საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 6, №1, 2012
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 6, no. 1, 2012

Physics

The Non-Perturbative Analytical Equation of State for the Gluon Matter

Vakhtang Gogokhia* **and Avtandil Shurgaia****

* *Hungarian Academy of Sciences, Central Research Institute of Physics, Wigner Institute, Department of Theoretical Physics, Budapest, Hungary; Razmadze Mathematical Institute of I. Javakhisvili Tbilisi State University*
** *Razmadze Mathematical Institute of I. Javakhisvili Tbilisi State University*

(Presented by Academy Member Anzor Khelashvili)

ABSTRACT. In order to derive the equation of state for the pure *SU*(3) Yang-Mills fields from first principles, it is proposed to generalize the effective potential approach for composite operators to non-zero temperature. It is essentially non-perturbative by construction, since it assumes the summation of an infinite number of the corresponding contributions. There is no dependence on the coupling constant, only a dependence on the mass gap, which is responsible for the large-scale structure of the QCD ground state. The equation of state generalizes the bag constant at non-zero temperature, while its nontrivial Yang-Mills part has been approximated by the generalization of the free gluon propagator to non-zero temperature, as a first necessary step. Even in this case we were able to show explicitly that the pressure may continuously change its regime at T* = 266.5 MeV. All the other thermodynamical quantities such as the energy density, entropy, etc. are to be understood to have drastic changes in their regimes in the close vicinity of T*. All this is in qualitative and quantitative agreement with thermal lattice QCD results for the pure Yang-Mills fields. We have firmly established the behaviour of all the thermodynamical quantities in the region of low temperatures, where thermal lattice QCD calculations suffer from big uncertainties. *© 2012 Bull. Georg. Natl. Acad. Sci.*

Key words: *temperature, QCD, Yang-Mills,gluon, bag constant.*

I. Introduction

The equation of state (EoS) for the Quark-Gluon Plasma (QGP) has been derived analytically in QCD up to the order $g^6\ln(1/g^2)$ by generalizing the perturbation theory (PT) method at non-zero temperature and density [1-4] and references therein). However, due to its non-analytical dependence on the QCD coupling constant g^2, nobody can trust its description of the QGP dynamics, apart from maybe at very high temperature. So there is an exact indication that the analytical EoS derived by thermal PT QCD is wrong.

At present, the only method to be used in order to investigate thermal QCD is the lattice QCD at finite temperature and baryon density which underwent a rapid recent progress [1,3,5-7] and references therein).

However, the lattice QCD, being a very specific regularization scheme, first of all is aimed at obtaining the well-defined corresponding expressions in order to get correct numbers and curves from them. So, one gets numbers and curves, but not understanding of what is the physics behind them. Such kind of understanding can only come from the dynamical theory which is continuous QCD. For example, any description of the QGP is to be formulated in the framework of the dynamical theory. The lattice thermal QCD is useless in this. The need in the analytical EoS remains, but, of course it should be essentially non-perturbative (NP), reproducing the thermal PT QCD results at a very high temperature only. Thus analytic NP QCD and lattice QCD approaches to finite-temperature QCD do not exclude each other, but on the contrary they should complement each other. Especially this is true for low temperatures where lattice QCD calculations suffer from big uncertainties [1,3,5-7]. There already exist interesting analytic approaches based on quasi-particle and liquid model pictures [8-17] to analyze the results of $SU(3)$ thermal lattice QCD calculations for the QGP EoS.

The formalism we are going to use in order to generalize it to non-zero temperature is the effective potential approach for composite operators [18-20]. It is essentially NP from the very beginning, since it is dealing with the expansion of the corresponding skeleton loop contributions (for a more detailed description see below). The main purpose of this paper is to derive EoS for the gluon matter by introducing the temperature dependence into the effective potential approach in a self-consistent way.

II. The Vacuum Energy Density

The quantum part of the vacuum energy density (VED) is determined by the effective potential approach for composite operators [18]. In the absence of external sources the effective potential is nothing but the VED. It is given in the form of the skeleton loop expansion, containing all the types of the QCD full propagators and vertices. So each vacuum skeleton loop itself is a sum of an infinite number of the corresponding PT vacuum loops, i.e., it contains the point-like vertices and free propagators (the figures of these expansions are explicitly shown in [20]). The number of the vacuum skeleton loops is equal to the power of the Planck constant $\hbar$.

Here we are going to formulate a general method of numerical calculation of the quantum part of the truly NP Yang-Mills (YM) VED in the covariant gauge QCD. The gluon part of the VED to leading order (the so-called log-loop level $\sim \hbar$) is given analytically by the effective potential for composite operators as follows [18]:

$$V(D)=\frac{i}{2}\int \frac{d^4q}{(2\pi)^4}\, Tr\{\ln(D_0^{-1}D)-D_0^{-1}D+1\}, \quad (1)$$

here $D(q)$ is the full gluon propagator and $D_0(q)$ is its free counterpart (see below). Traces over space-time and color group indices are assumed. Evidently, the effective potential is normalized to $V(D_0)=0$. Next-to-leading and higher order contributions (two and more vacuum skeleton loops) are suppressed at least by one order of magnitude in powers of $\hbar$.

The two-point Green's function, describing the full gluon propagator, is $D_{\mu\nu}(q) = -i\,\{ T_{\mu\nu}(q)d(-q^2,\xi) + \xi L_{\mu\nu}(q)\}(1/q^2)$, where $\hat{\imath}$ is the gauge-fixing parameter and $T_{\mu\nu}(q)=g_{\mu\nu}-(q_\mu q_\nu/q^2)=g_{\mu\nu}-L_{\mu\nu}(q)$. Its free counterpart $D_0\equiv D^0_{\mu\nu}(q)$ is obtained by putting the full gluon form factor $d(-q^2,\xi)$ simply to one, i.e., $D^0_{\mu\nu}(q) = -i\,\{T_{\mu\nu}(q)d(-q^2,\xi)+\xi L_{\mu\nu}(q)\}(1/q^2)$.

In order to evaluate the effective potential (1), we use the well-known expression

$Tr\{\ln(D_0^{-1}D) = 8\times 4 \ln det(D_0^{-1}D) = 32 \ln[(3/4)\, d(-q^2,\xi)+(1/4)]$. Going over to four-dimensional Euclidean space in Eq. (1), one obtains ($\varepsilon_g = V(D)$)

$$\varepsilon_g=-16\int \frac{d^4q}{(2\pi)^4}\,\{\ln[1+3\,d(q^2,\xi)]-\frac{3}{4}\,d(q^2,\xi)+a\}, \quad (2)$$

where the constant a = (3/4) – 2ln2 = -0.6363 and the integration over q^2 from zero to infinity is assumed. The VED ε_g derived in Eq. (2) is already a colorless quantity, since it has been already summed over color indices. Also, only the transversal ("physical") degrees of freedom of gauge bosons contribute to this equation (up to one skeleton loop order). So, there is no need for ghosts to cancel their longitudinal (unphysical) counterparts because of a normalization condition in this case.

However, the derived expression (2) remains rather formal, since it suffers from different types of PT contributions ("contaminations"). In order to define correctly the truly NP VED, let us make first the identical transformation of the full effective charge in Eq. (2) as follows: $d(q^2,\xi) = d(q^2,\xi) - d^{PT}(q^2,\xi) + d^{PT}(q^2,\xi) = d^{NP}(q^2) + d^{PT}(q^2,\xi)$, where $d^{PT}(q^2,\xi)$ correctly describes the PT structure of the full effective charge $d(q^2,\xi)$, including its behaviour in the ultra-violet (UV) limit compatible with asymptotic freedom (AF), otherwise remaining arbitrary. On the other hand, $d^{NP}(q^2)$ is assumed to reproduce correctly the NP structure of the full effective charge, including its asymptotic behaviour in the deep infrared (IR) limit. Evidently, both terms are valid in the whole energy/momentum range, i.e., they are not asymptotics. Let us also emphasize the principal difference between $d^{PT}(q^2,\xi)$ and $d^{NP}(q^2)$. The former is NP quantity "contaminated" by PT contributions, while the latter, being also NP, is, nevertheless, free of them. Thus the separation between the truly NP effective charge $d^{NP}(q^2)$ and its nontrivial PT counterpart $d^{PT}(q^2,\xi)$ is achieved. For example, if the full effective charge explicitly depends on the scale responsible for the truly NP dynamics in QCD, say Λ^2_{NP}, then one can define the subtraction $d^{NP}(q^2, \Lambda^2_{NP}) = d(q^2, \Lambda^2_{NP}) - d(q^2, \Lambda^2_{NP} = 0) = d(q^2, \Lambda^2_{NP}) - d^{PT}(q^2)$, which is obviously equivalent to the previous decomposition.

III. Generalization to Non-zero Temperature

Substituting the above-discussed exact decomposition into Eq. (2), introducing further the effective scale squared, separating the NP region from the PT one (soft momenta from hard momenta), and omitting some algebraic rearrangement (for details see [20]), one obtains:

$$\varepsilon_{YM} = - B_{YM} + B_{YM}(T) + P_{YM}(T) \qquad (3)$$

Here evidently $\varepsilon_g \equiv \varepsilon_{YM}$ and B_{YM} is the bag constant at zero temperature [20,21]. Also, $B_{YM}(T)$ and $P_{YM}(T)$ are explicitly given by the following expressions:

$$B_{YM}(T) = 16 \int^{q_{eff}} \frac{d^4 q}{(2\pi)^4} \cdot$$

$$\cdot \{\ln[1 + 3\alpha_s^{NP}(q^2)] - \frac{3}{4}\alpha_s^{NP}(q^2)\} \qquad (4)$$

and $P_{YM}(T)$ has more complicate form, namely

$$P_{YM}(T) = -16 \int \frac{d^4 q}{(2\pi)^4} \{ [\ln[1 +$$

$$+3\alpha_s^{PT}(q^2) + 3\alpha_s^{NP}(q^2)] - \frac{3}{4}[\alpha_s^{NP}(q^2) + \alpha_s^{NP}(q^2)] + a\} \qquad (5)$$

respectively, since it depends on both effective charges, $\alpha_s^{NP}(q^2) \equiv d^{NP}(q^2)$ and $\alpha_s^{PT}(q^2) \equiv d^{PT}(q^2)$. Precisely these expressions should be generalized to non-zero temperatures in order to get EoS for the pure YM fields. That is why we introduce the dependence on the temperature T in advance. Evidently, Eq. (4) will reproduce the temperature-dependent bag constant. In the expression for $P_{YM}(T)$ the integration is from zero to infinity, while in the integral for $B_{YM}(T)$ it is from zero to the effective scale squared q^2_{eff}, which is symbolically shown in Eq. (4). It is worth emphasizing that a so defined bag constant (4) is free of all types of PT contributions ("contaminations"), as it is required (this was a reason for the above-mentioned algebraic rearrangements and subtractions, see [20-22] and references therein).

The problem remaining to solve is to choose the truly NP effective charge $\alpha_s^{NP}(q^2)$. For the different truly NP effective charges we will get different analytical and numerical results. That is why the choice for its explicit expression should be physically and mathematically well justified. Let us choose the truly NP effective charge as follows:

$$\alpha_s^{NP}(q^2) = \frac{\Lambda_{NP}^2}{q^2}, \qquad (6)$$

where Λ_{NP}^2 is the mass scale parameter (the mass gap) responsible for the large-scale structure of the true QCD vacuum. It is well known that in continuous QCD it leads to a linear rising potential between heavy quarks, "seen" by lattice QCD [23] as well ($(q^2)^{-2}$-type behaviour for the full gluon propagator). Moreover, in [24-25] (see references therein as well) it has been explicitly shown that it is a direct nonlinear iteration solution of the transcendental equation for the full gluon propagator in the presence of a renormalized mass gap (see also Ref. [26]). The separation between the truly NP and the PT effective charges is both exact and unique, since the PT effective charge is always regular at zero, while the truly NP effective charge is singular at the origin. Let us also note that the chosen effective charge (6) does not depend explicitly on the gauge choice. It has been already used [20,21] in order to calculate the bag constant, which turned out to be in a very good agreement with such an important phenomenological parameter as the gluon condensate. It leads to many other desirable properties for the bag pressure at zero temperature [20]. Thus, our choice (6) is physically justified and mathematically confirmed.

In the imaginary time formalism [1,27] these expressions can be easily generalized to non-zero temperatures $T \equiv \beta^{-1}$ according to the prescription (let us recall that there is already Euclidean signature)

$$\int \frac{dq_0}{2\pi} \to T \sum_{n=-\infty}^{+\infty},$$

$$q^2 = \boldsymbol{q}^2 + q_0^2 = \boldsymbol{q}^2 + \omega_n^2 = \omega^2 + \omega_n^2, \ \omega_n = 2\pi n T, \qquad (7)$$

i.e., each integral over q_0 of the loop momentum is to be replaced by the sum over Matsubara frequencies labeled by n, which obviously assumes the replacement $q_0 \to \omega_n = 2\pi n T$ for bosons (gluons). In frequency-momentum space the truly NP effective charge becomes

$$\alpha_s^{NP}(q^2) = \alpha_s^{NP}(\boldsymbol{q}^2, \omega_n^2) = \frac{\Lambda_{NP}^2}{\omega^2 + \omega_n^2},$$

$$\alpha_s^{PT}(q^2) = \alpha_s^{PT}(\boldsymbol{q}^2, \omega_n^2) = \alpha_s^{PT}(\omega^2, \omega_n^2). \qquad (8)$$

Here and everywhere below $\omega = (\boldsymbol{q}^2)^{1/2}$ and $\boldsymbol{q}^2$ is the three-dimensional loop momentum squared in complete agreement with the relations (7).

IV. The Derivitons of $B_{YM}(T)$ and $P_{YM}(T)$

In frequency-momentum space the bag pressure (4) after the substitution of the expressions (7) and (8) becomes:

$$B_{YM}(T) = 16 \int \frac{d^3q}{(2\pi)^3} T \sum_{n=-\infty}^{+\infty} \Big[\ln\big[3\Lambda_{NP}^2 + \boldsymbol{q}^2 + \omega_n^2\big] - $$
$$ - \ln\big[\boldsymbol{q}^2 + \omega_n^2\big] - \frac{3}{4}\Lambda_{NP}^2 \left(\boldsymbol{q}^2 + \omega_n^2\right)^{-1} \Big], \qquad (9)$$

where the dependence on the effective scale ω_{eff} is omitted (see below), for simplicity. Here it is also convenient to introduce the following notation: $\omega' = (\boldsymbol{q}^2 + m'^2_{\text{eff}})^{1/2} = (\boldsymbol{q}^2 + 3\Lambda_{NP}^2)^{1/2} = (\omega^2 + 3\Lambda_{NP}^2)^{1/2}$. So it is possible to say that we have two sorts of gluons: massless ω and massive ω' with the effective mass $m'_{\text{eff}} = \sqrt{3}\Lambda_{NP}^2$. In this case the summation over the Matsubara frequencies ω_n can be easily done, as well as performing an almost trivial integration over angular variables.

Due to the above-mentioned normalization of the effective potential approach in Eq. (1), the investigation of the YM part (5) of the future gluon matter EoS makes sense to begin with putting first $\alpha_s^{PT}(q^2)=1$, i.e., approximating the nontrivial PT effective charge by its free PT counterpart. Then on account of the relations (7), the YM pressure (5) in frequency-momentum space becomes

$$P_{YM}(T) = -16 \int \frac{d^3q}{(2\pi)^3} T \sum_{n=-\infty}^{+\infty} \Big[\ln\Big[\frac{3}{4}\Lambda_{NP}^2 + \boldsymbol{q}^2 + \omega_n^2\Big] + $$
$$ + \ln\big[\boldsymbol{q}^2 + \omega^2\big] - \frac{3}{4}\Lambda_{NP}^2 \left(\boldsymbol{q}^2 + \omega_n^2\right)^{-1} \Big]. \qquad (10)$$

Comparing Eqs. (9) and (10) one can write down the final result directly. For this purpose, in the final evaluation of Eq. (9) one must change the overall sign and replace ω' by $\bar{\omega} = (\omega^2 + (3/4)\Lambda_{NP}^2)^{1/2}$. We should also integrate from zero to infinity. All the aspects of these derivations can be found in [22] in detail.

V. The Gluon Matter EoS

Denoting further $\varepsilon_{YM}(T) + B_{YM} = P_{GM}(T)$ in the left-hand-side of our EoS (3) and summing up all the results of the summation over the Matsubara frequencies in the expressions (9) and (10), one obtains that EoS (3) finally becomes

$$P_{GM}(\mathrm{T}) = \frac{6}{\pi^2}\Lambda_{NP}^2 P_1(T) + \frac{16}{\pi^2} T\{P_2(T) + P_3(T) - P_4(T)\}, \qquad (11)$$

where the dependence on the thermodynamical variable T is only shown explicitly and

$$P_1(T) = \int_{\omega_{eff}}^{\infty} d\omega \frac{\omega}{e^{\beta\omega} - 1}$$

$$P_2(T) = \int_{\omega_{eff}}^{\infty} d\omega\, \omega^2 \ln[1 - \mathrm{e}^{\beta\omega}]$$

$$P_3(T) = \int_{0}^{\omega_{eff}} d\omega\, \omega^2 \ln[1 - \mathrm{e}^{\beta\omega'}]$$

$$P_4(T) = \int_{0}^{\infty} d\omega\, \omega^2 \ln[1 - e^{\beta\bar{\omega}}]. \qquad (12)$$

In the formal PT limit $\Lambda_{NP}^2 = 0$ it follows $\bar{\omega} = \omega' = \omega$ and thus the gluon matter pressure (11) in this limit vanishes, i.e., it is truly NP, indeed. The effective potential has been normalized to zero in the $D \to D_0$ limit, which reproduces the case of the so-called Stefan-Boltzmann (SB) non-interacting (ideal) gas of massless particles (gluons) at high temperatures [1]. So the SB limit $P_{SB}(\mathrm{T}) = (8/45)\pi^2 T^4$ can be added (if necessary) to the truly NP pressure (11) in the $T \to \infty$ ($\beta \to 0$) limit only.

Other thermodynamical quantities. In quantum statistics the thermodynamical potential $\Omega(T)$ is nothing but the pressure $P(T)$ apart from the sign, so in our case we can put $\Omega(T) = -P_{GM}(T)$. In the quantum statistical theory all the important quantities such as energy density, entropy, etc. are to be expressed in terms of the thermodynamical potential. So the general formulae to be used are [1]: $\varepsilon(T) = -T\frac{\partial\Omega(T)}{\partial T} + \Omega(T)$, $s(T) = -\frac{\partial\Omega(T)}{\partial T}$, $c_V(T) = \frac{\partial\varepsilon(T)}{\partial T} = T\frac{\partial s(T)}{\partial T}$ for the pure YM fields, i.e., when the chemical potential is equal to zero. Evidently, here and everywhere below $\varepsilon(T)$ and $s(T)$ are the energy density and entropy, respectively, of the pure NP gluon matter, and one of the interesting thermodynamical characteristics of the QGP is the heat capacity $c_V(T)$. Their corresponding SB limits are: $\varepsilon_{SB}(T) = (24/45)\pi^2 T^4$, $s_{SB}(T) = (32/45)\pi^2 T^3$, $c_V^{SB}(T) = (96/45)\pi^2 T^3$ which should be added to our expressions in the high temperature $T \to \infty$ ($\beta \to 0$) limit only.

VI. The Scale-setting System

From the relations (7) it follows that in frequency-momentum space a possible free parameter of our approach is the effective scale $\omega_{eff} = \sqrt{q_{eff}^2 - \omega_c^2}$, where we introduced the constant Matsubara frequency ω_c, which is always positive. So ω_{eff} is always less than or equal to q_{eff} of the four-dimensional QCD, i.e., $\omega_{eff} \leq q_{eff}$. One then can conclude that q_{eff} is a very good upper limit for ω_{eff}. In this connection, let us recall now that the bag constant B_{YM} at zero temperatures has been successfully calculated at a scale $q_{eff} = 1$ GeV^2, in fair agreement with other phenomenological quantities such as gluon condensate [20]. So ω_{eff} is fixed as follows: $\omega_{eff} = q_{eff} = 1$ GeV^2. The mass gap squared

Λ^2_{NP} calculated just at this scale is equal to Λ^2_{NP} = 0.4564 GeV^2 [20]. Thus, we have no more free parameters in our approach. The confinement dynamics is nontrivially taken into account directly through the mass gap and the bag constant itself.

VII. Numerical Results And Discussion

All our numerical results are shown in Fig. 1. It is seen explicitly that the NP gluon pressure may continuously change its regime in the close neighborhood of a maximum at T^* = 266.5 MeV (which is obtained after the parameters ω_{eff} and Λ^2_{NP} have been fixed) in order to achieve the thermodynamical SB limit at high temperatures. For the displayed quantities in Fig. 1 the SB limits are the corresponding constants. At the same time, for all the other thermodynamical quantities such as the energy density, entropy and heat capacity this is impossible (none of their power-type fall off at this point can be smoothly transformed into constant behaviour at high temperatures). In order to achieve the thermodynamical SB limits at high temperatures their full counterparts should undergo drastic changes in their regimes in the close neighborhood of this point. As we already know from thermodynamics of $SU(3)$ lattice QCD [1], [2], [28] the energy and entropy densities have a discontinuity at about T_c =260 MeV, while the pressure remains continuous. Our characteristic temperature T^*=266.5 MeV is, surprisingly, very close to the same value. A clear evidence that something nontrivial in the behaviour of the thermodynamical quantities in the vicinity of our characteristic temperature T^* = 266.5 MeV should actually take place follows from the fact that at this point ε= 3P, which should be valid at very high temperatures only (SB limit). In other words, in order to derive EoS valid above T^*, and thus to provide a correct picture of thermodynamics of the gluon matter in the whole range of temperature, one needs a nontrivial approximation of the YM part (5), compatible with the asymptotic freedom phenomenon in QCD. This will be the subject of the subsequent paper.

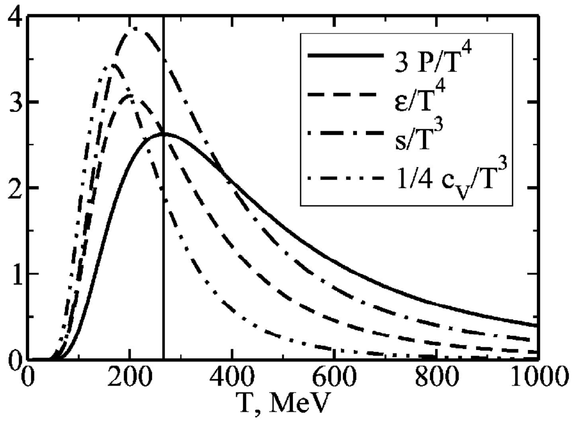

Fig. 1. The NP pressure P, energy density ε, entropy s and heat capacity c_V as functions of the temperature. The NP gluon pressure P has a maximum at T^* = 266.5 MeV.

It is worth emphasizing that we have no problems in describing and predicting the behaviour of all the important thermodynamical quantities at low temperatures below T^* (see Fig. 1). We do not expect any serious changes in the behaviour of the thermodynamical quantities in this region (exponential fall off or rise when the temperature goes down or up, respectively) even after taking into account the above-mentioned nontrivial approximation of the YM part (5), apart from the "non-physical" maximums which should disappear, of course. However, whatever changes may occur they will be under our control.

The confinement dynamics (6), generalized to non-zero temperatures in Eq. (8), is still important especially in the region of low temperatures even up to the temperature at which all the important thermodynamical quantities may undergo drastic changes in their behaviour (apart from pressure). From the structure of our EoS (see Eqs. (11)- (12)), it clearly follows that we have two massive gluonic excitations ω' and $\bar{\omega}$. The former can be interpreted as glueballs with masses $m'_{eff} = \sqrt{3}\ \Lambda_{NP}$ = 1.17 GeV, while the latter as gluons with effective masses $\bar{m}_{eff} = \frac{\sqrt{3}}{2}\Lambda_{NP}$ = 0.585 GeV. We have also two massless gluonic excitations propagating in accordance with the two first integrals in Eq.~(12). However, they are not free since in the formal PT $\Lambda_{NP}^2 = 0$ limit they vanish. So all our massive and massless gluonic excitations are of the NP dynamical origin. At the same time, the generalization of our formalism to non-zero temperature in order to introduce into the consideration topological objects like instantons and related issues (for example, tunneling) would be of great interest [29,30] (work is in progress).

Acknowledgments. One of the authors (V. Gogokhia) acknowledges support by the Hungarian National Fund OTKA - 77816 (P. Levai). We would like to thank V. Skokov for help in numerical calculations, especially for creating Fig. 1.

ფიზიკა

გლუონის მატერიის მდგომარეობის არაპერტურბაციული ანალიზური განტოლება

ვ. გოგოხია*, ა. შურღაია**

** უნგრეთის მეცნიერებათა აკადემია, ფიზიკის კვლევის ცენტრალური ინსტიტუტი, ვიგნერის ინსტიტუტი, ბუდაპეშტი, უნგრეთი; თბილისის ი. ჯავახიშვილის სახ. სახელმწიფო უნივერსიტეტი, ა. რაზმაძის სახ. მათემატიკის ინსტიტუტი*

*** თბილისის ი. ჯავახიშვილის სახ. სახელმწიფო უნივერსიტეტი, ა. რაზმაძის სახ. მათემატიკის ინსტიტუტი*

(წარმოდგენილია აკადემიის წევრის ა.ხელაშვილის მიერ)

ეფექტური პოტენციალის მიახლოება შედგენილი ოპერატორებისთვის განზოგადებულია არანულოვანი ტემპერატურებისთვის და მისი მეშვეობით, პირველადი პრინციპებიდან გამომდინარე, მიღებულია მდგომარეობის განტოლება SU(3) იანგ-მილსის ველებისთვის. ის არსებითად არაპერტურბაციული ხასიათისაა, რადგან უსასრულო რაოდენობის წევრების აჯამვას გულისხმობს. იგი დამოკიდებულია არა ბმის მუდმივაზე, არამედ მასურ ღრეჩოზე, რომელიც პასუხისმგებელია კვანტური ქრომოდინამიკის ძირითადი მდგომარეობის სტრუქტურაზე დიდ მანძილებზე. მდგომარეობის განტოლება არის ჩანთის მოდელის მუდმივას განზოგადება არანულოვანი ტემპერატურებისთვის, მაშინ როდესაც მისი იანგ-მილსის არატრივიალური ნაწილი აპროქსიმირებულია გლუონის თავისუფალი პროპაგატორით არანულოვანი ტემპერატურებისთვის, როგორც პირველი აუცილებელი ნაბიჯი. ამ შემთხვევაშიც კი ჩვენ შევძელით ცხადად გვეჩვენებინა, რომ წნევა უწყვეტად იცვლის თავის რეჟიმს T^*=266.5MeV ტემპერატურაზე. ყველა სხვა თერმოდინამიკური სიდიდეები, როგორიცაა ენერგიის სიმკვრივე, ენტროპია და სხვა, მკვეთრად იცვლიან თავიანთ რეჟიმებს T* ტემპერატურის მახლობლობაში. ყოველივე ეს თვისებრივ და რაოდენობრივ თანხმობაშია თერმული კვანტური ქრომოდინამიკის მესერული მიახლოების შედეგებთან წმინდა იანგ-მილსის ველებისთვის. ჩვენ მკაფიოდ დავადგინეთ ყველა თერმოდინამიკური სიდიდე დაბალი ტემპერატურების არეში, სადაც მესერეული მიახლოების გამოთვლები აწყდებიან დიდ არაცალსახობებს.

REFERENCES

1. *J.I. Kapusta, C. Gale* (2006), Finite-Temperature Field Theory. Cambridge University Press.
2. Quark Matter (2005), Edited by T. Csorgo, G. David, P. Levai, G. Papp, ELSEVIER, Amsterdam-...- St. Louis.
3. *M. Gyulassy, L. McLerran* (2005), Nucl.Phys.A750:30-63; (2004), arXiv:nucl-th/0405013.
4. *K. Kajantie, M. Lane, K. Rummukainen Y. Schroder* (2003), Phys. Rev., **D67**: 105008.
5. *Y. Aoki, Z. Fodor, S.D. Katz and K.K. Szabo* (2006), JHEP, **0601**: 089.
6. *C. Schmidt, Z. Fodor and S.D. Katz* (2006), PoS **LAT2005:** 163, (2005), arXiv:hep-lat/0510087.
7. *M. Cheng, N.H. Christ, S. Dalta et al.* (2008), Phys.Rev., D77, 014511; (2007), arXiv:0710.0354 [hep-lat].
8. *P. Levai, U. Heinz* (1998), Phys.Rev., **C 57**: 1879; (1997), arXiv:hep-ph/9710463.
9. *A. Peshier, B. Kampfer, O.P. Pavlenko and G.Soff* (1996), Phys Rev., **D54**: 2399.
10. *A. Peshier, B. Kampfer and G. Soff* (2000), Phys. Rev., **C61**: 045203; (1999), arXiv:hep-ph/9911474 .
11. *K.K. Szabo and A.I. Toth* (2003), JHEP, **0306**: 008; (2003), arXiv:hep-ph/0302255.
12. *M.A. Thaler, R.A. Schneider and W. Weise* (2004), Phys. Rev., **C**69: 035210.
13. *Yu.B. Ivanov, V.V. Skokov and V.D. Toneev* (2005), Phys. Rev., **D71:**1400.
14. *C. Ratti, S. Roessner, M.A. Thaler and W. Weise* (2007), Eur. Phys. J., **C49:**13.
15. *M. Bluhm, B. Kampfer and G. Soff* (2005), Phys. Lett., **B620:** 131.
16. *W. Cassing* (2007), Nucl. Phys., **A791:** 365.
17. *J. Letessier, J. Rafelski* (2003), Phys. Rev., C67: 031902; arXiv: 0301099
18. *J.M. Cornwall, R. Jackiw, E. Tomboulis* (1974), Phys. Rev., **D10:** 2428.
19. *T. Schafer* (2000), Nucl. Phys., **B575:** 269.
20. *G.G. Barnafoldi, V. Gogokhia* (2010), J. Phys. G: Nucl. Part. Phys., **37:** 025003; (2007), arXiv: 0708.0163.
21. *V. Gogokhia, Gy. Kluge* (2000), Phys. Rev., **D62:** 076008.
22. *V. Gogokhia, M. Vasuth* (2010), J. Phys. G: Nucl. Part. Phys., **37:** 075015; (2009), arXiv:0902.3901.
23. *K.D. Born et al.* (1994), Phys. Lett., **B329:**325.
24. *V.M. Miller et al.* (1994), Phys. Lett., **B335:** 71.
25. *V. Gogokhia* (2009), Int. Jour. Theor. Phys., **48:** 3470; (2009), arXiv:0907.0082.
26. *L.G. Vachnadze, N. A. Kiknadze, A. A. Khelashvili* (1995), Theor. Math. Phys., **102**: 34.
27. *L. Dolan, R. Jackiw* (1974), Phys. Rev., **D9:** 3320.
28. *G. Boyd, J. Engels, F. Karsch et al.*(1996), Nucl. Phys., **B469:** 419; (1996), arXiv:hep-lat/9602007.
29. *L.Q. Liang, H.J.W. Müller-Kirsten, D.K. Park, A.W. Shurgaia* (2000), Phys. Lett., **B483:** 225.
30. *A.V. Shurgaia* (2010), Mod. Phys. Lett., **A26:** 53.

Received November, 2011