JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 21, Number 3, Fall 2009

ANALYSIS OF SOME LOCALIZED BOUNDARY-DOMAIN INTEGRAL EQUATIONS

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Communicated by Simon Chandler-Wilde, Ivan Graham and J. Trevelyan

JIEA: A special Issue for the UKBIM6 Meeting

ABSTRACT. Some direct segregated localized boundary-domain integral equation (LBDIE) systems associated with the Dirichlet and Neumann boundary value problems (BVP) for a scalar "Laplace" PDE with *variable* coefficient are formulated and analysed. The parametrix is localized by multiplication with a radial localizing function. Mapping and jump properties of surface and volume integral potentials based on a localized parametrix and constituting the LBDIE systems are studied in a scale of Sobolev (Bessel potential) spaces. The main results established in the paper are the LBDIEs equivalence to the original variable-coefficient BVPs and the invertibility of the LBDIE operators in the corresponding Sobolev spaces.

1. Introduction. Partial Differential Equations (PDEs) with variable coefficients arise naturally in mathematical modelling of inhomogeneous media (e.g. functionally graded materials or materials with damage induced inhomogeneity) in solid mechanics, electromagnetics, thermo-conductivity, fluid flows trough porous media, and other areas of physics and engineering.

The Boundary Integral Equation Method/Boundary Element Method (BIEM/BEM) is a well established tool for solution Boundary Value Problems (BVPs) with constant coefficients. The main ingredient for reducing a BVP for a PDE to a BIE is a fundamental solution to the original PDE. However, it is generally not available in an analytical and/or cheaply calculated form for PDEs with variable coefficients.

Keywords and phrases. Partial Differential Equations, Variable coefficients, Boundary value problems, Parametrix, Localized Boundary-Domain Integral Equations, Pseudo-differential operators
Received by the editors on December 7, 2007.

DOI:10.1216/JIE-2009-21-3-405