Proceedings of A. Razmadze
Mathematical Institute
Vol. **135** (2004), 131–140

COMPOUND SUMS AND COUNTING PROCESSES

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Abstract. Compound sums of positive (non-negative)random variables are considered, i.e., sums of independent identically distributed random variables, whose number is also a random variable independent of summands. The main purpose of the paper is to introduce the so-called Conditional Binomial (CB) process which will make it possible to treat the compound sums as linear functionals of such a process. This counting process is the natural generalization of the well-known Binomial process for the case where a number of summands (indicators) is a random variable. In the paper the main properties of CB processes are described.

რეზიუმე. განიხილება დადებით (არაუარყოფით) შემთხვევით სიდიდეთა შედგენილი ჯამები ანუ დამოუკიდებელ ერთნაირად განაწილებულ შემთხვევით სიდიდეთა შემთხვევითი რაოდენობის ჯამები; როცა შესაკრებები დამოუკიდებულია შესაკრებთა რაოდენობისაგან. ნაშრომის მიზანია ე. წ. პირობითი ბინომური პროცესის შემოღება, რომელიც საშუალებას იძლევა შედგენილ ჯამებს შევხედოთ, როგორც წრფივ ფუნქციონალებს ამ პროცესიდან. მთვლელი პროცესი ბუნებრივად განაზოგადებს ბინომურ პროცესს, როცა შესაკრებთა (ინდიკატორთა) რაოდენობაც შემთხვევითი სიდიდეა. ნაშრომში შესწავლილია ასეთი პროცესების ძირითადი თვისებები.

1. Introduction

There are many objects of various fields of the human activities which are described in probabilistic and statistical terms as the compound sums of positive (non-negative) i.i.d. random variables. We mention here some of them from the insurance practice.

Let $N(t)$ denote the number of claims of insurance company at time t, and $Y_i(>0)$ denote the i-th claim. We assume that claims are independent random variables; they are independent of $N(t)$ and have the common absolutely continuous distribution function F with $F(0)=0$. Let $R>0$ denote

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2000 *Mathematics Subject Classification.* 60G50, 60G55, 91B30.
Key words and phrases. Compound sums, conditional binomial process, Panjer's class, martingales, Doob-Meyer's decomposition.

some “critical” value (retention level) and for some unction h we consider the following compound sum:

$$S(t) = \sum_{i=1}^{N(t)} h(Y_i; R). \tag{1}$$

As it will be seen below, this is a sufficiently general form of the sum with a random number of random variables, and one can describe many objects of interest by such sums. Indeed, assuming $y \geq 0$,

if $h(y; R) = y$, then $S(t)$ is a total amount of claims at time t;

for $h(y; R) = I\{y \geq R\}$, $S(t)$ is a number of “excess” claims at time t;

for $h(y; R) = y \cdot I\{y \geq R\}$ the process $S(t)$ describes a total amount of claims exceeding the critical level;

for $h(y; R) = (y - R) \cdot I\{y \geq R\}$ the process $S(t)$ denotes total claims reinsured for a company in an excess of loss reinsurance agreement.

To simplify further description, we fix the time t and omit it in the above notation. Introduce the process

$$Z_N(x) = \sum_{i=1}^{N} I\{Y_i \leq x\}, \tag{2}$$

where we naturally assume that $Z_N(x) = 0$ for all x if $N = 0$.

Then we can rewrite the sum in (1) in the form of the following linear functional:

$$S = \int_0^{+\infty} h(y; R)\, dZ_N(y). \tag{3}$$

This form is very useful for investigation of many classes of statistics when N, the number of summands in (2), is not random. As is well known, in the latter case the process (2) has binomial distribution with parameters $(N; F(x))$ and is called the Binomial Process (see, e.g., [1]). Thus in our case we naturally call the process defined by (2) the Conditional Binomial (CB) process, because it has the Binomial distribution (for each x) conditionally on N.

In Section 2 we present the main unconditional properties of the CB processes, and the martingale properties of these processes are given in Section 3.

2. Unconditional properties of Z_N

Note that for all x, (2) is by itself the simple compound sum: its summands are the Bernoulli variables $I\{Y_i \leq x\}, i = 1, 2, ...,$ with parameter $F(x)$. Denote $\xi(x) \equiv I\{Y \leq x\}$, where a random variable Y has the distribution function $F(x)$. Thus if

$$p_n \equiv P\{N = n\}, \quad n = 0, 1, 2, \dots, \tag{4}$$

is the distribution of N and $\varphi(t)$ denotes its probability generating function (pgf), then the pgf of $Z_N(x)$ is defined as follows:

$$\varphi_{Z}(t,x) = \varphi\bigl(\varphi_{\xi(x)}(t)\bigr) = \varphi\bigl(1 - F(x) + t\cdot F(x)\bigr)$$

and hence the distribution of the random variable $Z_N(x)$ is defined by

$$q_k(x) \equiv P\bigl\{Z_N(x) = k\bigr\} = \frac{\varphi_{Z}^{(k)}(0,x)}{k!} = F^k(x)\cdot\frac{\varphi^{(k)}(1-F(x))}{k!}. \qquad (5)$$

Now for the finite-dimensional distributions of the process $Z_{N}(\cdot)$, let $0 = x_0 < x_1 < \cdots < x_r$ denote any partition of the positive half-line, and the integers $k_1, k_2, \ldots, k_r$ satisfy the condition $0 \le k_1 \le k_2 \le \cdots \le k_r$. Then the following theorem is valid.

Theorem 1. *For the CB processes we have*

$$\begin{aligned} P\bigl\{Z_{N}(x_1) = k_1,\ \ Z_{N}(x_2) = k_2, \ldots, Z_{N}(x_r) = k_r\bigr\} &= \\ = \mathbf{M}(k_r; g_1, g_2, \ldots, g_r)\cdot P\bigl\{Z_{N}(x_r) = k_r\bigr\}&, \end{aligned} \qquad (6)$$

where $\mathbf{M}(k_r; g_1, g_2, \ldots, g_r)$ *denotes the multinomial distribution with parameters* k_r *and the vector of probabilities* $(g_1, g_2, \ldots, g_r)$*, where*

$$g_i = \bigl(F(x_i) - F(x_{i-1})\bigr)\big/F(x_r) = P\bigl\{x_{i-1} < Y \le x_i \big| Y \le x_r\bigr\}. \qquad (7)$$

Proof. It is clear that conditionally on N, the random vector

$$\bigl(Z_{N}(x_1), Z_{N}(x_2) - Z_{N}(x_1), \ldots, Z_{N}(x_r) - Z_{N}(x_{r-1}), N - Z_{N}(x_r)\bigr)$$

has the multinomial distribution with parameters N and the vector of probabilities

$$\bigl(F(x_1), F(x_2) - F(x_1), \ldots, F(x_r) - F(x_{r-1}), 1 - F(x_r)\bigr).$$

Hence we can write

$$\begin{aligned} &P\bigl\{Z_N(x_1) = k_1, Z_N(x_2) = k_2, \ldots, Z_N(x_r) = k_r\bigr\} = \\ &= P\bigl\{Z_N(x_1) = k_1, Z_N(x_2) = k_2, \ldots, Z_N(x_r) = k_r, N \ge k_r\bigr\} = \\ &= \sum_{n \ge k_r} P\bigl\{Z_N(x_1) = k_1, Z_N(x_2) = k_2, \ldots, Z_N(x_r) = k_r | N = n\bigr\}\cdot p_n = \\ &= \sum_{n \ge k_r} P\bigl\{Z_n(x_1) = k_1, Z_n(x_2) - Z_n(x_1) = k_2 - k_1, \ldots, Z_n(x_r) - \\ &\qquad - Z_n(x_{r-1}) = k_r - k_{r-1}\bigr\}\cdot p_n = \\ &= \frac{F^{k_1}(x_1)\cdot(F(x_2) - F(x_1))^{k_2-k_1}\cdots(F(x_r) - F(x_{r-1}))^{k_r-k_{r-1}}}{k_1!\cdot(k_2-k_1)!\cdots(k_r-k_{r-1})!}\cdot \\ &\qquad \cdot\sum_{n \ge k_r}\frac{(1 - F(x_r))^{n-k_r}}{(n-k_r)!}\cdot n!\cdot p_n = \end{aligned}$$

$$= \frac{F^{k_1}(x_1)\cdot(F(x_2)-F(x_1))^{k_2-k_1}\cdots(F(x_r)-F(x_{r-1}))^{k_r-k_{r-1}}}{k_1!\cdot(k_2-k_1)!\cdots(k_r-k_{r-1})!}\cdot$$
$$\cdot\varphi^{(k_r)}\big(1-F(x_r)\big).$$

Together with (5) and (7), the last equality gives us the assertion. □

We can see that the distribution of N participates in the definition of the distribution of the CB process in the last factor of (6), i.e., in the "point behavior" of the process $Z_N(\cdot)$ which we have already defined in (5).

By means of this theorem we immediately have the following properties:

Corollary 1. *For the CB processes we have*

$$P\big\{Z_N(x_1)=k_1,\, Z_N(x_2)=k_2,\dots,Z_N(x_{r-1})=k_{r-1}\big|Z_N(x_r)=k_r\big\}=$$
$$=\mathbf{M}(k_r;g_1,g_2,\dots,g_r). \tag{8}$$

Proof. Simply, (8) is another (conditional) form of (6). This corollary tells us that conditionally on the last point, the finite dimensional distributions of the CB process are multinomial regardless of what kind of distribution has the variable N (and whether N is random or deterministic). To compare this with the earlier known results, we can say that not only the Poisson process but all the CB processes have the above-mentioned property. □

Corollary 2. *For any distribution of N, the CB process is the Markov process.*

Proof. It suffices to show that

$$P\big\{Z_N(x_r)=k_r\big|Z_N(x_1)=k_1,Z_N(x_2)=k_2,\dots,Z_N(x_{r-1})=k_{r-1}\big\}=$$
$$=P\big\{Z_N(x_r)=k_r\big|Z_N(x_{r-1})=k_{r-1}\big\}. \tag{9}$$

We can easily see that the left-hand side of (9) is the ratio of (6) and of the same expression for r replaced by $r-1$. Thus we have

$$P\big\{Z_N(x_r)=k_r\big|Z_N(x_1)=k_1,Z_N(x_2)=k_2,\dots,Z_N(x_{r-1})=k_{r-1}=$$
$$=\big[P\big\{Z_N(x_r)=k_r\big\}/P\big\{Z_N(x_{r-1})=k_{r-1}\big\}\big]\cdot$$
$$\cdot C_{k_r}^{k_{r-1}}\cdot\big(F(x_r)-F(x_{r-1})\big)^{k_r-k_{r-1}}\cdot\frac{(F(x_{r-1}))^{k_{r-1}}}{(F(x_r))^{k_r}}.$$

By means of the same argument for the last two points, for the right-hand side of (9) we have

$$P\big\{Z_N(x_r)=k_r\big|Z_N(x_{r-1})=k_{r-1}\big\}=$$
$$=\big[P\big\{Z_N(x_r)=k_r\big\}/P\big\{Z_N(x_{r-1})=k_{r-1}\big\}\big]\cdot$$
$$\cdot P\big\{Z_N(x_{r-1})=k_{r-1}\big|Z_N(x_r)=k_r\big\}=$$

$$= \big[P\big\{Z_N(x_r) = k_r\big\}/P\big\{Z_N(x_{r-1}) = k_{r-1}\big] \cdot \frac{k_r!}{k_{r-1}! \cdot (k_r - k_{r-1})!} \cdot$$
$$\cdot \Big(\frac{F(x_{r-1})}{F(x_r)}\Big)^{k_{r-1}} \cdot \Big(\frac{F(x_r) - F(x_{r-1})}{F(x_r)}\Big)^{k_r - k_{r-1}},$$

which with the above expression provides us with the assertion. □

After the unconditional distribution of the CB processes is established in general, there arises the question of interest: which of the well-known counting processes have the same structure as the CB processes? In this direction it is not difficult to verify that the following theorem is valid.

Theorem 2. *If the probabilities p_n satisfy the recursion relation*

$$p_0 > 0, \quad p_k = p_{k-1} \cdot \big(a + b/k\big), \quad k \geq 1, \tag{10}$$

with some constants a and b, then for all x the probabilities $q_k(x)$ defined by (5) *satisfy the analogous relation*

$$q_k(x) = q_{k-1}(x) \cdot (a(x) + b(x)/k) \tag{11}$$

with

$$a(x) = a \cdot C_a\big(F(x)\big), \quad b(x) = b \cdot C_a\big(F(x)\big),$$

where the function C_a on $[0,1]$ *is defined by*

$$C_a(t) \equiv \frac{1}{1 - a \cdot (1 - t)}.$$

Proof. It is well-known (see, [2], [3]) that the only three non-degenerate distributions which satisfy (10) are Binomial, Poisson and Negative Binomial (the class of counting distributions satisfying (10) is called the Panjer's class), and the corresponding pgf has the form

$$\varphi(t) = \Big(\frac{1 - at}{1 - a}\Big)^{-\frac{a+b}{a}}. \tag{12}$$

Thus for high order derivatives of this function we have

$$\varphi^{(k)}(t) = k! \cdot p_k \cdot (1 - at)^{-k} \cdot \big(\varphi(t)/\varphi(0)\big). \tag{13}$$

Consider now the ratio $q_k(x)/q_{k-1}(x)$. By virtue of (5) and (13) we obtain

$$q_k(x)/q_{k-1}(x) = \frac{F(x)}{k} \cdot \frac{\varphi^{(k)}(1 - F(x))}{\varphi^{(k-1)}(1 - F(x))} =$$
$$= \frac{p_k}{p_{k-1}} \cdot \frac{F(x)}{(1 - a \cdot (1 - F(x))} = \frac{p_k}{p_{k-1}} \cdot C_a(F(x)).$$

Now substituting the ratio of probabilities defined by (10), we obtain the assertion. □

Thus from our theorem we find that the probabilities $q_k(x)$ are the members of the same class of distributions as the probabilities p_k (the distribution of N) and we conclude that the following corollary is valid.

Corollary 3. *For the CB processes defined by* (2) *we have:*

1) *if N is a* **Binomial** *with parameters* (M, θ)*, then Z_N is the* **Binomial Process** *with parameters* $(M, \theta \cdot F(x))$*;*

2) *if N is a* **Poisson** *with parameter λ, then Z_N is the* **Poisson Process** *with parameter $\lambda \cdot F(x)$;*

3) *if N is a* **Negative Binomial** *with parameters* (M, θ)*, then Z_N is the* **Polya** (**Negative Binomial**) **Process** *with parameters* $(M, c/(c+F(x)))$*, where $c = \theta/(1-\theta)$.*

Proof. By means of Theorem 2 we have only to establish the parameters of the processes.

It is easy to verify that for the case 1)

$$a = -\frac{\theta}{1-\theta}; \quad b = \frac{(M+1)\cdot\theta}{1-\theta}; \quad C_a(t) = \frac{(1-\theta)\cdot t}{1-\theta\cdot t}$$

hence

$$a(x) = \frac{-\theta\cdot F(x)}{1-\theta\cdot F}; \quad b(x) = \frac{(M+1)\cdot\theta\cdot F(x)}{1-\theta\cdot F(x)}$$

which correspond to the parameters in (10) assigned to the binomial distribution with parameters $(M, \theta F(x))$.

Analogously, for the case 2) we have

$$a = 0; \quad b = \lambda; \quad C_a(t) = t$$

and hence $a(x) = 0$ and $b(x) = \lambda \cdot F(x)$ which correspond to the parameters in (10) assigned to the Poisson distribution with parameter $\lambda \cdot F(x)$.

Finally, for the case 3) we have

$$a = 1-\theta; \quad b = (M-1)\cdot(1-\theta); \quad C_a(t) = \frac{t}{\theta+(1-\theta)\cdot t}$$

and so

$$a(x) = \frac{(1-\theta)\cdot F(x)}{\theta+(1-\theta)\cdot F(x)} \quad \text{and} \quad b(x) = \frac{(M-1)\cdot(1-\theta)\cdot F(x)}{\theta+(1-\theta)\cdot F(x)}$$

which correspond to the parameters in (10) assigned to the negative binomial distribution with parameters $(M, c/(c+F(x)))$ with $c = \theta/(1-\theta)$. □

At the end of this section it should be noted that the results similar to the statement of Corollary 3 are valid for other counting distributions, too. For example, this is true for the mixed Poisson case which is the most important for many applications. In this case

$$p_n = \int_0^\infty \frac{\lambda^n}{n!} \cdot e^{-\lambda} dH(\lambda)$$

with some absolutely continuous distribution function H (the structural distribution function), and by means of (5)

$$q_k(x) = \frac{F^k(x)}{k!} \cdot \varphi^{(k)}(1 - F(x)) = \int_0^\infty \frac{(\lambda F(x))^k}{k!} \cdot e^{-\lambda F(x)} dH(\lambda).$$

3. Martingale properties of Z_N

In this section we present the martingale properties (the Doob-Meyer's decomposition) of the CB processes. It is easy to verify that the following lemma is valid.

Lemma. *For the CB processes defined by* (2) *we have*

$$E\Big[\frac{N - Z_N(x)}{1 - F(x)} \big| Z_N(x)\Big] = \psi_{Z_N(x)}\big(1 - F(x)\big), \tag{14}$$

where

$$\psi_k(t) \equiv \varphi^{(k+1)}(t)/\varphi^{(k)}(t). \tag{15}$$

Proof. It is clear that

$$P\big\{N = n \big| Z_N(x) = k\big\} = \frac{p_n \cdot C_n^k \cdot F^k(x) \cdot (1 - F(x))^{n-k}}{\sum\limits_{m=k}^{\infty} p_m \cdot C_m^k \cdot F^k(x) \cdot (1 - F(x))^{m-k}}$$

and thus

$$E\big[N \big| Z_N(x) = k\big] = \frac{\sum\limits_{m=k}^{\infty} m \cdot p_m \cdot C_m^k \cdot F^k(x) \cdot (1 - F(x))^{m-k}}{\sum\limits_{m=k}^{\infty} p_m \cdot C_m^k \cdot F^k(x) \cdot (1 - F(x))^{m-k}}.$$

After some simple calculations from the last equality we finf that

$$E\big[N \big| Z_N(x) = k\big] = k + (k+1) \cdot \frac{1 - F(x)}{F(x)} \cdot \frac{P\{Z_N(x) = k+1\}}{P\{Z_N(x) = k\}}.$$

By means of (5) and (15) we have

$$E\big\{N \big| Z_N(x) = k\big] = k + \big(1 - F(x)\big) \cdot \psi_k\big(1 - F(x)\big),$$

which can be easily rewritten in the form of (14). □

By the above Lemma we obtain the Doob-Meyer's decomposition of the CB processes given by the following theorem.

Theorem 3. *The Doob-Meyer's decomposition of the submartingale* $\{Z_N(x), \mathcal{F}_x^{Z_N}\}$ *has the form*

$$M_Z(x) = Z_N^U\big(F(x)\big) - \int\limits_0^{F(x)} \psi_{Z_N^U(t)}(1-t)\,dt, \tag{16}$$

where

$$Z_N^U(t) = \sum_{i=1}^N \big\{F(Y_i) \le t\big\} \equiv \sum_{i=1}^N I\big\{U_i \le t\big\} \tag{17}$$

is the CB process with "uniform claims" and $\psi_k(\cdot)$ *is defined by* (15)*.*

Proof. Taking into account the fact that the CB processes are the Markov processes (see Corollary 2) and our Lemma, we easily find that

$$E\big[\Delta Z_N(x)\big|\mathcal{F}_x^{Z_N}\big] =$$
$$= E\big[\Delta Z_N(x)\big|Z_N(x)\big] = E\big[E\big[\Delta Z_N(x)\big|N, Z_N(x)\big]\big|Z_N(x)\big] =$$
$$= E\Big[\frac{N - Z_N(x)}{1-F(x)} \cdot \Delta F(x)\big|Z_N(x)\Big] = \psi_{Z_N(x)}\big(1-F(x)\big)\cdot \Delta F(x),$$

which obviously results in (15). □

Now let us consider the question what kind of transformation is (16) as that of Z_N into M_Z. We start with the consideration of some examples.

It is easy to see that the corollary below is valid.

Corollary 4. *If the distribution of* N *belongs to the Panjer's class of counting distributions, then the Doob-Meyer's decomposition of the CB processes has the form*

$$M_Z(x) = Z_N^U\big(F(x)\big) - \int\limits_0^{F(x)} \frac{a\cdot(Z_N^U(t)+1)+b}{1-a\cdot(1-t)}\,dt. \tag{18}$$

Proof. From (12) and (13) by definition (15) we can easily find that

$$\psi_k(t) = \frac{a\cdot(k+1)+b}{1-a\cdot t}, \tag{19}$$

and substituting it in (16) we obtain (18). □

Since due to (19) ψ_k is a linear function of k, we can see that (18) is a linear transformation of Z_N into M_Z. But this fact is not valid, in general, it sufficies to remind of the example of the mixing Poisson case considered above. Indeed, in this case we have

$$\psi_k \equiv \frac{\varphi^{(k+1)}(t)}{\varphi^{(k)}(t)} = \frac{\int_0^\infty \lambda^{k+1}\cdot e^{-\lambda(1-t)}dH(\lambda)}{\int_0^\infty \lambda^k \cdot e^{-\lambda(1-t)}dH(\lambda)}$$

which is, clearly, not a linear function of k.

Now we get back to another aspect of transformation (18) (in general (16)), namely, to the degeneration problem of the transformation. As it is easy to show, for the Panjer's class of counting distributions we have $a < 1$, hence the denominator, being a function of t in the integrand of (18), varies in the interval $[1-a; 1]$ when t varies in [0;1] (of course, we are interested in the case where $a > 0$). Thus the transformation (18) has no singular points.

For the explicit forms of (18) for particular distributions of this class we have the following three cases:

Binomial case:

$$M_Z^B(x) = Z_N^U\big(F(x)\big) - \theta \cdot \int\limits_0^{F(x)} \frac{M - Z_N^U(t)}{1 - \theta \cdot t}\, dt. \tag{18.1}$$

Poisson case:

$$M_Z^P(x) = Z_N^U\big(F(x)\big) - \lambda \cdot F(x). \tag{18.2}$$

Negative Binomial case:

$$M_Z^{NB}(x) = Z_N^U\big(F(x)\big) - \int\limits_0^{F(x)} \frac{M + Z_N^U(t)}{c + t}\, dt, \tag{18.3}$$

where as above $c = \theta/(1-\theta)$.

As we can see from (18.2) and (18.3), they are non-degenerate transformations of the CB process.

Now we return to the Binomial case (i.e., to (18.1)). In [1], the Doob-Meyer's decomposition of Binomial process was obtained in the form

$$M_Z^B(x) = Z_N^U\big(F(x)\big) - \int\limits_0^{F(x)} \frac{M - Z_N^U(t)}{1 - t}\, dt. \tag{20}$$

We see that (20) is the particular case of (18.1) when $\theta = 1$. Indeed, when $P\{N = M\} = 1$, i.e., when N is deterministic, we really must return from the CB process to the Binomial one. However, it is remarkable that in contrast to (18.1), the transformation (20) has a singular point when $F(x) = 1$. Now let us investigate the question whether this is a particular phenomenon or a result of changing deterministic N by a random variable (with a non-singular distribution).

Consider the deterministic case in general. Suppose $P\{N = M\} = 1$ for some $M > 0$. Then $\varphi(t) = t^M$, and as is easily seen $\psi_k(t) = I\{k < M\} \cdot (M-k)/t$. Thus for the integrand in (16) we have

$$\psi_{Z_N^U(t)}(1-t) = \frac{M - Z_N^U(t)}{1 - t}.$$

As a result we conclude that for $t = 1$ we have a singular point.

The situation diametrically changes when N is supported at the consecutive points $0, 1, 2 \ldots, M$ (for a finite M), or at the infinite sequence $0, 1, \ldots$. Then we have

$$\varphi(t) = \sum_{n=0}^{M} p_n \cdot t$$

for finite or infinite M, and

$$\psi_{Z_N^U(t)}(1-t) =$$

$$= \frac{\sum\limits_{n=Z_N^U(t)+1}^{M} \frac{n!}{(n-Z_N^U(t)-1)!} \cdot p_n \cdot (1-t)^{n-Z_N^U(t)-1}}{p_{Z_N^U(t)} + \sum\limits_{n=Z_N^U(t)+1}^{M} \frac{n!}{(n-Z_N^U(t))!} \cdot p_n \cdot (1-t)^{n-Z_N^U(t)}} \cdot I\big\{Z_N^U(t) < M\big\},$$

denominator of which is non-zero for all $t \in [0,1]$ iff $p_k > 0$ for all $k = 0, 1, dots, M < \infty$ or for all $k = 0, 1, \ldots$ if $M = \infty$.

Finally, we conclude that the following theorem is valid.

Theorem 4. *The transformation* (16) *has no singularity iff the random variable N takes all consequtive values $0, 1, \ldots, M < \infty$ or $0, 1, \ldots$ with positive probabilities.*

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(Received 12.11.2003)

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