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FINITE DIFFERENCE SCHEMES FOR BENJAMIN-BONA-MAHONY EQUATION

We consider an initial boundary-value problem for the Benjamin–Bona–Mahony (or Regularized Long Wave) equation

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}+\nu u\frac{\partial u}{\partial x}-\mu\frac{\partial^3 u}{\partial x^2\partial t}=0, \tag{1}$$

where ν and μ are positive constants, A three level one-parameter family of conservative difference schemes is studied on 9-point template. A two level scheme is used to find the values of the unknown functions on the first level. The obtained algebraic equations are linear with respect to the values of the unknown function for each new level. It is proved that the finite difference scheme converges with the rate $O(\tau^2+h^2)$ when the exact solution belongs to the Sobolev space W_2^3.

Physical boundary conditions require $u\to 0$ for $x\to\pm\infty$. While numerically solving the problem initially formulated on an unbounded domain, one typically truncates this domain, which necessitates setting the *artificial boundary conditions* (ABCs) at the newly formed external boundary. In the ideal case, the ABCs would be specified so that the solution on the truncated domain coincides with the corresponding fragment of the original infinite domain solution.

For the numerical solution of the equation (1) due to the initial condition $u(x,0)=u_0(x)$, artificial boundaries are chosen at some points $x=a,\ x=b,\ a<b$ and in the domain $Q_T:=(a,b)\times(0,T)$, the initial boundary-value problem with the following conditions

$$u(a,t)=u(b,t)=0,\ t\in[0,T),\quad u(x,0)=u_0(x),\ x\in[a,b]. \tag{2}$$

can be considered.

In the present paper, a numerical method for the selection of artificial boundary conditions is given, since the Dirichlet boundary conditions are zeros only within certain accuracy.

A certain value of a free parameter of the scheme is also given, which ensures a good accuracy of the approximate solution.

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Numerical experiments are carried out in order to check the accuracy, conservativity and convergence rate of method. Some calculations concerning interaction of two solitary waves are also done. In this case attention is given to the situation wherein a large solitary wave overtakes a smaller solitary wave.

An observer is easily able to distinguish the two individual solitons before their interaction, and again after their interaction. But during the collision this clear identity becomes confused [1].

When the higher wave overtakes the lower one, it appears that the first simply goes through the second and moves on ahead. In reality this is not the case. When such waves touch each other, the larger one slows down and diminishes while the smaller one accelerates and increases. When the smaller wave becomes as large as the original one, the waves detach and the ex-smaller one, which now is higher and faster, goes forward while the ex-bigger one is now moving behind at lower speed. The observable result of the collision is that the larger wave appears to have shifted ahead of the position that it would have occupied in uniform motion (without the collision), while the smaller one appears correspondingly to have shifted backwards. [2, p.35]

Fig.1 displays the trajectory of the peak of the large solitary wave and resulting phase shift, which corresponds to the critical value t_*.

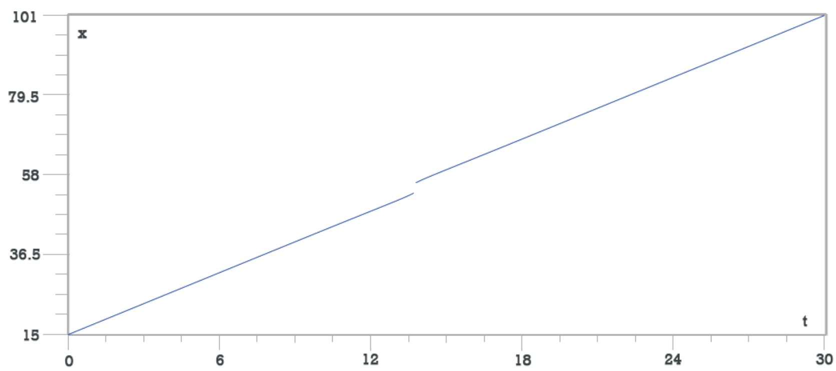

FIGURE 1. Trajectory of the peak of the large soliton

In order to examine the phase shift reasons, a profile of two solitons joined by nonlinear interaction is presented in Fig.2 for the moment t_*. Here the left peak, which was always visible until moment t_*, corresponds to the initial big (left) solitary wave.

A jump on the graph on Fig.1 (the phase shift) is conditioned by the fact that starting from the critical moment the part of the graph corresponds to the right (ex-smaller) solitary wave peak.

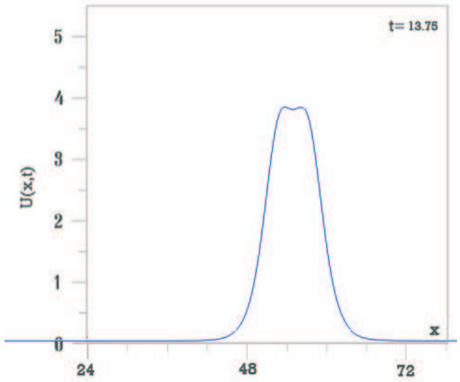

FIGURE 2. Peaks at critical moment

If we denote the abscissas of the left and right peaks at the moment t_* by x_l and x_r-it respectively, then the value of the phase shift will be $x_r - x_l$.

Note also, that the interaction of two solitary waves during overtaking collision is characterized by a mass exchange: unlike the collision of solid bodies, while colliding, the bigger wave not only transfers a certain amount of motion to the smaller one, but it also transfers a part of its mass to it.

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