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AN EXTENSION OF THE OCONE–HAUSSMANN–CLARK FORMULA FOR A CLASS OF NORMAL MARTINGALES

O. PURTUKHIA

Abstract. The space $\mathbf{D}_{p,1}$ $(1 < p < 2)$ is proposed for normal martingales and the integral representation formula of Clark (1970), Haussmann (1979) and Ocone (1984) is established for functionals in the space $\mathbf{D}_{p,1}$ $(1 < p < 2)$.

რეზიუმე. ნაშრომში განმარტებულია $\mathbf{D}_{p,1}$ $(1 < p < 2)$ სივრცე ნორმალური მარტინგალის შემთხვევაში და გამოყვანილია კლარკის (1970), ჰაუსმანისა (1979) და ოკონეს (1984) ინტეგრალური წარმოდგენის ფორმულა $\mathbf{D}_{p,1}$ $(1 < p < 2)$ სივრცის ფუნქციონალებისათვის.

Introduction

The past two decades have seen considerable interest in the application of stochastic calculus to problems of financial economics. Harrison and Pliska [4–5] were the first authors to show that the martingale representation theorem and Girsanov change of probability measure were the "keys" to understanding the option pricing in terms of the well-known Black–Sholes model. Subsequently, this method was successfully used in studying problems of valuation of American options (Bensoussan [6]), consumption/investment optimization (Karatzas, Lehoczky and Shreve [7]), term structure of interest rates (Artzner and Delbaen [8]), and equilibrium (Karatzas, Lehoczky and Shreve [9]).

For most stochastic optimization problems posed in general financial market models the above-mentioned method is highly successful in identifying closed-form expressions for quantities like optimal consumption and terminal wealth levels. However it is able to ascertain only the existence of associated portfolio strategies. In Ocone and Karatzas [10] a general representation formula is derived for the optimal portfolios associated with option pricing, maximizing utility from terminal wealth and consumption. Instrumental in obtaining these representations is an extension of the familiar

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Clark formula (Clark [1], Haussmann [2], Ocone [3])

$$F = E(F) + \int_0^T E\big[D_t^w F|\mathcal{J}_t\big]\,dw_t, \tag{0.1}$$

where w is the Wiener process on $[0,T]$, $\mathcal{J}_t := \sigma\{w_s : 0 \le s \le t\}$, D^w is the Malliavin derivative on the Wiener space, and F is an $\mathcal{J}_T$-measurable Wiener functional in the Sobolev space $\mathbf{D}^w_{2,1}$.

It was proved by Karatzas, Ocone and Li in [11] that (0.1) is also valid for functionals F from the Sobolev space $\mathbf{D}^w_{1,1}$. Ocone and Karatzas [10] found this extension useful for application to an optimal portfolio representation because it simplifies the technical hypotheses one needs to impose. In particular, this extension is therefore useful for avoiding unnecessarily restrictive moment bounds on F and DF. For example, if $F \in L^2(\overline{P})$ (where $d\overline{P} = \zeta dP$), it does not follow that $F\zeta \in L^2(P)$. However,

$$E|F\zeta|^p = \overline{E}(|F|^p\zeta^{p-1}) \le (\overline{E}F^2)^{p/2}(\overline{E}\zeta^{2(p-1)/(2-p)})^{(2-p)/2} < \infty,$$

if $1 \le p < 2$.

Ma, Protter and Martin have proposeding [12], an anticipating integral for the class of so-called normal martingales (a martingale M is called normal if $\langle M,M\rangle_t = t$) which have the chaos representation property. It is analogous to the Skorohod integral as developed by Nualart and Pardoux [13]. When M is Brownian motion, it is exactly the Skorohod integral.

There are many similarities between the above-mentioned martingale anticipating integral and the Skorohod integral, but there are also some important defferences. Many of these differences stem from one key fact: in the Brownian case $[B,B]_t = \langle B,B\rangle_t = t$, while in the normal martingale case only $\langle M,M\rangle_t = t$, and $[M,M]_t$ is random. For example, there are two ways to describe the variational derivative (also known as the Malliavin derivative in the Brownian case), and they are equivalent in the Brownian case but not in the martingale case. In [12] an example is given, which shows that in the martingale case one cannot define the derivative operator in the usual way to obtain the Sobolev space structure for the space $\mathbf{D}_{2,1}$. Indeed, this example somehow shows that the two definitions (Sobolev space and chaos expansion) are compatible if and only if $[M,M]$ is deterministic. Therefore in the martingale case the space $\mathbf{D}_{p,1}$ $(1 < p < 2)$ cannot be defined in the usual way (i. e., by closing the class of smooth functionals with respect to the corresponding norm).

The aim of this paper is to define the space $\mathbf{D}_{p,1}$ $(1 < p < 2)$ for the normal martingale and generalize the Ocone–Haussmann– Clark formula for functionals from the class $\mathbf{D}_{p,1}$ $(1 < p < 2)$ which was proved in [12] for functionals from the space $\mathbf{D}_{2,1}$. This extension is supposed to be useful for an optimal portfolio representation when the equation, which describes the evolution of assets, is driven by a normal martingale. We hope to address these issues in the future work.

Note that for proving the main result in [11] essential use was made of the so-called "good λ inequality" (cf. Lemma 42.3 [14]) for continuous martingales. The above-mentioned inequality makes, in the case of a continuous martingale, it possible to overestimate the probability of a predictable quadratic variation deviation by the probability of deviation of maximum of a martingale. In particular, $\sup\limits_{0\le t\le T}|M^n(t)| \xrightarrow{P} 0$ implies $\langle M^n, M^n\rangle_T \xrightarrow{P} 0$. This fact has given the authors of [11] a chance to generalize representation (0.1) to case of functionals from $\mathbf{D}^w_{1,1}$ for Brownian motion.

For martingales whose trajectories are not continuous (i.e., belong to the Skorohod space), the above-mentioned fact in general is not true. In particular, we have no chance, because of the convergence to zero in probability $\sup\limits_{0\le t\le T}|M^n(t)|$, to state that the sequence of quadratic variations converges to zero in probability. In the general case we have estimates only for mean values (in particular, the inequalities of Burkholder–Davis–Gundy's type), and these esimates allow us to generalize the representations obtained in [12] to the cases of functionals of the class $\mathbf{D}_{p,1}$ $(1 < p < 2)$. However, this extension could not cover the case $p = 1$, which is well-known in the Brownian case.

This paper is organized as follows. In Section 1 we recall the definitions and some elementary properties of a variational derivative operator and an anticipating integral with respect to a normal martingale and give the Ocone–Haussmann–Clark formula from [12]. In Section 2 the space $\mathbf{D}_{p,1}$ $(1 < p < 2)$ is defined for the class of normal martingales and the main result of this paper is proved.

1. Auxiliary Notation and Results

Let Σ_n be an increasing simplex of R^n_+:

$$\Sigma_n\big\{(t_1,\dots,t_n)\in R^n_+ : 0<t_1<\cdots<t_n\big\},$$

and extend a function f defined on Σ_n by making f symmetric on R^n_+. We can then define the multiple integral as

$$I_n(f) = n!\int\limits_{\Sigma_n} f(t_1,\dots,t_n)dM_{t_1}\cdots dM_{t_n}. \tag{1.1}$$

Let us define the class of functions for $n\in N$ and $T=[0,1]$ by $L^2_s(T^n) = \{f\in L^2(T^n) : f \text{ is symmetric in all variables}\}$.

The multiple integral with respect to M defined in (1.1) is assumed to be defined for every $f\in L^2_s(T^n)$ (or $L^2(\Sigma_n)$). It is known (see, for example, Meyer [15]) that for each $f \in L_s^2(T^n)$:

$$\|I_n(f)\|^2_{L^2(\Omega)} = (n!)^2\|f\|_{L^2(\Sigma)} = n!\|f\|^2_{L_s^2(T^n)}.$$

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Definition 1.1. (cf. Definition 3.2 [12]). Let $G = \sigma\{M_t; t \geq 0\}$ be the σ-field generated by a (normal) martingale M. Let H_n be the n-th homogeneous chaos, $H_n = I_n(f)$, where f ranges over all $f \in L^2(\Sigma_n)$. If $L^2(G, dP) = \bigoplus\limits_{n=0}^{\infty} H_n$ (the direct sum), then we say M possesses the chaos representation property (CRP).

Let $(\Omega, \mathcal{J}, \{\mathcal{J}_t\}, P)$ be a filtered probability space satisfying the usual conditions. In what follows we shall always assume that a normal martingale M with the CRP is given on the probability space $(\Omega, \mathcal{J}, \{\mathcal{J}_t\}, P)$, and that $\mathcal{J}$ is generated by M. Thus, for any random variable $F \in L^2(\mathcal{J}, dP) = L^2(L^2(\Omega))$ we have by the CRP that there exists a sequence of functions $f_n \in L_s^2(T^n)$, $n = 1, 2, \dots$, such that $F = \sum\limits_{n=0}^{\infty} I_n(f_n)$.

Consider the following subset $\mathbf{D}_{2,1} \subset L^2(\Omega)$:

$$\mathbf{D}_{2,1} := \Big\{F = \sum_{n=0}^{\infty} I_n(f_n) : \sum_{n=0}^{\infty} nn!\|f_n\|^2_n < \infty\Big\},$$

where $\|\cdot\|_n := \|\cdot\|_{L^2(T^n)}$. It is easily seen that $\mathbf{D}_{2,1}$ is dense in $L^2(\Omega)$ since every element in the finite Hilbert sum of chaoses belongs to $\mathbf{D}_{2,1}$.

The derivative operator is analogous to what is often called the Maliavin derivative in the Brownian case, and it is defined as a linear operator $D : \mathbf{D}_{2,1} \subset L^2(\Omega) \to L^2(T \times \Omega)$ by

$$D_t F := \sum_{n=1}^{\infty} nI_{n-1}\big(f_n(\cdot, t)\big), \quad t \in [0, 1],$$

whenever F has the chaos expansion $F = \sum\limits_{n=0}^{\infty} I_n(f_n)$. It is easy to see that

$$\|D_t F\|^2_{L^2(T\times\Omega)} = \sum_{n=1}^{\infty} nn!\|f_n\|^2_n < \infty,$$

for all $F \in \mathbf{D}_{2,1}$.

Note that since D is a densely defined operator, one can define its adjoint operator, denoted by δ, in the usual way. Let

$$\mathcal{R} := \Big\{u \in L^2(T \times \Omega) : \exists c > 0, \Big|E\int_0^1 u(t, \cdot) D_t F\, dt\Big| \leq c\|F\|, \forall F \in \mathbf{D}_{2,1}\Big\},$$

and the adjoint operator $\delta : L^2(T \times \Omega) \to L^2(\Omega)$ is defined by the equation:

$$E\big(\delta(u)F\big) = E\int_0^1 u(t, \cdot) D_t F\, dt, \quad \forall F \in \mathbf{D}_{2,1}, \quad u \in \mathcal{R}.$$

As the adjoint operator of D, δ is a densely defined, closed operator, which in turn shows that the operator D is closable. The operator δ is called the anticipating integral of the element in $\mathcal{R} = \mathrm{Dom}(\delta)$. If $u \in L^2(T \times \Omega)$ is predictable, then $u \in \mathrm{Dom}(\delta)$ and $\delta(u) = \int_0^1 u_t dM_t$, where the right-hand side is in the semimartingale (or Ito) sense (see Proposition 4.4 [12]).

Let M be a normal martingale with the CRP. For any $F \in L^2(\Omega)$, we can write

$$F = \sum_{n=0}^{\infty} I_n(f_n) = E(F) + \sum_{n=1}^{\infty} I_n(f_n) = E(F) + \int_0^1 u_t \, dM_t,$$

where u is a predictable process given by

$$u_t = \sum_{n=1}^{\infty} n! \int_{t_1 < t_2 < \cdots < t_{n-1} < t} f_n(t_1, \ldots, t_{n-1}, t) \, dM_1 \cdots dM_{t_{n-1}}.$$

In the Brownian case, the predictable process can be further described using the Malliavin derivative operator known as the Ocone–Haussmann–Clark formula (see (0.1)). In [12] an analogue of that formula is derived for the class of normal martingales with the CRP.

Theorem 1.1. (*cf. Theorem* 4.5[12]). *Let M be a normal martingale with the CRP, and let $F \in L^2(\Omega)$. If $F \in \mathbf{D}_{2,1}$, then*

$$F = E(F) + \int_0^1 {}^p(D_t F) \, dM_t,$$

where ${}^p(D_t F)$ denotes the predictable projection of the process $D_t F$.

2. An Ocone–Haussmann–Clark Type Formula

Consider a filtered probability space $(\Omega, \mathcal{J}, \{\mathcal{J}_t\}, P)$ satisfying the usual conditions. Assume that the normal martingale M with the CRP is given on the probability space $(\Omega, \mathcal{J}, \{\mathcal{J}_t\}, P)$, and that $\mathcal{J}$ is generated by M, $\mathcal{J} = \mathcal{J}_1$.

Definition 2.1. Fix $1 < p < 2$ and introduce the norm

$$\|F\|_{p,1} := E\big(|F|_{L^p} + \|DF\|_{L^2([0,1])}\big)$$

on $\mathbf{D}_{2,1}$, and denote by $\mathbf{D}_{p,1}$ $(1 < p < 2)$ the Banach space which is the closure of $\mathbf{D}_{2,1}$ under the norm $\| \cdot \|_{p,1}$.

Note that DF is well-defined on $\mathbf{D}_{p,1}$ $(1 < p < 2)$ by the closure. Given $F \in \mathbf{D}_{p,1}$ $(1 < p < 2)$ we can find a measurable process $(t, \omega) \longmapsto D_t F(\omega)$ such that for a.e., $\omega \in \Omega$, $D_t F(\omega) = DF(\omega)(t)$ holds for almost all $t \in [0, 1]$ (more precisely, $t \longmapsto D_t F(\omega)$ is in the equivalence class in $L^2([0, 1])$) defined by $DF(\omega)$). $D_tF(\omega)$ is defined uniquely up to sets of measure zero on $[0,1]\times\Omega$. (In general, if $X:\Omega\to L^2([0,1])$ is measurable, there exists a $B([0,1])\otimes\mathcal{J}$-measurable random variable, $\{\overline{X}(t,\omega);\,(t,\omega)\in T\times\Omega\}$, such that $\overline{X}(\cdot,\omega)=X(\omega)$ holds almost surely. We shall identify $X(\omega)(t)$ with $\overline{X}(t,\omega)$.

Remark 2.1. Obviously, the norm $\|\cdot\|_{p,1}$ and the space $\mathbf{D}_{p,1}$ can be defined for $p=1$ as well.

Theorem 2.1. *Let M be a normal martingale with the CRP, and let $F\in L^p(\Omega)$ $(1<p<2)$. If $F\in\mathbf{D}_{p,1}$, then*

$$F=E(F)+\int\limits_0^1 {}^p(D_tF)\,dM_t,$$

where ${}^p(D_tF)$ denotes the predictable projection of the process D_tF.

Proof. Consider a sequence $\{F_n\}_{n=1}^\infty\subset\mathbf{D}_{2,1}$ such that

$$\lim_{n\to\infty}\big\|F_n-F\big\|_{p,1}=0. \tag{2.1}$$

Let us introduce the following martingales:

$$N(t):=E\big(F|\mathcal{J}_t\big) \quad\text{and}\quad N_n(t):=E\big(F_n|\mathcal{J}_t\big),\quad n=1,2,\dots.$$

Then, by the well-known martingale representation theorem for normal martingales with the CRP, they admit respectively the representations

$$N(t)=E(F)+\int\limits_0^t u(s)\,dM_s$$

and

$$N_n(t)=E(F)+\int\limits_0^t u_n(s)\,dM_s,$$

where u is a predictable process.

By Theorem 1.1, $u_n(t)={}^p(D_tF_n)$.

By the Doob maximal submartingale inequality, using relation (2.1), we have

$$\big\{E\big[\sup_{0\le t\le1}\big|N_n(t)-N(t)\big|\big]^p\big\}^{1/p}\le\frac{p}{p-1}\sup_{0\le t\le1}\big[E\big|N_n(t)-N(t)\big|^p\big]^{1/p}=$$
$$=\frac{p}{p-1}\big[E\big|N_n(1)-N(1)\big|^p\big]^{1/p}=\frac{p}{p-1}\big|F_n-F\big|_{L^p}\le\frac{p}{p-1}\big\|F_n-F\big\|_{p,1}\to0,$$

as $n\to\infty$.

Therefore, by virtue the Burkholder–Davis–Gundy's inequality, we can write

$$E\big[N_n - N, N_n - N\big]_1^{p/2} = E\Big\{\int\limits_0^1 \big|u_n(t) - u(t)\big|^2 d[M,M]_t\Big\}^{p/2} \leq$$

$$\leq cE\big\{\sup_{0\leq t\leq 1} |N_n(t) - N(t)|\big\}^p \to 0,$$

as $n \to \infty$.

Then according to Lyapunov inequality we have

$$\lim_{n\to\infty} E\Big\{\int\limits_0^1 |u_n(t) - u(t)|^2 d[M,M]_t\Big\}^{1/2} \leq$$

$$\leq \lim_{n\to\infty} \bigg(E\Big\{\int\limits_0^1 \big|u_n(t) - u(t)\big|^2 d[M,M]_t\Big\}^{p/2}\bigg)^{1/p} = 0.$$

Hence,

$$P\!-\lim_{n\to\infty}\int\limits_0^1 \big|u_n(t) - u(t)\big|^2 \, d[M,M]_t = 0,$$

where $P\!-\lim\limits_{n\to\infty}$ denotes the limit in probability.

Furthermore, using the Kunita–Watanabe inequality, we conclude that

$$P\!-\lim_{n\to\infty}\int\limits_0^1 \big|u_n(t) - u(t)\big| \, d[M,M]_t \leq$$

$$\leq P\!-\lim_{n\to\infty}\bigg\{\bigg(\int\limits_0^1 \big|u_n(t)-u(t)\big|^2 \, d[M,M]_t\bigg)^{1/2} \cdot \big([M,M]_1\big)^{1/2}\bigg\} = 0. \quad (2.2)$$

On the other hand, the linearity of the predictable projection operator and Malliavin derivative operator imply that (2.3)

$$E\int\limits_0^1 \big|u_n(t) - {}^p(D_tF)\big| \, dt = E\int\limits_0^1 \big|{}^p(D_tF_n) - {}^p(D_tF)\big| \, dt =$$

$$= E\int\limits_0^1 \big|{}^p[D_t(F_n - F)]\big| \, dt. \quad (2.3)$$

Furthermore, by the proof of Theorem 14 from [17] one can see that if $X_t \leq Y_t$, then ${}^p(X_t) \leq {}^p(Y_t)$. Hence since $X_t \leq |X_t|$ and $-X_t \leq |X_t|$, we have ${}^p(X_t) \leq {}^p(|X_t|)$ and $-{}^p(X_t) = {}^p(-X_t) \leq {}^p(|X_t|)$. Thus we conclude that $|{}^p(X_t)| \leq {}^p(|X_t|)$. Moreover, it is obvious that $E[{}^p(X_t)] = E(X_t)$

(provided that we take $T = t < \infty$ in the above-mentioned theorem from [17]).

Therefore, using the Cauchy-Bunyakovski inequality, from relation (2.3) we obtain

$$E\int_0^1 \big|u_n(t) - {}^p(D_tF)\big|\,dt \le E\int_0^1 {}^p\big[|D_t(F_n - F)|\big]\,dt =$$
$$= E\int_0^1 \big|D_t(F_n - F)\big|\,dt \le E\bigg[\bigg(\int_0^1 \big|D_t(F_n - F)\big|^2\,dt\bigg)^{1/2}\bigg] \le$$
$$\le \big\|F_n - F\big\|_{p,1} \to 0. \tag{2.4}$$

as $n \to \infty$.

From here, because for every predictable process v:

$$E\int_0^1 \big|v(t)\big|\,dt = E\int_0^1 \big|v(t)\big|\,d\langle M, M\rangle_t = E\int_0^1 \big|v(t)\big|\,d[M, M]_t,$$

we ascertain that

$$\lim_{n\to\infty} E\int_0^1 \big|u_n(t) - {}^p(D_tF)\big|\,d[M, M]_t = 0 \tag{2.5}$$

and, moreover,

$$E\int_0^1 \big|{}^p(D_tF)\big|\,d[M, M]_t = E\int_0^1 \big|{}^p(D_tF)\big|\,dt < \infty.$$

According to relations (2.2) and (2.5) we can choose a sequence $\{n_k\}_{k\ge1}$ such that for almost all ω:

$$\lim_{k\to\infty}\int_0^1\big|u_{n_k}(t) - u(t)\big|d[M, M]_t = 0 \ \text{ and } \ \lim_{n\to\infty}\int_0^1\big|u_{n_k}(t) - {}^p(D_tF)\big|[M, M]_t = 0.$$

Therefore we conclude that for almost all ω:

$$u(\cdot, \omega) = {}^p(D.F)(\omega) \quad d[M, M]_t - \text{almost surely.}$$

Hence, for almost all ω, we have

$$\int_0^1 \big|u(t) - {}^p(D_tF)\big|\,d[M, M]_t = 0.$$

Thus, we obtain

$$E\int_0^1 \left|u(t) - {}^p(D_tF)\right| d[M,M]_t = 0.$$

From here, we ascertain that

$$u(t,\omega) = {}^p(D_tF)(\omega) \quad dP \otimes d[M,M]_t - \text{almost surely},$$

and, hence, dP-almost surely:

$$\int_0^1 u(t)\,dM_t = \int_0^1 {}^p(D_tF)\,dM_t.$$

□

Remark 2.2. From the proof of the Theorem 2.1 we see that

$$\int_0^1 \left|{}^p(D_tF)\right|^2 d[M,M]_t < \infty - \text{almost surely},$$

for every $F \in \mathbf{D}_{p,1}$ $(1 < p < 2)$. It does not seem possible to argue this fact using only that

$$E\|DF\|_{L^2([0,1])} = E\left\{\left(\int_0^1 \left|D_tF\right|^2 dt\right)^{1/2}\right\} < \infty.$$

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Author's address:
A. Razmadze Mathematical Institute
Georgian Academy of Sciences
1, M. aleksidze St., Tbilisi 0193
Georgia