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Sh. T. Tetunashvili

ON N. BARY'S ONE CONJECTURE

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The Cantor's theorem [1] on the uniqueness for the trigonometric series is well-known: if the series

$$\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos nx+b_n\sin nx$$

converges everywhere to zero, then

$$a_0=a_n=b_n=0 \quad (n=1,2,\dots).$$

V. Kozlov [2] proved that there exists the trigonometric series

$$\sum_{n=1}^{\infty} b_n\sin nx, \qquad \sum_{n=1}^{\infty} b_n^2>0 \tag{1}$$

possessing the following properties: if $S_n(x)$ are its partial sums, then there exists a subsequence $\{n_k\}$ for which $S_{n_k}(x)\to 0$ everywhere as $k\to\infty$, and uniformly on $[\delta,\pi-\delta]$ for any $\delta>0$.

N. K. Bary [3] has stated the conjecture that if trigonometric series (1) possesses all the above properties indicated in his result by V. Kozlov, then it is necessary that

$$\frac{n_{k+1}}{n_k}\to\infty \quad \text{as} \quad k\to\infty. \tag{2}$$

We give a negative answer to this conjecture. In particular, the following theorem is valid.

Theorem. *There exists trigonometric series* (1) *for which there is a subsequence* $\{n_k\}$ *such that*

(i) $S_{n_k}(x)\to 0$ everywhere as $k\to\infty$;

(ii) $S_{n_k}(x)\to 0$ uniformly on $[\delta,\pi-\delta]$ for any $\delta>0$;

(iii) $\sum\limits_{k=1}^{\infty}\frac{1}{n_k}=\infty$.

Remark. It is obvious that the last relation of the theorem implies

$$\varliminf_{k\to\infty}\frac{n_{k+1}}{n_k}=1,$$

which contradicts relation (2).

References

1. G. Cantor, Ueber die Ausdehnung eines Satzs aus der Theorie der trigonometrischen Reihen. (German) *Clebsch Ann.* **V**(1872), 123–133.
2. V. Kozlov, On complete systems of orthogonal functions. (Russian) *Mat. Sbornik N.S.* **26(68)**(1950), 351–364.

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3. N. K. Bary, On everywhere converging to zero subsequences of partial sums of a trigonometric series. (Rusian) *Izv. AN SSSR, Ser. Mathem.* **24**(1960), 531–548.

Author's address:
A. Razmadze Mathematical Institute
Georgian Academy of Sciences
1, M. Aleksidze St., Tbilisi 0193
Georgia

Technical University
77, Kostava St., Tbilisi 0175
Georgia