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ON SUMS OF EVERYWHERE CONVERGENT TRIGONOMETRIC SERIES

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Let $f(\tau)$ be a finite function on $\mathbb{R}^1 = (-\infty, +\infty)$. By $f(\tau+)$ and $f(\tau-)$ we denote one-sided limits of f at the point τ, i. e.

$$f(\tau+) = \lim_{h\to 0+} f(\tau + h) \text{ and } f(\tau-) = \lim_{h\to 0+} f(\tau - h).$$

Let $\overline{\lim\limits_{\tau\to\tau_0}} f(\tau)$ and $\underline{\lim\limits_{\tau\to\tau_0}} f(\tau)$ be upper and inferior limits of function f at some point τ_0 respectively.

The following theorems are valid

Theorem 1. *Let a trigonometric series*

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos 2\pi k\tau + b_k \sin 2\pi k\tau \tag{1}$$

converges everywhere to the finite function $f(\tau)$. *Then*

$$\underline{\lim_{\tau\to\tau_0}} f(\tau) \le f(\tau_0) \le \overline{\lim_{\tau\to\tau_0}} f(\tau), \quad \textit{for each } \tau_0 \in \mathbb{R}^1.$$

Theorem 2. *Let the series* (1) *converges everywhere to the finite function* $f(\tau)$ *and the point* τ_0 *is such that there exist* $f(\tau_0+)$ *and* $f(\tau_0-)$. *Then*

$$f(\tau_0) = \frac{f(\tau_0+) + f(\tau_0-)}{2}.$$

It should be noted that the analogous theorems for multidimensional trigonometric series also holds. We introduce some notations to formulate this theorems.

Let $d \ge 2$ be a natural number, $\mathbb{R}^d$ Euclidean space of dimension d, $\mathbb{Z}_0^d$ the set of points in $\mathbb{R}^d$ with nonnegative integer coordinates. We denote points of the set $\mathbb{Z}_0^d$ by $n = (n_1, \dots, n_d)$ and points of the $\mathbb{R}^d$ by $x = (x_1, \dots, x_d)$. We denote the trigonometric system defined on $[0, 1]$ by $T^1 = \{t_k(\tau)\}_{k=0}^{\infty}$,

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where $t_0(\tau) \equiv 1$, $t_{2k-1}(\tau) = cos2\pi k\tau$ and $t_{2k}(\tau) = sin2\pi k\tau$, $k = 1, 2, 3, \ldots$. For any $n \in \mathbb{Z}_0^d$ we denote

$$T_n(x) = \prod_{j=1}^{d} t_{n_j}(x_j), \qquad x \in [0,1]^d.$$

Let $f(x)$ be a finite function on $\mathbb{R}^d$. By $f^{(j)}(x+)$ and $f^{(j)}(x-)$ we denote one-sided limits of function f at the point x for variable x_j, i. e.

$$f^{(j)}(x+) = \lim_{h \to 0+} f(x_1, \ldots, x_{j-1}, x_j + h, x_{j+1}, \ldots, x_d)$$

and

$$f^{(j)}(x-) = \lim_{h \to 0+} f(x_1, \ldots, x_{j-1}, x_j - h, x_{j+1}, \ldots, x_d)$$

We consider d-multiple trigonometric series

$$\sum_{n=0}^{\infty} a_n T_n(x) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} a_{n_1}, \ldots, n_d \prod_{j=1}^{d} t_{n_j}(x_j). \tag{2}$$

Under convergence of the series (2) we mean the Pringsheim convergence.

Let $x^0 = \left(x_1^0, \ldots, x_d^0\right)$ be a point of $\mathbb{R}^d$. For multiple trigonometric series we have

Theorem 3. *Let the series* (2) *converges to the finite function* $f(x)$. *Then*

$$\underline{\lim}_{x \to x^0} f(x) \leq f(x^0) \leq \overline{\lim}_{x \to x^0} f(x), \quad \textit{for each} \quad x^0 \in \mathbb{R}^d.$$

Theorem 4. *Let the series* (2) *converges to the finite function* $f(x)$ *and the point* x^0 *is such that there exist* $f^{(j)}(x^0+)$ *and* $f^{(j)}(x^0-)$. *Then*

$$f(x^0) = \frac{f^{(j)}(x^0+) + f^{(j)}(x^0-)}{2}.$$

There are presented some results connected with everywhere convergent trigonometric series in [1].

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References

1. Sh. Tetunashvili, On the sums of convergent trigonometric series. *Proc. A. Razmazde Math. Inst.* **143**(2007), 115–119.

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