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ON DIVERGENCE OF FOURIER TRIGONOMETRIC SERIES BY SOME METHODS OF SUMMABILITY WITH VARIABLE ORDERS

Let

$$\frac{a_0}{2}+\sum_{k=1}^{\infty} a_k\cos kx+b_k\sin kx \tag{1}$$

be the Fourier series of summable function f and

$$\Lambda=\|\lambda_n(k)\|$$

be a triangular matrix.

Denote by $t_n(x;f,\Lambda)$ the Λ means of the series (1), i. e.

$$t_n(x;f,\Lambda)=\frac{a_0}{2}+\sum_{k=1}^{n}\lambda_n(k)(a_k\cos kx+b_k\sin kx) \tag{2}$$

Let $\{\alpha_n\}$ be a sequence of numbers from the interval $[0,1]$, and for $0\le k\le n$ the numbers $\lambda_n(k)$ be defined either by the equality

$$\lambda_n(k)=\frac{A_{n-k}^{\alpha_n}}{A_n^{\alpha_n}},\ \text{where } A_n^{\alpha_n}=\frac{(\alpha_n+1)(\alpha_n+2)\cdots(\alpha_n+n)}{n!} \tag{3}$$

or

$$\lambda_n(k)=\left(1-\frac{k}{n+1}\right)^{\alpha_n} \tag{4}$$

We introduce Cesaro summability method with variable orders denoted by $(C,\{\alpha_n\})$ which coincides with Λ summability method defined by (3) and Riesz summability method with variable orders denoted by $(R,\{\alpha_n\})$ which coincides with Λ summability method defined by (4).

The theorem below is well known.

Theorem A (Kolmogorov [1]). *There exists a summable function f, such that its Fourier trigonometric series* (1) *unboundedly diverges everywhere.*

If $\{\alpha_n\}$ is such that

$$\alpha_n=o\left(\frac{1}{\ln n}\right), \tag{5}$$

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then Theorem A holds for both $(C, \{\alpha_n\})$ and $(R, \{\alpha_n\})$ methods. Namely, in both cases (3) and (4) the following theorem is true:

Theorem. *Let a sequence* $\{\alpha_n\}$ *satisfies* (5). *Then there exists such summable function* f*, that means* (2) *of Fourier trigonometric series of* f *unboundedly diverge everywhere, i. e.*

$$\overline{\lim_{n\to\infty}} |t_n(x; f, \Lambda)| = +\infty$$

at any point x.

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References

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