ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 5, November 2014

Hedging of One European Option of Integral Type in Black-Scholes Model

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***Abstract*— the problem of European Option hedging in the case of Black-Scholes market model is considered. It is well-known, that the Clark-Ocone integral representation formula is the effective tool for the solving of hedging problem. But in this case there are some difficulties to use this formula directly, because integrand of payoff is not differentiable by Malliavin. So, we solve this hedging problem using the local time of the risky asset price process and its relationship with the payoff of option.**

Key words or phrases: **Black-Scholes model, Clark-Ocone representation, local time, Trotter-Meyer Theorem, hedging problem.**

I. INTRODUCTION AND PRELIMINARIES

We consider the European Option of integral type in the case of Black-Scholes market model. We develop the method of hedging of option based on the using the local time of the risky asset price S. We give the Clark representation of local time and then using the relation between payoff of option and local time based on the stochastic type Fubini theorem we obtain the Clark integral representation of payoff our option. Therefore we solve the hedging problem. The method will be useful in cases when there are difficulties to use directly the Clark-Ocone integral representation ([1], [6]).

Let on the probability space $(\Omega, \Im, P)$ be given the Wiener process $w = (w_t)$, $t \in [0,T]$ and $(\Im_t^w)$, $t \in [0,T]$ be the natural filtration generated by the Wiener process w. Consider the Black-Scholes market model with risk-free asset price evolution described by

$$dB_t = rB_t dt\,, \quad B_0 = 1, \qquad (1.1)$$

where $r \geq 0$ is interest rate and risky asset price evolution

$$dS_t = \mu S_t dt + \sigma S_t dw_t\,, \qquad (1.2)$$

where $\mu \in R$ is appreciation rate and $\sigma > 0$ is volatility coefficient.

Let

$$Z_T = \exp\{-\frac{\mu - r}{\sigma} w_T - \frac{1}{2}(\frac{\mu - r}{\sigma})^2 T\}$$

and $\overline{P}_T$ is the measure on $(\Omega, \Im_T^w)$ such that $d\overline{P}_T = Z_T dP$.

From Girsanov's Theorem it follows (see [8]) that under this measure (martingale risk neutral measure)

$$\overline{w}_t = w_t + \frac{\mu - r}{\sigma} t$$

is the standard Wiener process and

$$dS_t = rS_t dt + \sigma S_t d\overline{w}_t\,, \qquad (1.3)$$

or

$$S_t = S_0 \exp\{\sigma \overline{w}_t + (r - \sigma^2 / 2)t\}.$$

Consider the problem of "replication" the European Option with the payoff of integral type

$$G = \int_0^T I_{\{a \leq S_t \leq b\}} S_t^2 dt\,, \qquad (1.4)$$

where a and b are some positive constants, $a < b$.

Remark 1. It is well-known, that the Clark-Ocone stochastic integral representation formula is the effective tool for the solving of hedging problem. But in our case there are some difficulties to use this formula directly, because integrands of integral type payoff are not differentiable by Malliavin. In the Malliavin theory it is well-known that the indicator of event A is Malliavin differentiable if and only if probability $P(A)$ is equal to zero or one. Hence, for all t the indicator $I_{\{a \leq S_t \leq b\}}$ has not Malliavin derivative. Earlier, by us it was proved one result: if square integrable random processes u_t has the Wiener-Chaos decomposition with kernels $f_{u,n}^t(\cdot)$, measurable in all their variables, then the Lebesgue average process with respect to dt, has the Wiener-Chaos decomposition with kernels coinciding to the Lebesgue average of $f_{u,n}^t(\cdot)$ with respect to dt. On this basis we established that if the square integrable random process is not stochastic differentiable, then the Lebesgue average (wih respect to dt) process also is not stochastic differentiable. Hence, the functional G (from (1.4)) considered by us is not stochastic differentiable, and, therefore application of the Clark-Ocone integral representation formula is impossible. In this paper, we try to obtain the Clark integral representation formula with known integrand applying a nonconventional method.

Our main goal is to find a trading strategy $\pi = (\beta_t, \gamma_t)$, $t \in [0,T]$ such that the capital process

$$X_t = \beta_t B_t + \gamma_t S_t\,, \quad X_T = G \qquad (1.5)$$

under the self-financing condition

$$dX_t = \beta_t dB_t + \gamma_t dS_t\,. \qquad (1.6)$$

Let for simplicity $r = 0$ and $S_0 = 1$. Then, on the one hand, from (1.6) and (1.3) we have

$$X_t = X_0 + \sigma \int_0^t \gamma_u S_u d\overline{w}_u\,.$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 5, November 2014

On the other hand, from (1.5) one have

$$G = X_T = X_0 + \sigma \int_0^T \gamma_u S_u d\overline{w}_u . \qquad (1.7)$$

Our problem is to find the trading strategy (γ_t), $t \in [0,T]$. From (1.7) we see that this problem equivalent to finding a martingale representation of the payoff G with explicit form of integrand. Note that G is square integrable functional (it will be checked below) of Wiener process $\overline{w} = (\overline{w}_t)$, $t \in [0,T]$ and therefore we try to obtain the Clark integral representation with known integrand.

II. AUXILIARY RESULTS

Consider the local time of stochastic process $S = (S_t)$, $t \in [0,T]$. By definition (see [7], (IV.44.1)) local time of S at point $x \in R$ is

$$l_t^x(S) = |S_t - x| - |S_0 - x| - \int_0^t \operatorname{sgn}(S_u - x) dS_u . \quad (2.1)$$

For any measurable and bounded real function ψ it is true (see [7], Trotter, Meyer Theorem IV.45.1) the following relation

$$\int_0^T \psi(S_t) d\langle S \rangle_t = \int_{-\infty}^{\infty} l_T^x(S) \psi(x) dx , \qquad (2.2)$$

where $\langle S \rangle_t$ is the predictable square variation of martingale S.

Suppose

$$\psi(x) = I_{\{a \le x \le b\}} .$$

Note also that in considered case ($r = 0$), according to the Ito's formula, we have

$$dS_t^2 = 2\sigma S_t^2 d\overline{w}_t + \sigma^2 S_t^2 dt ,$$

and, therefore

$$\langle S \rangle_t = \int_0^t \sigma^2 S_u^2 du .$$

Then, from (2.2) we obtain

$$\int_0^T I_{\{a \le S_t \le b\}} \sigma^2 S_t^2 dt = \int_a^b l_T^x(S) dx , \quad (2.3)$$

and, hence, the payoff G of our option will become the following form

$$G = \frac{1}{\sigma^2} \int_a^b l_T^x(S) dx . \qquad (2.4)$$

Theorem 1. The Clark integral representation of local time $l_T^x(S)$ is the following

$$l_T^x(S) = \overline{E}(|S_T - x|) - |1 - x| +$$

$$+ \sigma \int_0^T \{1 - 2\Phi[\frac{\ln x - \sigma \overline{w}_t - \sigma^2 (T/2 - t)}{\sigma \sqrt{T - t}}]\} S_t d\overline{w}_t$$

$$- \sigma \int_0^T \{\operatorname{sgn}(S_t - x)\} S_t d\overline{w}_t . \qquad (2.5)$$

Proof. In our case ($r = 0$, $S_0 = 1$) from the relations (1.3) and (2.1) we have:

$$l_T^x(S) = |S_T - x| - |1 - x| - \sigma \int_0^T \operatorname{sgn}(S_t - x) S_t d\overline{w}_t . \quad (2.6)$$

Using the Clark-Ocone representation formula we find the integral representation of $|S_T - x|$. Note that the Malliavin derivative of $|S_T - x|$ is (see [5], Proposition 1.2.4 or [4], Theorem 2)

$$D_t(|S_T - x|) = D_t[(S_T - x)^+ + (S_T - x)^-] =$$
$$= \sigma S_T I_{\{S_T \ge x\}} - \sigma S_T I_{\{S_T < x\}}$$

and consequently the integrand of Clark-Ocone integral representation will be

$$\overline{E}[D_t(|S_T - x|) \mid \Im_t^{\overline{w}}] = \overline{E}[\sigma S_T I_{\{S_T \ge x\}} \mid \Im_t^{\overline{w}}] -$$
$$- \overline{E}[\sigma S_T I_{\{S_T < x\}} \mid \Im_t^{\overline{w}}] := J_1 - J_2 ,$$

where $\Im_t^{\overline{w}}$ is the natural filtration of Wiener process $\overline{w}$.

According to the well-known properties of Wiener process and conditional mathematical expectation, it is not difficult to see that

$$J_1 := \overline{E}[\sigma S_T I_{\{S_T \ge x\}} \mid \Im_t^{\overline{w}}] = \sigma \overline{E}[\exp\{\sigma \overline{w}_T - \sigma^2 T/2\}$$
$$\cdot I_{\{\exp\{\sigma \overline{w}_T - \sigma^2 T/2\} \ge x\}} \mid \Im_t^{\overline{w}}] =$$
$$= \sigma \overline{E}[\exp\{\sigma(\overline{w}_T - \overline{w}_t) + \sigma \overline{w}_t - \sigma^2 T/2\}$$
$$\cdot I_{\{\sigma(\overline{w}_T - \overline{w}_t) + \sigma \overline{w}_t - \sigma^2 T/2 \ge \ln x\}} \mid \Im_t^{\overline{w}}] =$$
$$= \sigma \exp\{\sigma \overline{w}_t - \sigma^2 T/2\} \overline{E}[\exp\{\sigma(\overline{w}_T - \overline{w}_t)\}$$
$$\cdot I_{\{\sigma(\overline{w}_T - \overline{w}_t) + \sigma \overline{w}_t - \sigma^2 T/2 \ge \ln x\}} \mid \Im_t^{\overline{w}}] =$$
$$= \sigma \exp\{\sigma \overline{w}_t - \sigma^2 T/2\} \overline{E}[\exp\{\sigma(\overline{w}_T - \overline{w}_t)\}$$
$$\cdot I_{\{\sigma(\overline{w}_T - \overline{w}_t) + \sigma \overline{w}_t - \sigma^2 T/2 \ge \ln x\}} \mid \overline{w}_t] =$$
$$= \sigma S_t \exp\{-\sigma^2 (T - t)/2\} \{\overline{E}[\exp\{\sigma(\overline{w}_T - \overline{w}_t)\}$$
$$\cdot I_{\{\sigma(\overline{w}_T - \overline{w}_t) \ge \ln x - \sigma y + \sigma^2 T/2\}}]\} |_{y = \overline{w}_t} :=$$
$$:= \sigma S_t \exp\{-\sigma^2 (T - t)/2\} \{J_1(y)\} |_{y = \overline{w}_t}$$

For the purpose of calculation the last multiplier $J_1(y)$ above we will use the fact that the random variable $\sigma(\overline{w}_T - \overline{w}_t)$ has the normal distribution with the parameters 0 and $\sigma^2 (T - t)$. Then, due to the standard technique of integration, we easily obtain[1]

[1] Here and below $\Phi_{\nu,\sigma}$ is the normal distribution function with parameters ν and σ, and $\varphi_{\nu,\sigma}$ is its density function; $\Phi := \Phi_{0,1}$ and $\varphi := \varphi_{0,1}$.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 5, November 2014

$$J_1(y):=\overline{E}[\exp\{\sigma(\overline{w}_T-\overline{w}_t)\}I_{\{\sigma(\overline{w}_T-\overline{w}_t)\geq \ln x-\sigma y+\sigma^2 T/2\}}]=$$

$$=\frac{1}{\sqrt{2\pi\sigma^2(T-t)}}\int_{\ln x-\sigma y+\sigma^2 T/2}^{\infty}\exp\{v\}\exp\{-\frac{v^2}{2\sigma^2(T-t)}\}dv=$$

$$=\frac{1}{\sqrt{2\pi\sigma^2(T-t)}}\exp\{\sigma^2(T-t)/2\}$$

$$\cdot\int_{\ln x-\sigma y+\sigma^2 T/2}^{\infty}\exp\{-\frac{[v-\sigma^2(T-t)]^2}{2\sigma^2(T-t)}\}dv=$$

$$=\exp\{\sigma^2(T-t)/2\}$$

$$\cdot[1-\Phi_{\sigma^2(T-t),\sigma^2(T-t)}(\ln x-\sigma y+\sigma^2 T/2)]=$$

$$=\exp\{\sigma^2(T-t)/2\}$$

$$\cdot\{1-\Phi[\frac{\ln x-\sigma y-\sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}.$$

Combining the above-obtained relations, we conclude that

$$J_1=\sigma S_t\exp\{-\sigma^2(T-t)/2\}\exp\{\sigma^2(T-t)/2\}$$

$$\cdot\{1-\Phi[\frac{\ln x-\sigma\overline{w}_t-\sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}=$$

$$=\sigma S_t\{1-\Phi[\frac{\ln x-\sigma\overline{w}_t-\sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}. \qquad (2.7)$$

Further, by arguments similar to those used in proving of relation (2.7), it is not difficult to verify that

$$J_2:=\overline{E}[\sigma S_T I_{\{S_T<x\}}\mid\Im_t^{\overline{w}}]=\sigma\overline{E}[\exp\{\sigma\overline{w}_T-\sigma^2 T/2\}$$

$$\cdot I_{\{\exp\{\sigma\overline{w}_T-\sigma^2 T/2\}<x\}}\mid\Im_t^{\overline{w}}]=$$

$$=\sigma S_t\Phi[\frac{\ln x-\sigma\overline{w}_t-\sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]. \qquad (2.8)$$

Hence, due to the Clark-Ocone integral representation formula, taking into account the relations (2.7) and (2.8), we ascertain that

$$|S_T-x|=\overline{E}(|S_T-x|)+\int_0^T\overline{E}[D_t(|S_T-x|)\mid\Im_t^{\overline{w}}]d\overline{w}_t=$$

$$=\overline{E}(|S_T-x|)$$

$$+\sigma\int_0^T\{1-2\Phi[\frac{\ln x-\sigma\overline{w}_t-\sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}S_t d\overline{w}_t. \qquad (2.9)$$

Summing up the relations (2.6) and (2.9), we complete the proof of theorem.

III. INTEGRAL REPRESENTATION THEOREM AND HEDGING THE OPTION WITH PAYOFF G

Recall that, our aim is to find the stochastic integral representation of the Wiener functional G from (2.4). At first, we will check that it is a square integrable functional. Indeed, due to the Helder's inequality, we have

$$\overline{E}(G^2)=\overline{E}\{\int_0^T I_{\{a\leq S_t\leq b\}}S_t^2 dt\}^2\leq\overline{E}\{T\int_0^T I_{\{a\leq S_t\leq b\}}S_t^4 dt\}\leq$$

$$\leq\overline{E}\{T\int_0^T b^4 dt\}=T^2 b^4<\infty.$$

Theorem 2. The following integral representation formula is fulfilled

$$G=\frac{1}{\sigma^2}\int_a^b[\overline{E}(|S_T-x|)-|1-x|]dx+\int_0^T v_t d\overline{w}_t,$$

where

$$v_t=\frac{1}{\sigma}S_t\int_a^b\{1-2\Phi[\frac{\ln x-\sigma\overline{w}_t-\sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}dx-$$

$$-\frac{1}{\sigma}S_t\int_a^b\{\mathrm{sgn}(S_t-x)\}dx. \qquad (3.1)$$

Proof. At beginning, let us rewrite the relation (2.5) in the form

$$l_T^x(S):=C(T,x)+\int_0^T g(\sigma,t,x,\overline{w}_t)S_t d\overline{w}_t. \qquad (3.2)$$

According to the Clark formula, there exists the adapted (to the filtration of $\overline{w}$) and square integrable process $h(t,x)$ such that the functional G admits the representation

$$G=\overline{E}G+\int_0^T h(t,x)d\overline{w}_t. \qquad (3.3)$$

Suppose that u_t is any adapted an square integrable process on the $[0,T]\times\Omega$ and let us denote

$$F:=\int_0^T u_t d\overline{w}_t.$$

Then, on the one hand, we have

$$\overline{E}(GF)=\overline{E}\{\int_0^T h(t,x)u_t dt\}. \qquad (3.4)$$

On the other hand, using the stochastic Fubiny Theorem (see [3], Lemma III.4.1 or [2], Corollary of the Lemma IV.2.4), due to the relation (2.3) it is easy to see that

$$G=\frac{1}{\sigma^2}\int_a^b\{C(T,x)+\int_0^T g(\sigma,t,x,\overline{w}_t)S_t d\overline{w}_t\}dx=$$

$$=\frac{1}{\sigma^2}\int_a^b C(T,x)dx+\frac{1}{\sigma^2}\int_0^T\{S_t[\int_a^b g(\sigma,t,x,\overline{w}_t)dx]\}d\overline{w}_t.$$

Therefore we can write

$$\overline{E}(GF)=\frac{1}{\sigma^2}[\int_a^b C(T,x)dx]\cdot\overline{E}(F)+$$

$$+\frac{1}{\sigma^2}\overline{E}\int_0^T[u_t S_t\int_a^b g(\sigma,t,x,\overline{w}_t)dx]dt=$$

$$=\frac{1}{\sigma^2}\overline{E}\int_0^T\{[\int_a^b g(\sigma,t,x,\overline{w}_t)dx]S_t u_t\}dt. \qquad (3.5)$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 5, November 2014

Due to the relations (3.4) and (3.5), for any $u \in L_2([0,T]\times\Omega)$ we have

$$\overline{E}(\int_0^T \{h(t,x) - \frac{1}{\sigma^2}[\int_a^b g(\sigma,t,x,\overline{w}_t)dx]S_t\}u_t dt) = 0 .$$

From here we conclude that for almost all (t,x,ω):

$$h(t,x) = \frac{1}{\sigma^2}[\int_a^b g(\sigma,t,x,\overline{w}_t)dx]S_t . \qquad (3.6)$$

Combining now the relations (2.5), (1.4), (3.2), (3.3) and (3.6) we easily ascertain that the representation (3.1) is fulfilled.

The result of Theorem 2 give to us the possibility to find the component γ_t of the hedging strategy $\pi = (\beta_t, \gamma_t)$, $t \in [0,T]$ which is defined by integrand of representation (3.1) and is equal

$$\gamma_t = \frac{v_t}{\sigma S_t} = \frac{1}{\sigma^2}\int_a^b \{1 - 2\Phi[\frac{\ln x - \sigma\overline{w}_t - \sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}dx -$$

$$-\frac{1}{\sigma^2}\int_a^b \mathrm{sgn}(S_t - x)dx . \qquad (3.7)$$

Remark 2. Using the integration by parts formula, due to the well-known properties of integration, it is not difficult to see that

$$\int_a^b \{1 - 2\Phi[\frac{\ln x - \sigma\overline{w}_t - \sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}dx =$$

$$= b - a - 2\{x\Phi[\frac{\ln x - \sigma\overline{w}_t - \sigma^2(T/2-t)}{\sigma\sqrt{T-t}}]\}\Big|_a^b +$$

$$+2\exp\{\sigma\overline{w}_t + \sigma^2(T-3t/2)\}$$

$$\cdot\{\Phi[\frac{\ln x - \sigma\overline{w}_t - \sigma^2(3T/2-2t)}{\sigma\sqrt{T-t}}]\}\Big|_a^b .$$

On the other hand, it is not difficult to see that the last integral in (3.7) is equal

$$\int_a^b \mathrm{sgn}(S_t - x)dx = (x \wedge S_t)\Big|_a^b - (x \vee S_t)\Big|_a^b =$$

$$= (a-b)I_{\{S_t \le a\}} + (2S_t - a - b)I_{\{a < S_t \le b\}} + (b-a)I_{\{S_t > b\}} .$$

Further, using the result of Theorem 2, we can find the capital process

$$X_t = \overline{E}[G \mid \Im_t^{\overline{w}}] = \overline{E}G + \int_0^t v_s d\overline{w}_s . \qquad (3.8)$$

It is well-known (see [8], or (1.5) in this paper) that the second component β_t of hedging strategy π:

$$\beta_t = X_t - \gamma_t S_t . \qquad (3.9)$$

Therefore hedging strategy $\pi = (\beta_t, \gamma_t)$, $t \in [0,T]$ in problem of "replication" of integral type European Option with payoff G given by (1.4) in case of Black-Scholes model, is defined by relations (3.9), (3.8) and (3.7) and the price C of this option

$$C = \overline{E}G = \frac{1}{\sigma^2}\int_a^b [\overline{E}(|S_T - x|) - |1-x|]dx .$$

Remark 3. By the definition of a mathematical expectation, due to the well-known properties of the normal distribution, using the standard technique of integration, we easily conclude that

$$\int_a^b [\overline{E}(|S_T - x|)]dx = \int_a^b [\overline{E}(|\exp\{\sigma\overline{w}_T - \sigma^2 T/2\} - x|)]dx =$$

$$= \int_a^b \frac{1}{\sqrt{2\pi T}} \int_{-\infty}^{\infty} |\exp\{\sigma u - \frac{\sigma^2}{2}T\} - x|\exp\{-\frac{u^2}{2T}\}dudx =$$

$$= \int_a^b \frac{1}{\sqrt{2\pi T}} \int_{-\infty}^{\frac{\ln x + \sigma^2 T/2}{\sigma}} (x - \exp\{\sigma u - \frac{\sigma^2}{2}T\})\exp\{-\frac{u^2}{2T}\}dudx +$$

$$+ \int_a^b \frac{1}{\sqrt{2\pi T}} \int_{\frac{\ln x + \sigma^2 T/2}{\sigma}}^{\infty} (\exp\{\sigma u - \frac{\sigma^2}{2}T\} - x)\exp\{-\frac{u^2}{2T}\}dudx =$$

$$= \int_a^b x[2\Phi(\frac{\ln x + \sigma^2 T/2}{\sigma\sqrt{T}}) - 1]dx -$$

$$-\int_a^b \Phi(\frac{\ln x - \sigma^2 T/2}{\sigma\sqrt{T}})dx +$$

$$+\int_a^b [1 - \Phi(\frac{\ln x - \sigma^2 T/2}{\sigma\sqrt{T}})]dx =$$

$$= \{x^2[\Phi(\frac{\ln x + \sigma^2 T/2}{\sigma\sqrt{T}}) - \frac{1}{2}]$$

$$-\exp\{\sigma^2 T\}\Phi(\frac{\ln x - 3\sigma^2 T/2}{\sigma\sqrt{T}})\}\Big|_a^b -$$

$$-\{x[2\Phi(\frac{\ln x - \sigma^2 T/2}{\sigma\sqrt{T}}) - 1] - 2\exp\{\sigma^2 T\}$$

$$\cdot\Phi(\frac{\ln x - 3\sigma^2 T/2}{\sigma\sqrt{T}})\}\Big|_a^b =$$

$$= (x - \frac{x^2}{2})\Big|_a^b + \{x^2\Phi(\frac{\ln x + \sigma^2 T/2}{\sigma\sqrt{T}})\}\Big|_a^b -$$

$$-2\{x\Phi(\frac{\ln x - \sigma^2 T/2}{\sigma\sqrt{T}})\}\Big|_a^b +$$

$$+\exp\{\sigma^2 T\}\{\Phi(\frac{\ln x - 3\sigma^2 T/2}{\sigma\sqrt{T}})\}\Big|_a^b .$$

Moreover, it is easy to see that

$$\int_a^b |1-x|dx = \begin{cases} [(b-1)^2 - (a-1)^2]/2, & a > 1; \\ [(a-1)^2 + (b-1)^2]/2, & a \le 1 < b; \\ [(a-1)^2 - (b-1)^2]/2, & b \le 1. \end{cases}$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 5, November 2014

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Faculty of Mechanics and Mathematics of Tbilisi State University, 1980. Post-graduate student of Tbilisi State University (TSU) and Moscow State University, 1980-1984.

Scientific Degree:

Canditate of Physical and Mathematical Sciences, Moscow State University, 1984. Head of Department of Mathematics, Associate Professor, Tbilisi State University, since 2006

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2012- **Head** of Department of mathematics at TSU. 2006- Associate professor Faculty of Exact and Natural Sciences of TSU. 2009- Senior Specialist at the Office of Academic Process Management & Scientific Researches. 2000-2009 Senior staff scientist of A. Razmadze Mathematical Institute. 2006-2007 Chief of Educational-Scientific Mathematical Institute of TSU. 1990-2006 Senior staff scientist Laboratory of stochastic analysis and statistical decisions of TSU. 1985-2000 Staff scientist of A. Razmadze Mathematical Institute.

Publications and research work (more 50).

List of main publications in last 5 years:

Glonti O., and **Purtukhia O.** “Clark`s Representation of Wiener functional and Hedging of the Barrier Option”, Bulletin of the Georgian National Academy of Sciences, vol. 8, no. 1, pp. 32-39, 2014.

Dochviri B., **Purtukhia O.,** Sokhadze G., and Tkemaladze G. “On the Stochastic Model of a Chemical Reaction”, Bulletin of the Georgian National Academy of Sciences, vol. 7, no. 2, pp. 92-96, 2013.

Purtukhia O. “Sobolev and logarithmic Sobolev type inequalities”, Applied Mathematics, Informatics and Mechanics - AMIM, vol. 17, no 2, pp. 26-39, 2012. Nadaraya E., **Purtukhia O.,** and Sokhadze G. “On the Cramer-Rao inequality in an infinite dimensional space”, Proceedings of A. Razmadze Mathematical Institute, vol. 160, pp. 121-134, 2012. **Purtukhia O.,** and Jaoshvili V. “Ito type formula for Poisson anticipating integral”, Reports of Enlatged Session of the Seminar of I. Vekua Institute of Applied Mathe-matics, vol. 25, pp. 103-108, 2011. Dochviri B., Nadaraya E., **Purtukhia O.,** and Sokhadze G. “On one problem of Hypotheses testing”, Reports of Enlarged Session of the Seminar of I. Vekua Institute of Applied Mathematics, vol 24, pp. 31-34, 2010. **Purtukhia O.,** and Jaoshvili V. “The Stein’s identity and Poisson functionals”, Reports of Enlarged Session of the Seminar of I. Vekua Institute of Applied Mathematics, vol. 24, pp. 113-118, 2010. **Purtukhia O.,** and Jaoshvili V. “An Extension of the Ocone-Haussmann-Clark Formula For the Compensated Poisson Processes,” SIAM Journal Theory Probab. Appl. Volume 53, issue 2, pp. 316-321, 2009. **Participation in research grants projects in last 5 years:** 2006-2009 Georgian National Science Foundation (grant # 337/07, 06-223-3-104). 2010-2012 Georgian National Science Foundation (grants # 09_383_3-106 and grant # 09_471_3-104). 2013-2015 Shota Rustaveli National Scientific Foundation, # FR/69/5-104/12. 2013-2016 Shota Rustaveli National Scientific Foundation, # FR/308/5-104/12.