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Central Series for Groups with Action and Leibniz Algebras

The well-known construction of Witt defines a functor from the category of groups to the category of Lie algebras [5], [4]. The aim of this paper is to define a category and to give an analogue of Witt's construction for its objects, which will lead us to the category of Leibniz algebras. This problem was stated by J.-L. Loday; later an analogous question for the possibly defined partial Leibniz algebras was proposed, which was inspired by the work of Baues and Conduché [1]. Since the main interest lied in absolute case, the author decided to begin with this one.

We consider the category of groups with action on itself $\text{Gr}^\bullet$, the category of abelian groups with action on itself $\text{Ab}^\bullet$ and a category of groups with bracket operation $\text{Gr}^{[1]}$. This kind of groups are Ω -groups in the sense of [2]. We construct adjoint pairs of functors relating categories $\text{Gr}^\bullet$, $\text{Ab}^\bullet$, $\text{Gr}^{[1]}$, Gr . We define ideals and commutators for the objects of $\text{Gr}^\bullet$ (similarly for $\text{Gr}^{[1]}$) and show that these notions are equivalent to the special case of the known definitions for Ω -groups [2]. We define central series of groups with action on itself, a category of Lie-Leibniz algebras LL and the category of groups with action on itself satisfying a certain condition Gr^c . We give an analogue of Witt's construction [5] and prove that it defines a functor $LL : \text{Gr}^c \rightarrow \text{LL}$; in particular this gives a functor $\text{Gr}^c \rightarrow \text{Leibniz}$. In a similar way one can construct a functor $\text{Ab}^c \rightarrow \text{Leibniz}$, which is actually the restriction of LL on Ab^c . The functorial relations with the classical situation ($\text{Gr} \rightarrow \text{Lie}$) is considered, namely by the restriction of LL on Gr we obtain the result of Witt [5], [4].

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