Proc. A. Razmadze Math. Inst. 129(2002), 158–159

G. Oniani

On the Non-Compactness of Maximal Operators

(Reported on June 26, 2002)

Notation. A mapping B defined on $\mathbb{R}^n$ is said to be a *differentiation basis in* $\mathbb{R}^n$ (see e.g., [1]) if for every $x\in\mathbb{R}^n$, $B(x)$ is a family of open bounded sets containing the point x such that there exists a sequence $\{R_k\}\subset B(x)$ with $\dim R_k\to 0$ $(k\to\infty)$.

Under M_B we mean the *maximal operator corresponding to the differentiation basis* B, that is,

$$M_B(f)(x)=\sup_{R\in B(x)}\frac{1}{|R|}\int\limits_R|f|\quad \Big(f\in L_{loc}(\mathbb{R}^n),\quad x\in\mathbb{R}^n\Big).$$

The basis B is said to *differantiate the integral of the function* f, if for almost every $x\in\mathbb{R}^n$ the integral mean $\frac{1}{|R|}\int_R f$ tends to $f(x)$ when $R\in B(x)$, $\dim R\to 0$.

The basis B is called: *density basis*, if B differentiates an integral of a characteristic function of every measurable function; *convex*, if for every $x\in\mathbb{R}^n$ the collection $B(x)$ consists of convex sets.

The basis B we'll call quasi-density basis if it contains some density basis, i.d. if there exists a density basis H such that $H(x)\subset B(x)$ $(x\in\mathbb{R}^n)$.

Denote by $\mathbb{Q}$ the differentiation basis for which: $\mathbb{Q}(x)(x\in\mathbb{R}^n)$ consists of all cubic intervals containing x. Recall that $M_{\mathbb{Q}}$ is named as Hardy-Littlewood maximal operator.

Let (X,S,μ) be a measure space,Δ be the class of all μ-measurable functions defined on X. Normed function space (briefly: function space)E is said to be *ideal* (see e.g., [2]) if $x\in\Delta$, $y\in E$, $|x|\le|y|$ μ-a.e., $\Rightarrow x\in X$ and $\|x\|_E\le\|y\|_E$.

Function space E on (X,S,μ) is said to be *symmetric* if it is ideal and $x\in\Delta$, $y\in E$, (x is equimeasurable with y) $\Rightarrow x\in X$ and $\|x\|_E=\|y\|_E$.

Let E be a symmetric space on $\mathbb{R}^n$ with respect to Lebesgue measure and w is locally integrable and a.e., positive function on $\mathbb{R}^n$. Denote by E_w the set of all measurable functions f for which there is a function $g\in E$ such that

$$|\{|f|>t\}|_w=|\{|g|>t\}|\quad (t>0)\tag{1}$$

where $|\cdot|_w=wdx$ and $|\cdot|=dx$. The norm in E_w is defined as follows: for $f\in E_w$ $\|f\|_{E_w}=\|g\|_E$, where g is a some function from E satisfeing (1). E_w is called as *the space E with respect to the weight w*. Note that E_w is the symmetric space on measure space $(\mathbb{R}^n,\, wdx)$.

Result. Edmunds and Meskhi proved that: *for any $1<p<\infty$ and any weights w and v on $\mathbb{R}^n$, $M_{\mathbb{Q}}$ acts non-compactly from $L^p_w(\mathbb{R}^n)$ to $L^p_v(\mathbb{R}^n)$.*

It is true the following generalization of this result.

Theorem. *If B is a convex quasi-density differentiation basis, then for any symmetric space E on $\mathbb{R}^n$ with respect to Lebesgue measure and any weights w and v on $\mathbb{R}^n$, M_B acts non-compactly from E_w to E_v.*

2000 *Mathematics Subject Classification:* 42B25.

Key words and phrases. Maximal operators, compactness, weight symmetric spaces.

References

1. M. de Guzman, Differantiation of integrals in $\mathbb{R}^n$. *Springer,* 1975.

2. S. Krein, Y. Petunin and E. Semenov, Interpolation of linear operators. *Nauka, Moscow,* 1978.

3. D. E. Edmunds and A. Meskhi, On a measure of non-compactness for maximal operators. *Math. Nachr.* (accepted for publication) 1999.

Author's address:
A. Tsereteli Kutaisi State University
55, Queen Tamar St., Kutaisi 384000
Georgia