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ON THE OCCASION OF BORIS KHVEDELIDZE'S 100TH BIRTHDAY ANNIVERSARY

This year we mark the centenary of Boris Khvedelidze, one of the brilliant representatives of the Georgian mathematical school, outstanding scientist, academician of the Georgian National Academy of Sciences.

Boris Khvedelidze was born on November 7, 1915 in the town of Chiatura (Georgia). His father Vladimir Khvedelidze and mother Olga Berishvili were doctors.

In 1931, upon graduation from the Tbilisi pedagogical technical college he worked in the fundamental library of the Georgian Polytechnical Institute as librarian. In 1933, he continued his education at the faculty of physics and mathematics of the Tbilisi State University. During his studies at this faculty he was deeply impressed by the lectures delivered by professors Levan Gokieli, Archil Kharadze, Niko Muskhelishvili, Ilya Vekua and Levan Magnaradze whom he always recalled with a great warmth.

Having graduated with honours from the University, in 1938 Boris Khvedelidze was successfully enrolled in the post-graduate course at the Institute of Mathematics of the Georgian branch of Academy of Sciences of the USSR. His supervisor was Ilya Vekua. Under his guidance B. Khvedelidze set about investigation of the Poincaré boundary value problem for the second order differential equation of elliptic type. His first research work in this subject has been presented by N. Muskhelishvili for publication in "Transactions of the Academy of Sciences of the USSR".

The years of B. Khvedelidze's post-graduate studies coincided with the period when mathematical research work in Georgia were effectively developing. Under the N. Muskhelishvili's supervision the seminar in the theory of Cauchy integrals and their applications to the boundary value problems of analytic and harmonic functions was working intensively. This seminar has played an important role in the formation of many Georgian mathematicians who have in the sequel carried out the well-known investigations dealt with the boundary value problems of the function theory of a complex variable and with singular integral equations. At one of the seminar sessions the supervisor put the question on the extension of the known results obtained for the Riemann problem in a simply-connected domain to a multiply connected domain. This problem was successfully solved by Boris Khvedelidze.

In 1942 B. Khvedelidze defended his Candidate's thesis. About the results obtained in this dissertation N. Muskhelishvili in his monograph "Singular Integral Equations" (M., 1968) wrote: "The first complete solution of the problem

$$A(s)\frac{\partial u}{\partial n} + B(s)\frac{\partial u}{\partial s} + c(s)u = f(s)$$

has been given by B. Khvedelidze". The results of this work and those obtained by I. Vekua were applied in Holland to study mathematical problems arisen after the destructive flood in 1953. On the mathematical method employed to this event, professor Dantzig at the International Congress of Mathematicians in Amsterdam declared: "These methods have been discussed in detal at the Tbilisi School under the supervision of N. Muskhelishvili. For our aims, the results obtained by I. Vekua and B. Khvedelidze are of particular importance".

In 1957 B. Khvedelidze defended his Doctoral dissertation under the title "Linear Boundary Value Problems of the Function Theory, Singular Integral Equations and Some Their Applications". Its content is presented in detail in his monograph under the same title (Proceedings of Tbilisi A. Razmadze Mathematical Institute, vol. 23, 1956). In this work B. Khvedelidze developed the method of Cauchy type integrals with density from $L^p(\Gamma)$, $p > 1$ to solve that group of boundary value problems which he called discontinuous (i.e., the problems when an unknown function may have on the boundary an infinite set of singularities). Towards this end, it was, first of all, necessary to study the problem of continuity of the operator generated by a singular Cauchy integral in Lebesgue spaces. In the case, where the line of integration is a Lyapunov's curve, he proved that the operator in the weighted space $L^p(\Gamma, \rho)$, $p > 1$ is continuous when ρ is the power function

$$\rho(t) = \prod_{k=1}^{n} |t - t_k|^{\alpha_k}, \quad t_k \in \Gamma, \quad t_i \neq t_j, \quad \text{when} \quad i \neq j, \quad -\frac{1}{p} < \alpha_k < \frac{1}{p'}.$$

Such a result for conjugate functions and Hilbert transformation (which are the Cauchy singular operators for a circumference and a straight line, respectively) was, for the first time, stated by Hardy and Littlewood. This result in the case of general rectifiable curves has found wide applications and, seemingly, therefore the above weighted function is frequently called in literature as Khvedelidze's weight.

Further, the method of Cauchy type integrals has been effectively used by B. Khvedelidze for inversion of a singular integral and for solution of boundary value problems of the theory of analytic functions. This is, first of all, concerns with the problem of linear conjugation in Privalov's statement, that is, with the solution of the problem in the class of Cauchy type integrals with density from $L^p(\Gamma)$. He has achieved "essential progress in a class of free terms of a boundary condition and in a class of admissible solutions" (F. D. Gakhov, in: "Investigation of Modern Problems of the Theory of Functions of a Complex Variable", M., 1964, p. 361).

With the same success B. Khvedelidze studied singular integral equations with the Cauchy kernel in Lebesgue spaces. In his book "Singular Integral Equations" (M., 1960, 404-405) N. Muskhelishvili writes: "At last we note one more of the results of great interest from the point of view of the questions dealt with in the present book". B. Khvedelidze has shown that singular integral equations considered in this chapter have the same solutions in the class H^*, as well as in classes $L^p(\Gamma)$, $p > 1$.

The above-mentioned results obtained by B. Khvedelidze were later on developed by him, his pupils and collaborators and also by many followers beyond Georgia. A part of the results obtained in this direction have been skillfully expounded in his paper "The Method of Cauchy Type Integrals in Discontinuous Boundary Value Problems of the Theory of Holomorphic Functions of One Complex Variable" (Modern Problems of Mathematics, vol. 7, M., 1975, 5-162; English translation in: "J. Sov. Math.", 7(1977), 309-414). This and the above-mentioned work published in "Proceedings of A. Razmadze Mathematical Institute" are up to the present days the handbooks of many specialists engaged in this area. It is difficult to find research works in the boundary value problems of the function theory and singular integral equations lack of references to the works of Boris Khvedelidze. Besides numerous works in this subject, there are more than ten monographs.

In 1967 B. Khvedelidze was elected Corresponding Member and in 1983 Full Member of the Georgian Academy of Sciences.

In is not easy to list his vast scientific, pedagogical and public activity he led for many dozens of years. From 1957 to 1986 he headed department of Function Theory and Functional Analysis at A. Razmadze Mathematical Institute, and from 1986 to the end of his life he was the head of organized by his initiative department of the Methods of Complex Analysis.

B. Khvedelidze was the head of Organizing Committee of the Georgian Mathematical Society and repeatedly he was elected its Vice-President. For many years he was at the head of the Chair of Higher Mathematics at Georgian Polytechnical Institute. B. Khvedelidze made an important contribution to the formation of Abkhazian State University and Sukhumi branch of the Tbilisi State University, being one of the leading professors from the day of its foundation.

Together with A. Kharadze, V. Chelidze and I. Kartsivadze, he was the author of the course in mathematical analysis in Georgian language which has played an important role in formation of many generations of Georgian mathematicians. It is no less important to mention B. Khvedelidze's remarkable human gualities-tenderness, benevolence, willingness to render assistance. B. Khvedelide was always strong-willed, with fortitude he endured vital confusions. In the years of Soviet repressions his family was deported to the South Kazakhstan (his nephew after the war has stayed in France). For a year and a half he was a teacher at zooveterinary technical school in the town of Kaplanbec. Thanks to his friends, who supplied him with the needed literature, he was able to continue scientific work even in exile. With a great gratitude he recalled the fact that his own library gifted by him before exile to the Tbilisi State University has been given him back after exile.

It should be noted that in days when scientific circles mark Boris Khvedelidze's centenary, officials of A. Razmadze Mathematical Institute intensively continue research work in new aspects of boundary value problems of the function theory, theory of operators and singular integral equations. Recently, several monographs in this direction have been published by the research workers of the institute.

Boris Khvedelidze passed away on March 27, 1993.

His name once again reminds us of his veritable professionalism, rear pedagogical talent, devotion to his people and work.

Blessed memory on the well-known scientist and wonderful person will for a long time remain in the hearts.

V. Kokilashvili and V. Paatashvili

List of Publications of B. Khvedelidze

(i) Monographs and Memoirs

1. Linear discontinuous boundary problems in the theory of functions, singular integral equations and some of their applications. (Russian) *Akad. Nauk Gruzin. SSR. Trudy Tbiliss. Mat. Inst. Razmadze* **23** (1956), 3–158.
2. The method of Cauchy type integrals in discontinuous boundary value problems of the theory of holomorphic functions of a complex variable. (Russian) *Current problems in mathematics, Vol. 7 (Russian)*, pp. 5–162 (errata insert). *Akad. Nauk SSSR Vsesojuz. Inst. Nauchn. i Tehn. Informacii, Moscow*, 1975.

(ii) Papers

3. On the Poincare boundary value problem of the logarithmic potential theory. (Russian) *Dokl. Akad. Nauk SSSR* **30** (1941), No. 3.
4. On the Poincare boundary value problem of the logarithmic potential theory for multi-connected domains. (Russian) *Soobshch. Akad. Nauk Gzuz. SSR* **2** (1941), No. 7, 571–578.
5. On the Poincare boundary value problem of the logarithmic potential theory. Second announcement. (Russian) *Soobshch. Akad. Nauk Gzuz. SSR* **2** (1941), No. 10.
6. Solution of one boundary value problem of the Newton potential theory by the method of Acad. N. I. Muskhelishvili. (Russian) *Trudy Tbiliss. Gos. Univ.* **23** (1942), 65–177.
7. On one Riemann linear boundary value problem for a system of analytic functions. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **4** (1943), No. 4, 289–296.
8. Poincare problem for the second order linear differential equation of elliptic type. (Russian) *Trudy Tbiliss. Gos. Univ.* **12** (1943).
9. Some properties of improper integrals in the sense of the Cauchy–Lebesgue principal value. (Russian) *Soobshcheniya Akad. Nauk Gruzin. SSR* **8** (1947), 283–290.
10. Singular integral equations in improper Cauchy–Lebesgue integrals. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **8** (1947), No. 7, 424–434.
11. On an inversion formula (with I. N. Kartsivadze). (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **10** (1949), 587–591.
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13. On the problem of linear conjunction in the theory of analytic functions. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **76** (1951), 177–180.
14. On linear singular integral equations with a singular kernel of Cauchy type. (Russian) *Dokl. Akad. Nauk SSSR(N.S.)* **76** (1951), 367–370.
15. On an integral of Cauchy type (with I. N. Kartsivadze). (Georgian) *Akad. Nauk Gruzin. SSR. Trudy Tbiliss. Mat. Inst. Razmadze* **20** (1954), 211–244.
16. On a class of singular integral equations with kernels of Cauchy type. (Russian) *Soobshch. Akad. nauk Gruz. SSR* **15** (1954), 401–405.
17. Some composition formulas for singular integrals and their applications to the inversion of a Cauchy-type integral. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **16** (1956), 81–88.
18. On the Riemann–Privalov problem in the theory of analytic functions. (Russian) *Uspekhi Mat. Nauk (N.S.)* **10** (1955), no. 3(65), 165–171.
19. On a discontinuous problem of Riemann–Privalov in the theory of analytic functions. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **102** (1955), 1081–1084.

20. On the discontinuous boundary problem of Riemann–Privalov with coefficients having critical pointss. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **111** (1956), 40–43.
21. Singular integral equations with Cauchy kernels in the class of functions that possess weighted sums. (Russian) *Dokl. Akad. Nauk SSSR (N.S.)* **111** (1956), 304–307.
22. On the discontinuous problem of Riemann–Privalov for several unknown functions. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **17** (1956), 865–872.
23. On singular integral equations with Cauchy type kernels in a class of summable with weight functions. (Russian) *Trudy 3-go Vsesojuzn. S'ezda* **I** (1956).
24. On systems of singular integral equations with Cauchy kernels. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **18** (1957), No. 2, 129–136.
25. A remark on my work "Linear discontinuous boundary problems in the theory of functions, singular integral equations and some of their applications". (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **21** (1958), 129–130.
26. On the Riemann–Privalov problem with continous coefficients (with G. F. Manjavidze). (Russian) *Dokl. Akad. Nauk SSSR* 123(1958), 791–796 .
27. The discontinuous Riemann–Privalov problem with given displacement. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **21** (1958), 3850–389.
28. Regularization problem in the theory of integral equations with Cauchy kernel. (Russian) *Dokl. Akad. Nauk SSSR* **140** (1961), 66–68.
29. Some notes to the theory of singular equations with a Cauchy type kernel (with D. F. Kharazov). (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **28** (1962), 129–135.
30. The Riemann–Privalov boundary-value problem with a piecewise continuous coefficient. (Russian) *Gruzin. Politehn. Inst. Trudy* **1962** (1962), No. 1 (81), 11–29.
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33. A certain singular integral operator (with E. G. Gordadze). (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **71** (1973), No. 1, 33–36.
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35. Singular integral operators, and the regularization problem (with E. G. Gordadze). (Russian) Collection of articles on the theory of functions, Vol. 7. *Sakharth. SSR Mecn. Akad. Math. Inst. Šrom.* **53** (1976), 15–37.
36. On singular integral operators (with E. G. Gordadze). *Function theoretic methods in differential equations*, pp. 132–157. Res. Notes in Math., No. 8, *Pitman, London*, 1976.
37. The problem of linear conjugacy and of characteristic singular integral equations. (Russian) *Complex analysis and its applications (Russian)*, pp. 577–585, 671, *Nauka, Moscow*, 1978.
38. A discontinuous problem of linear conjugation with a piecewise-continuous coefficient. (Russian) *Soobshch. Akad. Nauk Gruz. SSR* **97** (1980), No. 3, 529–532.
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