denotes any of the points a_k ($k = 1, 2, \dots, 2n+4$)). In the neighborhood of the point $\zeta = \infty$ we have

$$\omega(\zeta) = c_0 + \frac{c_1}{\zeta} + \frac{c_2}{\zeta^2} + \dots, \quad (7)$$

where c_k ($k = 0, 1, \dots$) are the constants.

The change of the variable $z = \omega(\zeta)$ results in

$$e^{2i\alpha(t)} = -\frac{\omega'(\xi)}{\overline{\omega'(\xi)}}. \quad (8)$$

Taking in (2), (3) and (4) into account the fact that $z = \omega(\zeta)$ and also the condition (1), we easily obtain

$$\begin{aligned} \operatorname{Im} \Phi_0(\xi) &= 0 \quad \text{on } l_1 + l_2, \\ \operatorname{Re} \Phi_0(\xi) &= \frac{1}{4}(k+p) \quad \text{on } l_0, \end{aligned}$$

where $\Phi_0(\zeta) = \Phi(\omega(\zeta))$.

Taking into consideration (5), (6) and (7), we establish that $\Phi_0(\zeta)$ is bounded as $\zeta \rightarrow \infty$ and in the neighborhood of the points a_k ($k = 1, 2, \dots, 2n+4$) satisfies the conditions

$$|\Phi_0(\zeta)| < \frac{\operatorname{const}}{|\zeta - c|^\delta}, \quad 0 \leq \delta < \frac{1}{2}. \quad (9)$$

Consider the function $F(\zeta) = \Psi_0(\zeta) - \frac{1}{4}(k+p)$. To define $F(\zeta)$, in view of (9), we get

$$\begin{aligned} \operatorname{Im} F(\xi) &= 0 \quad \text{on } l_1 + l_2, \\ \operatorname{Re} F(\xi) &= 0 \quad \text{on } l_0. \end{aligned} \quad (10)$$

$F(\zeta)$ has the same estimates as $\Phi_0(\zeta)$.

(11) is, in fact, the homogeneous Keldysh-Sedov's problem for the half-plane, and under the above conditions this problem has only a trivial solution $F(\zeta) = 0$. Therefore

$$\Phi_0(\zeta) = \frac{1}{4}(k+p). \quad (11)$$

Taking into account (12) and (8), the boundary conditions (2) and (3) take the form

$$a \cdot \omega(\zeta) + \overline{\omega'(\xi) \cdot \Psi_0(\xi)} = 0 \quad \text{on } l_0, \quad (12)$$

$$\operatorname{Im} \Psi_0(\xi) = 0 \quad \text{on } l_1 + l_2. \quad (13)$$

Obviously,

$$\operatorname{Re} \omega(\xi) = A(\xi) \quad \text{on } l_2, \quad \operatorname{Im} \omega(\xi) = B(\xi) \quad \text{on } l_1, \quad (14)$$

where $a = \frac{1}{2}(k-p)$, $A(\xi)$ and $B(\xi)$ are the piecewise constant functions, in particular, $A(\xi) = x_0$ for $\xi \in (a_{2n+3}, a_{2n+4})$, $A(\xi) = -x_0$, for $\xi \in (a_1, a_2)$, $B(\xi) = y_0$ for $\xi \in (-\infty, a_1) \cup (a_{2n+4}, +\infty)$, $B(\xi) = 0$ for $\xi \in \bigcup_{k=1}^{n+2} (a_{2k}, a_{2k+1})$

Introduce the notation

$$U(\zeta) = \frac{1}{2}\omega'(\zeta)\left[1 + \frac{1}{a}\Psi_0(\zeta)\right], \quad V(\zeta) = \frac{1}{2}\omega'(\zeta)\left[1 - \frac{1}{a}\Psi_0(\zeta)\right]. \quad (15)$$

It is not difficult to state that $|\zeta| \rightarrow \infty$, as

$$U(\zeta) = O(\zeta^{-2}), \quad V(\zeta) = O(\zeta^{-2}), \quad (16)$$

and in the neighborhood of the points a_k ($k = 1, 2, \dots, 2n+4$) they satisfy the conditions

$$|U(\zeta)| \leq \frac{\text{const}}{|\zeta - c|^\delta}, \quad |V(\zeta)| \leq \frac{\text{const}}{|\zeta - c|^\delta}, \quad 0 \leq \delta \leq \frac{1}{2}. \quad (17)$$

Taking into account (13)–(18), to find $U(\zeta)$ and $V(\zeta)$ we obtain the boundary problem: Find the functions $U(\zeta)$ and $V(\zeta)$, analytic in the upper half-plane ($\eta > 0$), which on the axis $\eta = 0$ satisfy the boundary conditions

$$\text{Re } U(\xi) = 0 \quad \text{on } l_0 + l_2, \quad \text{Im } U(\xi) = 0 \quad \text{on } l_1, \quad (18)$$

$$\text{Im } V(\xi) = 0 \quad \text{on } l_0 + l_1, \quad \text{Re } V(\xi) = 0 \quad \text{on } l_2, \quad (19)$$

and also the conditions (17) and (16) in the neighborhoods of the points $\zeta = \infty$ and a_k ($k = 1, 2, \dots, 2n+4$).

The problems (17)–(20) are, in fact, the homogeneous Keldysh-Sedov's problems for a half-plane. The solution of these problems is given by the formulas ([7])

$$U(\xi) = X_1(\zeta) P_n(\zeta), \quad V(\xi) = X_2(\zeta) Q_0(\zeta), \quad (20)$$

where

$$\begin{aligned} X_1(\zeta) &= \left[\prod_{k=1}^{2n+4} (\zeta - a_k) \right]^{-\frac{1}{2}}, \\ X_2(\zeta) &= \left[\prod_{k=0}^1 (\zeta - a_{(2n+2)k+1}) (\zeta - a_{(2n+2)k+2}) \right]^{-\frac{1}{2}}, \end{aligned} \quad (21)$$

$P_n(\zeta) = \sum_{k=0}^n C_k \zeta^k$, $Q_0(\zeta) = D_0$, $C_0, C_1, \dots, C_n$ and D_0 are the real constants, under $X_1(\zeta)$ and $X_2(\zeta)$ are understood the one-valued branches satisfying the conditions $X_k(\zeta) < 0$ ($k = 1, 2$), when $\eta = 0$ and $\xi > a_{2n+4}$.

From formulas (16) we find that

$$\omega'(\zeta) = U(\zeta) + V(\zeta), \quad \Psi_0(\zeta) = a \frac{U(\zeta) - V(\zeta)}{U(\zeta) + V(\zeta)}. \quad (22)$$

Taking into account the fact that $\Psi_0(\zeta) = \Psi(\omega(\zeta)) = \psi'(\omega(\zeta))$ and $\Phi_0(\zeta) = \Phi(\omega(\zeta)) = \varphi'(\omega(\zeta))$, from (12) and (23) we obtain

$$\begin{aligned}\omega(\zeta) &= \int_{\zeta_0}^{\zeta} [U(\xi) + V(\xi)] d\xi + \omega(\zeta_0), \\ \psi(\omega(\zeta)) &= a \int_{\zeta_0}^{\zeta} [U(\xi) - V(\xi)] d\xi + \psi(\omega(\zeta_0)), \\ \varphi(\omega(\zeta)) &= b\omega(\zeta) + C,\end{aligned}\tag{23}$$

where ζ_0 is an arbitrary point of the domain S_1 , $b = \frac{1}{4}(k+p)$, and C is the constant to be defined.

Consider in detail the following particular cases.

1. Two Holes. Suppose that the domain S is likewise symmetric with respect to the oy -axis. In this case we can put $n = 2$, $\zeta_0 = 0$, $a_5 = -a_4 = \lambda_1$, $a_6 = -a_3 = \lambda_2$, $a_7 = -a_2 = 1$, $a_8 = -a_1 = \gamma$, $\omega(0) = 0$, $\omega(-\infty) = \omega(+\infty) = iy_0$.

λ_1 , λ_2 and γ are the unknown real constants. Taking all this into account, from formulas (22) for $X_1(\zeta)$ and $X_2(\zeta)$ we obtain

$$\begin{aligned}X_1(\zeta) &= [(\zeta^2 - \lambda_1^2)(\zeta^2 - \lambda_2^2)(\zeta^2 - 1)(\zeta^2 - \gamma^2)]^{-\frac{1}{2}}, \\ X_2(\zeta) &= [(\zeta^2 - 1)(\zeta^2 - \gamma^2)]^{-\frac{1}{2}}.\end{aligned}\tag{24}$$

It can be easily verified that

$$X_1(-\xi) = \overline{X_1(\xi)}, \quad X_2(-\xi) = \overline{X_2(\xi)}.\tag{25}$$

Owing to the symmetry, we have $\omega(-\xi) = \overline{-\omega(\xi)}$. Taking in these equalities into account the first one, from (24) and (26) we obtain $P_2(-\xi) = P_2(\xi)$. Thus $C_1 = 0$, $P_2(\zeta) = C_0 + C_2\zeta^2$ and

$$\omega(\zeta) = \int_0^{\zeta} [(C_0 + C_2\xi^2)X_1(\xi) + D_0X_2(\xi)] d\xi.\tag{26}$$

For the sake of definiteness, we assume that $\omega'(0) > 0$ i.e., $\arg(\omega'(0)) = 0$. Then we should have $\text{Im}(\omega(\lambda_1)) = 0$ and $\text{Im}(\omega(\lambda_2)) = 0$. These conditions imply that $C_0 = -C_2m_1$, where

$$m_1 = \int_{\lambda_1}^{\lambda_2} \xi^2 X_1(\xi) d\xi \cdot \left[\int_{\lambda_1}^{\lambda_2} X_1(\xi) d\xi \right]^{-1}.\tag{27}$$

Here $\lambda_1 < m_1 < \lambda_2$. Moreover, $X_2(\xi) > 0$ for $\xi \in (\lambda_1, \lambda_2)$, hence $D_0 > 0$. On the other hand, since $X_1(0) < 0$, therefore $C_0 < 0$, and we find that

$C_2 > 0$. Taking all this into account in (24)–(27) and introducing the notation $\frac{D_2}{C_2} = m_2 > 0$, $C_2 = R > 0$, we finally get

$$\begin{aligned}\omega(\zeta) &= R \int_0^\zeta [(\xi^2 - m_1)X_1(\xi) + m_2 \cdot X_2(\xi)] d\xi, \\ \varphi(\omega(\zeta)) &= b\omega(\zeta) + C, \\ \psi(\omega(\zeta)) &= a \int_0^\zeta [(\xi^2 - m_1)X_1(\xi) + m_2 \cdot X_2(\xi)] d\xi + C_1,\end{aligned}\tag{28}$$

where C and C_1 are the constants.

The well-known Kolosov-Muskhelishvili's formulas ([6]) yield

$$\begin{aligned}\operatorname{Re} e^{-i\alpha(t)} (\chi\varphi(t) - t\overline{\varphi'(t)} - \overline{\psi(t)}) &= 2\mu u_n \quad \text{on } L_1 + L_2, \\ \varphi(t) + t\overline{\varphi'(t)} + \overline{\psi(t)} &= pt \quad \text{on } L_0.\end{aligned}\tag{29}$$

If we take into account that $t = \omega(\xi)$, the above formulas will take the form

$$\begin{aligned}\operatorname{Re} ((\chi - 1)b\omega(\xi) + \chi C - \overline{\psi(\omega(\xi))}) &= 2\eta u_1 \quad \text{for } \xi \in (1, \gamma), \\ a\omega(\xi) + C + \overline{\psi(\omega(\xi))} &= 0 \quad \text{for } \xi \in (\lambda_1, \lambda_2), \\ \operatorname{Im} ((\chi - 1)b\omega(\xi) + \chi C - \overline{\psi(\omega(\xi))}) &= f(\xi) \\ &\quad \text{for } \xi \in (0, \lambda_1) \cup (\lambda_2, 1) \cup (\gamma, +\infty),\end{aligned}\tag{30}$$

where $f(\xi) = 2\mu u_2$ for $\xi \in (\gamma, +\infty)$ and $f(\xi) = 0$ for $\xi \in (0, \lambda_1) \cup (\lambda_2, 1)$, $b = \frac{1}{4}(k + p)$.

From the conditions (23) we immediately find that $\operatorname{Im} C = 0$ and $C_1 = -a\omega(\lambda_1) - C$. Since $\operatorname{Im}(a\omega(\lambda_1)) = 0$, therefore

$$\psi(\omega(\zeta)) = a \int_0^\zeta [(\xi^2 - m_1)X_1(\xi) - NX_2(\xi)] d\xi - a\omega(\lambda_1) - C,$$

where C is the real constant.

The constants k and C can by virtue of (3) be expressed in terms of λ_1 , λ_2 and γ as follows:

$$k = -p \frac{K_0}{K} + u_2 \frac{8\mu}{K},\tag{31}$$

Here

$$K = (\chi - 1)y_0 + 2 \int_1^\gamma [(\xi^2 - m_1)|X_1(\xi)| - m_2|X_2(\xi)|] d\xi,$$

$$\begin{aligned}
K_0 &= (\chi - 1)y_0 - 2 \int_1^\gamma [(\xi^2 - m_1)|X_1(\xi)| - m_2|X_2(\xi)|] d\xi, \\
C &= \frac{1}{\chi + 1} \left(2\mu u_1 - \frac{1}{4}(k + p)(\chi - 1)x_0 + \right. \\
&\quad \left. + \frac{1}{2}(k - p) \int_0^1 [(\xi^2 - m_1)X_1(\xi) - m_2X_2(\xi)] d\xi \right). \tag{32}
\end{aligned}$$

Taking into account that $\int_1^\gamma |\omega'(\xi)| = y_0$ and $\int_\gamma^\infty |\omega'(\xi)| = x_0$, we obtain

$$\begin{aligned}
R &= x_0 \left(\int_\gamma^\infty [(\xi^2 - m_1)|X_1(\xi)| + m_2|X_2(\xi)|] d\xi \right)^{-1}, \tag{33} \\
m_2 &= \left(R_0 \int_1^\gamma (\xi^2 - m_1)|X_1(\xi)| d\xi - \int_\gamma^\infty (\xi^2 - m_1)|X_1(\xi)| d\xi \right) \times \\
&\quad \times \left(\int_\gamma^\infty |X_2(\xi)| d\xi - R_0 \int_1^\gamma |X_2(\xi)| d\xi \right)^{-1},
\end{aligned}$$

where $R_0 = \frac{x_0}{y_0}$.

Formulas (30), (34) and (35) provide us with the solution of the above-formulated problem.

For different values of the constants λ_1 , λ_2 and γ satisfying the conditions $0 < \lambda_1 < \lambda_2 < 1 < \gamma$, $m_2 > 0$ we obtain different variants of equi-strong hole contours.

2. One Hole. Suppose that in this case the domain occupied by the body is symmetric with respect to the ox - and oy -axes. Thus we can put $n = 1$, $\zeta_0 = 0$, $a_4 = -a_3 = \lambda$, $a_5 = -a_2 = 1$, $a_6 = -a_1 = \gamma$, $\omega(0) = ir$, $\omega(-\infty) = \omega(+\infty) = iy_0$. Here r , λ and γ are the unknown real constants. The functions $X_1(\zeta)$ and $X_2(\zeta)$ are expressed by the formulas

$$\begin{aligned}
X_1(\zeta) &= [(\zeta^2 - \lambda^2)(\zeta^2 - 1)(\zeta^2 - \gamma^2)]^{-\frac{1}{2}}, \\
X_2(\zeta) &= [(\zeta^2 - 1)(\zeta^2 - \gamma^2)]^{-\frac{1}{2}}.
\end{aligned}$$

Again, we take here those branches for which $X_1(\xi), X_2(\xi) < 0$ for $\xi > \gamma$.

It can be easily verified that the relations $X_1(-\xi) = -\overline{X_1(\xi)}$ and $X_2(-\xi) = \overline{X_2(\xi)}$ hold, and therefore $P_1(\zeta) = C_1\zeta$ ($C_0 = 0$). This result can likewise be obtained from the condition (28) owing to which $m_1 = 0$. Taking all this into account, we obtain the formulas, analogous to formulas

(30):

$$\begin{aligned}\omega(\zeta) &= R \int_0^\zeta [\xi X_1(\xi) + m_2 \cdot X_2(\xi)] d\xi + ir, \quad \varphi(\omega(z)) = b\omega(\zeta) + C, \\ \psi(\omega(\zeta)) &= a \int_0^\zeta [\xi X_1(\xi) + m_2 \cdot X_2(\xi)] d\xi - a\omega(\lambda) - C.\end{aligned}\quad (34)$$

In our case $\text{Im } \omega(\lambda) = 0$, whence we have

$$r = R \int_0^\lambda \xi |X_1(\xi)| d\xi. \quad (35)$$

Applying the conditions (31) to our case, formulas (33), (34) and (35) take the form

$$\begin{aligned}k &= -p \frac{K_0}{K} + u_2 \frac{8\mu}{K}, \\ K &= (\chi - 1)y_0 + 2 \int_1^\gamma [\xi |X_1(\xi)| - m_2 |X_2(\xi)|] d\xi, \\ K_0 &= (\chi - 1)y_0 - 2 \int_1^\gamma [\xi |X_1(\xi)| - m_2 |X_2(\xi)|] d\xi, \\ C &= \frac{1}{\chi + 1} \left(2\mu u_1 - \frac{1}{4}(k + p)(\chi - 1)x_0 + \right. \\ &\quad \left. + \frac{1}{2}(k - p) \int_0^1 [\xi X_1(\xi) - m_2 X_2(\xi)] d\xi \right), \\ R &= x_0 \left(\int_\gamma^\infty [\xi |X_1(\xi)| + N |X_2(\xi)|] d\xi \right)^{-1}, \\ m_2 &= \left(R_0 \int_1^\gamma \xi |X_1(\xi)| d\xi - \int_\gamma^\infty \xi |X_1(\xi)| d\xi \right) \times \\ &\quad \times \left(\int_\gamma^\infty |X_2(\xi)| d\xi - R_0 \int_1^\gamma |X_2(\xi)| d\xi \right)^{-1}.\end{aligned}\quad (36)$$

Formulas (36), (37) and (38) provide us with the solution of the above-formulated problem.

For different values of the constants λ and γ satisfying the conditions $0 < \lambda < 1 < \gamma$, $m_2 > 0$, we obtain different variants of equiv-strong hole contours.

Assume there is a square $x_0 = y_0$, and also $u_1 = u_2 = u$. Then there exists the cyclic symmetry with the angle $\beta = \frac{\pi}{2}$ (a picture of stress distributions does not change if we turn the coordinate system by the angle, equal to β). As is known ([4]), the functions Φ and Ψ in this case satisfy the conditions $\Phi(e^{i\beta}z) = \Phi(z)$ and $\Psi(e^{i\beta}z)e^{2i\beta} = \Psi(z)$. Using the second formula from the above ones, we obtain $\Psi(\omega(1)) = -\Psi(\infty)$ and $\Psi(-\gamma) = -\Psi(\gamma)$. These relations allow us to find $\gamma = \sqrt{1 + \sqrt{1 - \lambda^2}}$ and $m_2 = \frac{1}{\sqrt[3]{1 - \lambda^2}}$. Thus it remains only one parameter λ , $0 < \lambda < 1$.

The structure of mapping functions makes it possible to see that the form of the equiv-strong contour does not depend on whether the stressed state is induced by the action of normal stress $\sigma_n = p$ on the hole contour, or by normal displacement $u_n = u$ on the square sides, or by the both factors together.

It should also be noted that we can choose σ_n and u_n is such a way that on the hole boundary σ_1 is equal to zero.

In Fig. 2 we can see equiv-strong contours corresponding to different values of λ . Our calculations show that for fixed values of the elastic constants and sides length of the square the value $\sigma_1 = k$ decreases as the hole size increases, and vice versa, it increases as the hole size decreases (the hole size is denoted by r). In the figure are given the values of r and the corresponding values of k for $x_0 = 5$, $p = 1$, $u = 0$, $\chi = 2$ and $\mu = 10^5$.

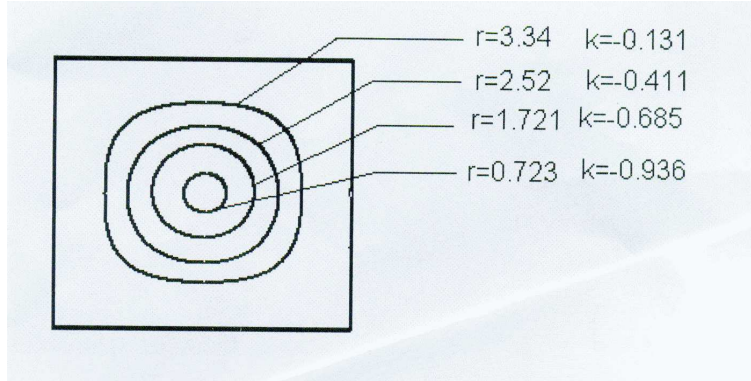


Fig. 2

All calculations have been performed on the Personal Computer by using the program Mathcad 2000 professional.

$$\lambda := 0.999, \quad \gamma := \sqrt{1 + \sqrt{1 - \lambda^2}}, \quad x_0 := 5, \quad p := 1, \quad \chi := 2,$$

$$\begin{aligned}
u &:= 0, \quad \mu := 10^5, \quad X_1(x) := [(x^2 - \lambda^2)(x^2 - 1)(x^2 - \gamma^2)]^{-0.5}, \\
X_2(x) &:= [(x^2 - 1)(x^2 - \gamma^2)]^{-0.5}, \quad m_2 := (1 - \lambda^2)^{-0.2}, \\
I(x, y, j) &:= \left[\int_x^y (x|X_1(x)| + jm_2|X_2(x)|) dx \right], \\
R &:= x_0 I(\gamma, \infty, 1)^{-1}, \quad q := 2I(1, \gamma, -1)[I(1, \gamma, 1)(x - 1)]^{-1}, \\
q_1 &:= 8\mu u[(\chi - 1)x_0 + 2RI(1, \gamma, -1)]^{-1}, \\
k &:= -p(1 - q)(1 + q)^{-1} + q_1, \quad t := 0,001 \dots \lambda, \\
x(t) &:= Rm_2 \int_0^1 |X_2(t)| dt, \quad y(t) := R \int_0^\lambda t|X_1(t)| dt - R \int_0^t t|X_1(t)| dt.
\end{aligned}$$

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