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The Strong Amalgamation Property and Codescent Morphisms

We describe codescent morphisms in regular categories satisfying the strong amalgamation property. Let us recall that a category $\mathbf{C}$ is said to satisfy the strong amalgamation property if for any monomorphisms f, g with the same domain there exists a pullback $qf = gr$ with q and r being monomorphisms.

Theorem. *Let $\mathbf{C}$ be a regular category with pushouts and let $\mathbf{C}$ satisfy the strong amalgamation property. Then, for a monomorphism $f : B \rightarrow B'$ the following conditions are equivalent:*

- (i) *f is a codescent morphism;*
- (ii) *for any kernel pair $(s, s') : R \rightarrow B \times B$ and the kernel pair (t, t') of the coequalizer of (fs, fs') , the pullback of (t, t') along $f \times f$ is (s, s') .*

Each (regular) monomorphism of $\mathbf{C}$ is a codescent morphism if and only if $\mathbf{C}$, in addition, satisfies the congruence extension (or, equivalently, the transferability) property.

In the case of varieties of universal algebras condition (ii) clearly takes the form: any congruence R on B can be extended to B' .

As is known [1], among varieties satisfying the strong amalgamation property are the categories of M -sets (for a monoid M), groups, not necessarily associative rings, Lie algebras (over a field), quasi-groups, commutative quasi-groups, Steiner quasi-groups, medial quasi-groups, semilattices, lattices, weakly associative lattices, Boolean algebras and Heyting algebras. Moreover, the categories of M -sets, semilattices, Boolean algebras and Heyting algebras satisfy the congruence extension property and, thus, every monomorphism is a codescent morphism in these categories.

REFERENCES

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