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**Functorial Properties of Generalized Relative Cohomology Groups of
Adamson–Hochschild cohomologies**

Let (G, X) be a pair, where G is any group and X is a non-empty set with G -action from the left. For any G -module A consider the following abelian group

$$\text{Der}(G, X, A) = \{f : G \rightarrow A \mid f(g_1 g_2) = g_1 f(g_2) + f(g_2), f|_{H_X} = 0, g_1, g_2 \in G\},$$

which defines a left exact functor from the category of G -modules to the category of abelian groups.

Definition. The n -th right relative derived functor of the functor $\text{Der}(G, X, A)$ with respect to proper classes of (G, X) exact sequences will be called the relative cohomology group of the pair (G, X) with coefficients in the G -module A and will be denoted by $\hat{H}^{n+1}(G, X, A)$.

Proposition 1. For any G -module A one has the following isomorphism:

$$\text{Hom}_G(Z, J_{(G, X)}(A)) \cong \text{Der}(G, X, A).$$

Theorem 1. For any G -module A one has the following isomorphism:

$$\hat{H}^{n+1}(G, X, A) \approx H^n(G, X, A), \quad n \geq 2.$$

Proposition 2. For any G -module A there is the following long exact sequence:

$$0 \rightarrow A^G \rightarrow A^{H_X} \rightarrow \hat{H}^1(G, X, A) \rightarrow H^1(G, A) \rightarrow H^1(H_X, A) \rightarrow \hat{H}^2(G, X, A) \rightarrow \dots$$

Corollary. If the set X has no G -invariant elements, i. e., if $gx = x$ implies $g = 1$ for any $x \in X$ (in other words $H_X = 1$), then $\hat{H}^n(G, X, A)$ is isomorphic to the usual cohomology group $H^n(G, A)$ ($n \geq 2$).

Let (G_1, X) and (G_2, X) be pairs, where the groups G_1 and G_2 act on the set X from the left. Denote by $G_1 *_X G_2$ the amalgamated product of groups G_1 and G_2 with respect to the common subgroup $H_X = H_X^{G_1} \cap H_X^{G_2}$, where $H_X^{G_1}$ and $H_X^{G_2}$ respectively are subgroups of G_1 and G_2 of all elements acting trivially on any $x \in X$.

Theorem 2. Let $G = G_1 *_X G_2$. Then for any G -module A there is an isomorphism:

$$\hat{H}^n(G, X, A) \approx \hat{H}^n(G_1, X, A) \oplus \hat{H}^n(G_2, X, A), \quad n \geq 1.$$

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