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Semigroups $B_X(D)$ determined by join X -semilattices of class $\Sigma_1(X, 6)$

Let X be any non-empty set and $Z_i, i = \overline{0, 5}$, be subsets of X . Consider the semilattice $D = \{Z_0, Z_1, Z_2, Z_3, Z_4, Z_5\}$, where:

$$\begin{aligned} Z_4 \cup Z_5 &= Z_3, \quad Z_4 \cup Z_2 = Z_0, \quad Z_5 \cup Z_2 = Z_0, \quad Z_3 \cup Z_2 = Z_0, \\ Z_1 \cup Z_2 &= Z_0, \quad Z_4 \subset Z_3, \quad Z_5 \subset Z_3, \quad Z_3 \subset Z_1, \quad Z_1 \subset Z_0, \quad Z_2 \subset Z_0. \end{aligned}$$

Denote by $\Sigma_1(X, 6)$ the set of all such semilattices. One has the following

Lemma 1. *The characteristic set $C(D) = \{C, P_1, P_2, P_3, P_4, P_5\}$ of any semilattice from the class $\Sigma_1(X, 6)$ satisfies the conditions:*

$$\begin{aligned} |C| &\geq 0, \quad |P_i| \geq 1, \quad C \cap P_i = \emptyset, \quad P_i \cap P_j = \emptyset \quad (i, j = \overline{1, 5}, i \neq j), \\ Z_0 &= C \cup P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5, \\ Z_1 &= C \cup P_2 \cup P_3 \cup P_4 \cup P_5, \\ Z_2 &= C \cup P_1 \cup P_3 \cup P_4 \cup P_5, \\ Z_3 &= C \cup P_2 \cup P_4 \cup P_5, \\ Z_4 &= C \cup P_2 \cup P_5, \\ Z_5 &= C \cup P_2 \cup P_4. \end{aligned}$$

Corollary 2. *Elements of the semilattice $\Sigma_1(X, 6)$ and the set X satisfy the following condition:*

$$|Z_0| \geq 5, \quad |Z_1| \geq 4, \quad |Z_2| \geq 4, \quad |Z_3| \geq 3, \quad |Z_4| \geq 2, \quad |Z_5| \geq 2, \quad |X| \geq 5.$$

Theorem 3. *If $D \in \Sigma_1(X, 6)$, then the semigroup $B_X(D)$ has no right unit.*

Theorem 4. *Let $D \in \Sigma_1(X, 6)$ and D' be a subsemilattice of D . Then the only subsemigroups $B_X(D')$ of $B_X(D)$ which have right units are those for which the corresponding semilattice D' is a chain.*

This result yields description of the regular and idempotent elements of the semigroup $B_X(D)$.

Theorem 5. *Every element of the set $B = \{a \in B_X(D) \mid V(X^*, a) \geq 5\}$ is externally adjoined to the semigroup $B_X(D')$.*

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