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ON THE GLOBAL SOLVABILITY OF THE CAUCHY CHARACTERISTIC PROBLEM FOR ONE CLASS OF NONLINEAR SECOND ORDER HYPERBOLIC SYSTEMS

In the Euclidean space $\mathbb{R}^{n+1}$ of the independent variables $x_1, x_2, \dots, x_n, t$ consider a nonlinear hyperbolic system of the form

$$\Box u_i + \lambda \frac{\partial}{\partial u_i} G(u_1, \dots, u_N) = F_i(x,t), \ \ i = 1, \dots, N, \tag{1}$$

where λ is a given real constant, G is a given real scalar function, $F = (F_1, \dots, F_N)$ is a given, and $u = (u_1, \dots, u_N)$ is an unknown real vector-functions, $n \geq 2$, $N \geq 2$, $\Box := \frac{\partial^2}{\partial t^2} - \Delta$, $\Delta := \sum\limits_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$. We assume that $G \in C^2(\mathbb{R}^N)$.

Consider the Cauchy characteristic problem on finding in the light cone of the future $D_T : t > |x|$ a solution $u(x,t)$ of the system (1) by the boundary condition

$$u|_{\partial D} = 0, \tag{2}$$

where $\partial D : t = |x|$ is the conic surface, characteristic for the system (1).

Definition. The system (1) can be rewritten in the form of one vector equation

$$L_\lambda u := \Box u + \lambda \nabla_u G(u) = F(x,t),$$

where $u = (u_1, \dots, u_N), F = (F_1, \dots, F_N), \nabla_u = (\frac{\partial}{\partial u_1}, \dots, \frac{\partial}{\partial u_N})$. Let $F \in L_{2,\mathrm{loc}}(D)$ and $F\big|_{D_T} \in L_2(D_T)$ for any $T > 0$, where $D_T = D \cap \{t < T\}$. The vector-function $u = (u_1, \dots, u_N) \in \overset{0}{W}{}^1_{2,\mathrm{loc}}(D)$ is called a global strong generalized solution of the problem (1), (2) of the class W_2^1, if for any $T > 0$ $u|_{D_T}$ belongs to the space $\overset{0}{W}{}^1_2(D_T, S_T) := \{u \in W_2^1(D_T) : u|_{S_T} = 0\}$, where $W_2^1(D_T)$ is the well-known Sobolev space, $S_T = \partial D \cap \{t \leq T\}$ and there exists a sequence of vector-functions $u^m \in \overset{0}{C}{}^2(\overline{D}_T, S_T) := \{u \in C^2(\overline{D}_T) : u|_{S_T} = 0\}$ such that $u^m \to u$ in the space $\overset{0}{W}{}^1_2(D_T, S_T)$ and $L_\lambda u^m \to F$ in the space $L_2(D_T)$.

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For the function G from the system (1) consider the following conditions

$$G(u) \geq -M_1|u|^2 - M_2,\ |u| = ||u||_{\mathbb{R}^n},\ M_i = const \geq 0,\ i = 1, 2, \quad (3)$$

$$\Big|\frac{\partial^2}{\partial u_i \partial u_j} G(u)\Big| \leq a + b|u|^\gamma,\ 1 \leq i, j \leq N;\ a, b, \gamma = const \geq 0. \quad (4)$$

Theorem. *Let $\lambda > 0$, $0 \leq \gamma < \frac{2}{n-1}$ and the conditions (3), (4) be fulfilled. Then for any $F \in L_{2,\mathrm{loc}}(D_\infty)$ such that $F\big|_{D_T} \in L_2(D_T)$ for each $T > 0$, the problem (1), (2) has a unique global strong generalized solution $u \in \overset{0}{W}{}^1_{2,\mathrm{loc}}(D)$ of the class W_2^1.*

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