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On A_1-weights for Maximal Operators Corresponding to Translation Invariant Bases Formed of Intervals

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A mapping B defined on $\mathbb{R}^n$ is said to be a *differentiation basis in* $\mathbb{R}^n$ if for every $x \in \mathbb{R}^n$, $B(x)$ is a family of open bounded sets containing the point x such that there exists a sequence $\{R_k\} \subset B(x)$ with $\operatorname{diam} R_k \to 0$ $(k \to \infty)$.

Under M_B we mean the *maximal operator corresponding to the differentiation basis* B, that is,

$$M_B(f)(x) = \sup_{R \in B(x)} \frac{1}{|R|} \in l_R|f| \quad (f \in L_{\text{loc}}(\mathbb{R}^n), \quad x \in \mathbb{R}^n).$$

Further instead of n-dimensional interval we'll say simply interval.

Denote by $\mathbb{I}$ and $\mathbb{Q}$ the differentiation bases for which: $\mathbb{Q}(x)(x \in \mathbb{R}^n)$ consists of all cubic intervals containing x and $\mathbb{I}(x)(x \in \mathbb{R}^n)$ consists of all intervals containing x, respectively. Recall that $M_{\mathbb{Q}}$ and $M_{\mathbb{I}}$ are named as Hardy-Littlewood and strong maximal operators, respectively.

We say that the differentiation basis B is:

1) *translation invariant*, if for every $x \in \mathbb{R}^n$;

2) *formed of intervals*, if for every $x \in \mathbb{R}^n$ the collection $B(x)$ consists of intervals.

Let T be an operator defined on some class X of measurable functions in $\mathbb{R}^n$. The function $\omega \in X$ is said to be an A_1-*weight* for T (see e.g., [1]) if there is $C > 0$ such that $|T\omega(x)| \le C\omega(x)$ a.e.. The class of all A_1-weights of T denote by $A_1(T)$.

Coifman and Rochberg [2] proved that if f is locally integrable function with $M_{\mathbb{Q}}f(x) < \infty$ a.e., then $(M_{\mathbb{Q}}f)^\delta \in A_1(M_{\mathbb{Q}})$ for every $0 < \delta < 1$.

Let B be a differantiation basis.Further we'll say that the operator M_B has a *Coifmann-Rochberg property*, if for every locally integrable function f with $M_Bf(x) < \infty$ a.e., we have that $(M_Bf)^\delta \in A_1(M_B)$ for every $0 < \delta < 1$.

Garsia-Cuerva and Rubio de Francia [3, p. 472] posed the problem: *whether the strong maximal operator has a Coifman-Rochberg property*? The negative answer to this question was given by Soria [1].

Thus $M_{\mathbb{Q}}$ has a Coifman-Rochberg property and $M_{\mathbb{I}}$ does not have. In this connection naturally arisis question: *what kind of must be a trunslation invariant basis B formed of intervals in order it's corresponding maximal operator M_B to have a Coifman-Rochberg property*?

Below we will give the answer to this question.

Let Δ be a collection of intervals. We call Δ:

1) *monotone*, if for every I, $J \in \Delta$, either $I \subset J$ or $J \subset I$;

2) *complete*, if there exist $C_1, C_2 > 1, C_1 < C_2$ such that for every $I \in \Delta$ there exists $J \in \Delta$ with the properties: $I \subset J$ and $C_1|I| \le |J| \le C_2|I|$.

For the interval $I = (a_1, b_1) \times \cdots \times (a_n, b_n)$ denote $\inf I = (a_1, \ldots, a_n)$.

Let B be a translaition invariant basis formed of intervals. For every $I \in B(0)$ let's denote by I^* the smallest among intervals that are consentric with I, contain I and have

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the edges with the lengths of the form $2^m (m \in \mathbb{Z})$, and assume that $\Delta_B = \{I^* - \inf I^* : I \in B(0)\}$.

Theorem. *Let B be a translation invariant basis formed of intervals. Then the following two conditions are equivalent:*
1) *M_B has a Coifman-Rochberg property;*
2) *Δ_B is an union of finite number of monotone and complete collections.*

References

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