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DIFFERENT APPROACHES IN THE CONSTRUCTIVE MARTINGALE REPRESENTATION OF BROWNIAN FUNCTIONALS

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Annotation. *New approaches to the problem of stochastic integral representation of nonsmooth Brownian functionals are considered, including both the methods of nonanticipative stochastic analysis without using the Malliavin calculus and the anticipative stochastic analysis using the Malliavin calculus.*

In the theory of stochastic processes, the representation of functionals of Brownian motion by stochastic integrals, also known as the martingale representation theorem, states that a functional that is measurable with respect to the filtration generated by a Brownian motion can be written in terms of Ito's stochastic integral with respect to this Brownian motion. In the 80th of the past century, it turned out (Harison and Pliska, 1981) that the martingale representation theorems (along with the Girsanov's measure change theorem) play an important role in the modern financial mathematics. In particular, using the integrand of the stochastic integral appearing in the integral representation, one can construct hedging strategies in the European options of different type. Here we will explore the stochastically non-smooth Brownian functionnals that can be considered in the future as a payoff function of a certain exotic European Option and study the issues of their stochastic integral representation.

In the theory of stochastic integration, in contrast to the standard integration theory, besides the fact that the integrand is the measurable function of two variables, it should be the adapted (nonanticipated) process. Skorokhod (1975) replaced this requirement with the requirement of smoothness in some sense of the integrand. Later, Gaveau and Trauber have proved that the Skorokhod operator of stochastic integration coincides with the conjugate operator of a stochastic derivative (with the so-called Malliavin's) operator.

One of the important properties of the Ito stochastic integral is that the Ito stochastic integral as a process of a square integrable adapted integrand is a martingale. On the other hand, according to the well-known Clark formula ([1]), the inverse statement is also true: if F is a $\Im_T^B = \sigma\{B_s : 0 \le s \le T\}$-measurable random variable with $EF^2 < \infty$, then there exists the $\Im_t^B$-adapted process $\psi(\cdot,\cdot) \in L_2([0,T]\times\Omega)$, such that ($P$-a.s.) the integral representation: $F = EF + \int_0^T \psi(t,\omega)dB_t(\omega)$ holds.

However, this result says nothing on finding the process $\psi(t,\omega)$ explicitly. In this direction we are familiar with one sufficiently general result, the so-called Clark-Ocone's formula ([2]) by which for the Brownian functionals: $\psi(t,\omega) = E[D_t^B F \mid \Im_t^B](\omega)$, where $D_t^B F$ is the stochastic derivative of the functional F. It should be noted that application of the Clark-Ocone's formula needs as a rule, on the one hand, essential efforts, and, on the other hand, in the cases if the functional F has no stochastic derivative, its application is impossible.

In many papers using Malliavin calculus or some kind of differential calculus for stochastic processes, the results are quite general but unsatisfactory from the explicitness point of view: the integrands in the stochastic integral representations always involve predictable projections or conditional expectations and some kind of gradients. A different method for finding the process $\psi(t,\omega)$ was proposed by Shiryaev, Yor and Graversen (2003, 2006), which was based on the Ito (generalized) formula and the Levy theorem for the Levy martingale $M_t = E[F \mid \Im_t^B]$ associated with F. Later on, using the Clark-Ocone formula, Renaud and Remillard (2006) have established explicit martingale representations for path-dependent Brownian functionals.

Our approach with prof. Jaoshvili (2005-2009) in the framework of the classical Ito calculus, on the basis of the standard L_2 theory and the theory of weighted Sobolev spaces (without using the Malliavin calculus), made it possible to construct an explicit formula for the itegrand when the functional does not have the above-mentioned smoothness. Further, it has turned out that the requirement of smoothness of functional can be weakened by the requirement of smoothness only of its conditional mathematical expectation. We (with prof. O. Glonti, 2017) considered Brownian functionals which are not stochastically differentiable. In particular, we generalized the Clark-Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method of finding of integrand.

Theorem 1 (Theorem 2.1. [3]). *Suppose that* $g_t := E[F \mid \Im_t^B]$ *is Malliavin differentiable* ($g_t \in D_{2,1}^B$) *for almost all* $t \in [0,T)$. *Then we have the stochastic integral representation* $g_T = F = EF + \int_0^T \nu_u dB_u$ (P*-a.s.*), *where* $\nu_u := \lim_{t \to T} E[D_u g_t \mid \Im_u^B]$ *in the* $L_2([0,T] \times \Omega)$.

Next, we have considered functionals which didn't satisfy even these weakened conditions. To such functionals belong, for example, Lebesgue integral (with respect to time variable) from stochastically non smooth square integrable processes.

Theorem 2 (Corollary 2.2. [4]). *For* $F = (\int_0^1 B_s ds - K)^+$ *the following stochastic integral representation holds*

$$F = \varphi(K\sqrt{3})/\sqrt{3} - K[1 - \Phi(K\sqrt{3})] +$$
$$+ \int_0^1 (1-t)\{1 - \Phi(\sqrt{3(1-t)^{-3}}[K - \int_0^t (1-s)dB_s])\}dB_t ,$$

where Φ *is the standard normal distribution function and* φ *ist density.*

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