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Maximal Subgroups of Some Classes of Complete Semigroups of Binary Relations

In this paper, we describe all maximal subgroups of those complete semigroups of binary relations $B_X(D)$ for which the semilattice D belongs to one of the following classes of complete join X -semilattices:

1) to the class of all nodal join X -semilattices (see [1, 2]).

Recall that D belongs to the class of all nodal join X -semilattices if $D = \{\check{D}'\} \cup D'$, where the set D' of subsets of the set X satisfies the condition $D' = \{Z_1, Z_2\}$, where

$$Z_1 \neq \emptyset, \quad Z_2 \neq \emptyset, \quad Z_1 \cup Z_2 = \check{D}', \quad Z_1 \cap Z_2 = \emptyset;$$

2) to the class of all elementary join X -semilattices (see [3]).

Recall that D belongs to the class of all elementary join X -semilattices if $D = \{\check{D}'\} \cup D'$, where the set of subsets D' of the set X satisfies the following conditions:

- $|D'| \geq 2$, $\check{D}' = \cup D'$;
- the set-theoretic union of any two different elements of the set D' is equal to $\check{D}'$;
- the set-theoretic intersection of any two elements of the set D' is different from a nonempty set;

3) $D = D' \cup \{Z_0, \check{D}'\}$, where the set $\{\check{D}'\} \cup D'$ is an elementary X -semilattice of unions, and Z_0 is the smallest element of the semilattice D (see [3]).

Further, we denote by Σ_i ($i = 1, 2, 3$) those classes of complete join X -semilattices which are defined in the subitems 1), 2) and 3). If $D \in \Sigma_i$, then the diagram of the semilattice D coincides with the diagrams shown in the figures 1, 2, 3.

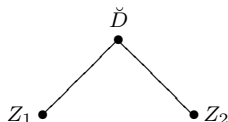


Fig. 1

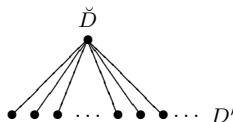


Fig. 2

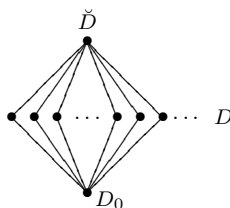


Fig. 3

In what follows $G_X(D; \epsilon)$ denotes the maximal subgroup of the semigroup $B_X(D)$ whose unit is an idempotent binary relation ϵ of the semigroup $B_X(D)$.

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Theorem 1. *If $D \in \Sigma_1$ or $D \in \Sigma_3$, then for any idempotent binary relation ε from the semigroup $B_X(D)$ the group $G_X(D; \varepsilon)$ is a cyclic group whose order is not greater than two.*

Theorem 2. *If D is an elementary join X -semilattice (i. e., $D \in \Sigma_2$), then for any idempotent binary relation ε from the semigroup $B_X(D)$ the group $G_X(D; \varepsilon)$ is the unit group.*

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